"ESTIMATING THE CORRELATION OF INTERNATIONAL EQUITY MARKETS WITH MULTIVARIATE EXTREME AND GARCH MODELS"

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Abstract

In this paper we study the dependence structure of extreme realization of returns between seven Southeast Asian stock markets and the U.S. Methodologically we apply the Multivariate Extreme Value theory that best suits to the problem under investigation. The main advantage of this approach is that it generates dependence measures even if the multivariate Gaussian distribution does not apply, as the case is for the tails of the high frequency stock index returns distributions. The empirical evidence suggests that Constant and Dynamic Conditional Correlation *GARCH(1,1)* models produce estimates of the correlation coefficient with a similar ranking to the ones produced from the Multivariate Extreme Value theory. This evidence is substantiated from a formal clustering analysis. The policy implication of our study is that the benefits from portfolio diversification with assets from the Southeast Asian stock markets are not eroded during crisis periods.

JEL Classification: G15; C10; F30.

Keywords: Extreme Value Dependence; Multivariate GARCH; Emerging markets.

1. Introduction

Recent research on domestic and international stock markets indicates the presence of correlation asymmetry, i.e. computed correlations differ substantially and they are considerably greater in downside markets. Correlation asymmetry has severe implications for the use of portfolio diversification as a method of reducing the risk on a portfolio for a given level of expected return. Portfolio managers who fail to take this into consideration face the possibility to be over-exposed to risky assets when the benefits of diversification are most needed.

A large literature now exists that has tested the existence of correlation asymmetry in international equity markets. Longin and Solnik (1995) study the correlation of monthly excess returns for seven major economies over the period 1960-1990. They estimate a multivariate GARCH(1,1) model and conclude that there is a positive time trend in conditional correlation for all countries even after the variance terms have been modeled with a GARCH parameterization. They also use a Threshold GARCH model where they condition the correlation on both the sign and the magnitude of past shocks. They find that the correlation increases in periods of high turbulence but is no more sensitive to negative than to positive shocks. Karolyi and Stulz (1996) use daily returns between Japanese and US stocks and find that large absolute returns to broad-based market indices positively impact both the magnitude and the persistence of return correlations. They also demonstrate that macroeconomic announcements, foreign exchange and interest rate shocks do not significantly affect comovements which are found to be time varying. Bracker and Koch (1999) analyze the economic determinants of the correlation structure between ten markets. They use daily data and conclude, among others, that correlations are positively related to the world market volatility, negatively related to world market returns and that the presence of a positive trend can not be rejected. Bekaert, Harvey and Ng (2005) follow a more structural approach that disassociates the notion of contagion from the increased correlation. In this framework contagion is defined as the excess correlation that is not explained by higher factor volatility. They apply a two factor model with time varying loadings to stock market returns in three regions, Europe, Southeast Asia and Latin America. Their results indicate the presence of contagion around the Southeast Asian crisis only which is also extended to Latin America as well.[1]

Although the close connection between correlation and high volatility has been documented for different time periods and between a great number counties it is not safe to conclude that the "true" correlation is changing over time. Boyer *et.al.* (1999) show that from a completely statistical perspective one would expect higher correlations during volatile periods and therefore the policy of conditioning the correlation on a specific rule (e.g. bear or bull market conditions) is not the appropriate one for studying the "correlation breakdown" problem. The valid approach would be to specify the distribution of the conditional correlation under the null hypothesis and test whether it changes in volatile periods.[2] Alternatively, Forbes and Rigobon (2002) correct the correlation index for conditional biases and argue that there is no evidence of contagion surrounding the three most recent crises.

Notwithstanding the difficulties in the estimation of the correlation coefficient over crisis periods, a more critical issue appears to be the suitability of correlation as a dependence measure. This reservation stems from the fact that the Pearson correlation coefficient will represent the dependence measure between two variables only if the dependence structure is Gaussian over the whole distribution. This is however rather unlikely considering the distribution properties of high frequency stock market returns. Recently, a number of studies have implemented asymptotic results from the multivariate extreme value theory (MEVT) in order to estimate the dependence of international equity returns under extreme market conditions. The attractive feature of the MEVT is that its results hold for a wide range of parametric distributions of returns and not only for the multivariate normal. Longin and Solnik (2001) model the multivariate distribution of positive and negative monthly return exceedances, which are linked to high values of corresponding thresholds, of the five largest stock markets. They conclude that the assumption of multivariate normality cannot be accepted (rejected) for large negative (positive) returns. The estimated correlation coefficients are always higher in the case of return exceedances for negative thresholds and they tend to increase with the absolute size of the threshold.[3] Poon *et. al.* (2004) argue that traditional tests for asymptotic extremal dependence bias the results in favor of this hypothesis and they suggest an additional measure of extremal dependence for variables that are asymptotically independent. They apply the pair of dependence measures on daily data of stock index returns of the five largest stock markets and they conclude that the asymptotic dependence between the European countries (United Kingdom. Germany and France) has increased over time but that the asymptotic independence between Europe, United States and Japan best characterizes their stock markets behavior. Deminer and Charnes (2003) apply the MEVT in order to model the dependence structure between spot and futures returns and then to calculate optimal hedge ratios that minimize a given measure of risk.

In this paper we apply the MEVT in order to estimate the dependence structure of extreme realizations of equity returns between mature (USA, Japan) and emerging Southeast Asian stock markets (Hong Kong, Taiwan, Malaysia, Indonesia, Singapore and Thailand). The results are compared to those obtained from two classes of MVGARCH(1,1) models: the constant conditional correlation (CCC) model proposed by Bollerslev (1990) and the dynamic conditional correlation (DCC) model by Engle (2002). The above testing methodology for the dependence structure stands in stark contrast to the classical multivariate analysis which is performed jointly for the marginal distributions and the dependence structure by considering the complete covariance matrix (e.g. MVGARCH models). So in the so-called Copula approach we analyze separately the main diagonal elements (scatter parameters) of the covariance matrix from the dependence structure contained in the off-diagonal elements that are not “contaminated” by the scatter parameters.

In the next section we offer a brief presentation of the copula methodology that allows the extraction of the dependence structure of a set of variables independently of the marginal distributions, which might refer to a wide class of models. Then the MEVT and the MGARCH(1,1) approaches are applied on a rather popular in the relevant literature data set that comprises of daily stock market returns of most of the Southeast Asian emerging capital markets. Moreover, we have also included the S&P 500 as well as the Nikkei 225 indices.

Dependence measures are estimated for all possible pairs of series and the results are discussed in the third part of the paper. The main evidence is that the case for the existence of correlation asymmetry does not appear to be supported empirically. The left tail dependence estimates from the MEVT are only marginally greater than those obtained from the right tail and at the same time they are not dramatically different from the MVGARCH(1,1) correlation estimates. The average correlation estimate from the DCC model is almost equal to the one estimated from the CCC model while both of them are found to be very close to the unconditional correlation measures. Also, we find that the inclusion or not of the October 1987 crash period does not affect our estimates in any systematic way. Overall, our results are rather relaxing towards the risk of having suboptimal portfolios that fail to diversify the risk in stressful periods. Finally, we attempt to investigate whether there are different clusters of markets on the basis of the information obtained from the correlation estimates. The classification of the markets in different clusters does not depend on the method of estimation while at the same time the Southeast Asian emerging equity markets do not seem to belong in a distinct cluster.

2. The dependence structure of multivariate extremes and multivariate GARCH models

Copulas, or dependence functions, represent a way of trying to extract the dependence structure from the joint distribution. This is being accomplished by separating the joint distribution into a part that describes the dependence structure and a part that describes the marginal behaviour only. Let us consider a *q*-dimensional vector of random returns denoted by $Y^t = (Y_1, Y_2, ...Y_q)^t$ with marginal distributions $F_1, ..., F_q$. The joint distribution function C of $(F_1(Y_1), ...F_q(Y_q))^t$ is then called the copula of the random vector $Y^t = (Y_1, Y_2, ...Y_q)^t$. It follows then that:

$$F(y^*{}_1, ...y^*{}_q) = \Pr ob[Y_1 \leq y^*{}_1, ..., Y_q \leq y^*{}_q] = C(F_1(y^*{}_1), ..., F_q(y^*{}_q)), \qquad (1)$$

where $y^*_i = u_i + y_i$ and y_i refers to the exceedance of Y_i over the threshold u_i.

Once the problem is to study the dependence structure of extreme returns, the multivariate return exceedances distribution must be defined. The possible limit non-degenerate distribution however must satisfy two properties; first, the fat-tails feature of univariate returns and second the empirical regularity that correlations rise at crisis periods. The first property is satisfied by the Generalized Pareto Distribution (*GPD*) function that is given by

$$G(y) = 1 - \{1 + \xi y / \sigma\}^{-1/\xi}, \xi \neq 0, \qquad (2)$$
$$G(y) = 1 - e^{(-y)}, \xi = 0$$

where ξ is the tail index, $\sigma > 0$ the scale parameter and the support is $y \geq 0$ when $\xi > 0$ and $0 < y < -(\sigma / \xi)$ when $\xi < 0$. Essentially all the common continuous distributions of statistics belong in this class of distributions. For example the case $\xi > 0$ corresponds to heavy tailed distributions such as the Pareto and Student-*t*. The case $\xi = 0$ corresponds to distributions like the normal or the lognormal whose tails decay exponentially. The short-tailed distributions with a finite endpoint such as the uniform or beta correspond to the case $\xi < 0$, (Pickands, 1975).

The second property is satisfied by the logistic model in the bivariate extreme value family that is given by:

$$C(s,t) = \Pr(S \leq s, T \leq t) = e^{\left[-\left\{s^{-(1/a)} + t^{-(1/a)}\right\}^{a}\right]}, 0 < a \leq 1, \qquad (3)$$

(Poon *et. al.* (2004), Longin and Solnik (2001)). In order to disassociate the correlation structure from the marginal distributions the bivariate return exceedances have been transformed to unit Fréchet margins

$$S = -1/\log F_{u_1}(y_1), \quad T = -1/\log F_{u_u}(y_2)$$

where $F_{u_i}(y_i)$ is the *GPD* of exceedance y_i. The asymptotic dependence of *(S,T)* is defined by:

$$\chi = \lim_{s \to \infty} \Pr(T \succ s / S \succ s), \quad (4)$$

where $0 \le \chi \le 1$, and the two variables are termed asymptotically dependent if $\chi > 0$, perfectly dependent if $\chi = 1$ and asymptotically independent if $\chi = 0$.[4] The relationship between the coefficient α, of eq. (3), and χ is given by $\chi = 2 - 2^{\alpha}$ so when the variables are exactly independent $\alpha = 1$ while when $\alpha < 1$the variables are asymptotically dependent to a degree depending on α.

Once we have chosen the thresholds, the bivariate distribution of return exceedances is described by seven parameters: the two tail probabilities, the dispersion parameters, the tail indexes of each variable, and the dependence parameter of the logistic function. The parameters of the model are estimated by the maximum likelihood method. In the bivariate case, the correlation coefficient of extremes is related to the coefficient of dependence by (Tiago de Oliveira, 1973; Longin and Solnik, 2001):

$$\rho = 1 - \alpha^2, \quad (5)$$

In order to investigate the empirical implications of those two different estimation philosophies we have also chosen to estimate the correlation indices from multivariate volatility models. The first model we estimate is the one suggested by Bollerslev (1990) that handles the high dimensionality of the parameter space of the variance – covariance matrix by adopting the assumption of constant contemporaneous correlations (CCC). In the CCC-GARCH(1,1) specification the conditional variance matrix is specified as $H_t = D_t R D_t$, where H_t takes the form:

$$H_t = \begin{bmatrix} \sqrt{h_{11,t}} & 0 \\ 0 & \sqrt{h_{22,t}} \end{bmatrix} \begin{bmatrix} 1 & \rho_{12} \\ \rho_{21} & 1 \end{bmatrix} \begin{bmatrix} \sqrt{h_{11,t}} & 0 \\ 0 & \sqrt{h_{22,t}} \end{bmatrix}, \quad (6)$$

In this model the correlation matrix *R* is time invariant. For the bivariate GARCH(1,1) case the CCC model contains only 7 parameters compared to 21 encountered in the full VECH model and the positive definiteness of the variance – covariance matrix is easily satisfied

($|\rho| < 1$). In this framework the asymmetric behavior of the conditional covariances in bull and bear markets is guaranteed by the proper parameterization of the conditional variances. In our case we apply the Glosten-Jagannathan-Runkle (1993) GJR-GARCH(1,1) model:

$$h_{ii,t}^{2} = \omega + \beta Y_{t-1}^{2} + \gamma h_{ii,t-1}^{2} + \delta Y_{t-1}^{2} I_{t-1}, \qquad (7)$$

where $\omega \geq 0, \beta \geq 0, \gamma \geq 0, \delta \geq 0$,$I_{t-1}=1$ when $Y_{t-1}<0$ and zero otherwise.

The assumption that the conditional correlations are constant may seem unrealistic in many empirical applications like the dependence of international equity returns. Engle (2002) extends the CCC estimator by allowing the conditional correlations to be time varying, that is the conditional variance is $H_t = D_t R_t D_t$. The dynamic conditional estimator (DCC) is obtained in two stages. In the first stage univariate GJR-GARCH(1,1) models are estimated for each return series. The standardized residuals from the first stage, $n_{i,t} = (\varepsilon_{i,t} / \sqrt{h_{ii,t}})$, are used in the second stage in the estimation of the correlation parameters. The correlation structure R is also the correlation of the original data and is given by $R_t = Q_t^{*-1} Q_t Q_t^{*-1}$, where Q^* is a diagonal matrix whose elements are the square root of the diagonal elements of the covariance matrix Q that is specified by a GARCH process as below:

$$Q_t = S(1 - \lambda - \mu) + \lambda(n_{t-1} n_{t-1}^{'}) + \mu Q_{t-1}. \qquad (8)$$

where the sum of λ and μ measures the long -run persistence. Q is calculated as a weighted average of S, the unconditional covariance of the standardized residuals, a lagged function of the standardized residuals and the past realization of the conditional variance (Engle, 2002).

3. Empirical evidence

We have applied the competing models on a data set consisting of daily returns of the following equity indices: S&P 500 Composite (USA), Nikkei 255 Stock Average (Japan), Hang Seng Price Index (Hong Kong), the Stock Exchange Weighted Price Index of Taiwan, KLCI Composite Price Index (Malaysia), the Jakarta Stock Exchange Composite Price Index

(Indonesia), the Straits Times (New) Price Index (Singapore) and the SET 100 Basic Industries Index (Thailand). The data cover the period 5/1/87 – 31/12/04. Daily index returns are generated by taking first differences of the logarithmic indices that have been obtained from Datastream. The U.S. market is the latest to close on any particular day among the eight stock markets in our sample. This means that any shock in the U.S. stock market will impact on the other stock markets on the following day. Hence, we use the previous day's U.S. return whenever the returns pair involves the S&P 500 index (Poon *et. al.,* 2000). Estimates of the dependence coefficients have been obtained over two sub-periods, 5/1/87-5/3/01 and 2/11/90 – 31/12/04, since we intend to check the sensitivity of our estimates on the inclusion or not of the turbulent period surrounding the October 1987 stock exchange crisis.

The summary statistics for the log differenced return series of the eight stock markets are given in Table 1. All series have a negative skewness and a kurtosis that is significantly greater than three. Exceptions are the series for Indonesia and Thailand which are positively skewed and the series for Taiwan with a near Normal kurtosis. The Jacque-Bera test for normality rejects normality for all series. Q(16) is the Ljung -Box Q-statistic to test for the hypothesis of no autocorrelation up to order (16). This hypothesis is rejected for all the series.

The MEVT is applied on the exceedances of the return series from high enough, positive or negative, thresholds *(Peak over Threshold, POT, method).*[5] In order to estimate the threshold, u, we follow Neftci (2000) according to whom $u = 1.176\sigma_n$. σ_n is the standard deviation of $(Y_t)_{t=1}^{n}$ and $1.176 = F_t^{-1}(0.10) = 1.44\sqrt{(\nu-2)/\nu}$ when a Student-*t* *(ν=6)* distribution, F, is being assumed. This implies that the excesses over the threshold belong to the 10% tails. The thresholds we use are shown in table 2 and they are symmetric for the right and left tail.

The maximum likelihood estimates of the tail index, with their corresponding standard errors, and the scale parameters are presented in table 2. The estimated tail index values range between -0.172 (Taiwan, left tail, first sub-period) and 0.427 (Indonesia, right tail, first sub-period). For U.S., Japan and Taiwan we are unable to reject, except for two cases in the first sub-period, which the tail indices are different from zero and this implies that the *GPD* corresponds to the exponential distribution. The left tail index is greater than the right tail index, during the first sub-period, in five out of the eight cases. Therefore, high losses are more likely than similar gains in those markets. The evidence from the second sub-period is different. There are a greater number of cases where the right tail index is higher than the left tail one. Notwithstanding the above evidence and irrespectively of the period we look at, if we take into account the standard errors of the estimates then the left tail index estimates are not statistically different from the right tail ones.[6,7]

In table 3 we present the correlation coefficients from the *MEVT* and the two *MVGARCH(1,1)* models. In the case of the DCC(1,1) model we report both the *average* correlation estimate over the entire estimation period (in parenthesis) and the last estimate. The highest correlation from the *MEVT* models is between the negative returns of Malaysia and Singapore while the lowest one between Taiwan and Indonesia. Both of them are estimated over the first sub-period. Differences between the two sub-periods are not observed in the sense that no trend is being observed that would allow one to claim that the markets are getting more integrated or not. The correlation estimates from average DCC(1,1) and CCC(1,1) model are very similar while the lowest and the highest estimates are observed for the same pairs of markets that we found in the *MEVT* models. This last result is representative of all the other estimates and therefore the ranking of the strength of the correlations is similar between the two different estimation methods. This further seems to imply that volatility is the major contributing factor to the between-series extremal dependence [8]. In our study the close proximity of the correlation estimates from the MEVT and MVGARCH(1,1) weakens the argument that there has been a contagion effect among the Southeast Asian markets during the most recent crises. Since from a completely statistical perspective, one would expect higher correlation during periods of high volatility, contagion is not simply increased correlation during a crisis period (Bekaert and Harvey, 2003).

In order to classify the various pairs of capital markets into different groups on the basis of the estimated dependence measures, we apply a clustering analysis that assigns each estimate to the cluster having the nearest mean. K-means is one of the simplest unsupervised learning algorithms that solve the well known clustering problem. The procedure follows a simple and easy way to classify a given data set through a certain number of clusters (assume k clusters) fixed a priori. The main idea is to define k centroids, one for each cluster. Group membership is determined by calculating the centroid for each group (*the multidimensional version of the mean*) and assigning each observation to the group with the closest centroid, (MacQueen, 1967). The evidence appears in table 4. The main result is that the classification of the estimated correlations into low, medium and high dependence groups is very similar between the *MEVT,* the *CCC(1,1),* and the average *DCC(1,1)* estimates. The last (i.e., 5/3/01 or 31/12/04) *DCC(1,1)* correlation estimates are more sensitive, as expected, to the last observation included in the sample and this accounts for the different classification of the pairs of countries that is produced. Moreover, the classification of the correlation coefficients of extreme positive and of extreme negative returns is very similar. Finally, we examine whether there is any validity to the argument that the Southeast Asian capital markets belong to a distinct cluster of markets where the other two are the U.S. and the European markets. If this argument was correct then we would expect to find that the correlation indices between the Southeast Asian markets would be always classified to the high correlation cluster. A simple inspection of table 4 shows that this is not the case. If we concentrate our attention on the medium and high correlation clusters only then we can claim that there exists an integrated capital market in Southeast Asia consisting of Hong Kong, Singapore, Malaysia and Thailand. Taiwan appears to be an "outlier" since it exhibits systematically low correlation with all the other neighbouring markets and the same applies, to a smaller extent, for Indonesia. Japan and U.S. exhibit varying degrees of correlation with the other markets and therefore their investors can benefit from diversifying their portfolios with assets from the Southeast Asian stock markets.

4. **Concluding remarks**

In this paper we studied the dependence structure of the extreme realization of returns between seven Southeast Asian stock markets and the USA. Methodologically, we applied the Multivariate Extreme Value theory that best suits to the problem under investigation. The main advantage of this approach is that it generates dependence measures even if the multivariate Gaussian distribution does not apply, as the case is for the tails of high frequency stock index returns. The empirical evidence suggests that the more conventional Constant and Dynamic Conditional Correlation *GARCH(1,1)* models produce estimates of the correlation coefficient with a similar ranking to the ones produced from the MEVT. This evidence is substantiated from a formal clustering analysis. Moreover, the point estimates of the correlations for extreme negative returns are not significantly higher, in most cases, than the ones obtained for the extreme positive returns. The policy implications of our study are that the benefits from portfolio diversification with assets from the Southeast Asian stock markets are not eroded during crisis periods and that the extreme correlations should be attributed to the increased volatility in turbulent periods.

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Footnotes

[1] The existence of correlation asymmetry has also been empirically verified between domestic equity portfolios and the aggregate market. Correlations are greater in "bear" markets than in "bull" markets, (Ang and Chen, 2002).

[2] A valid alternative procedure would be to employ models representative of the data generating process, which build in the possibility of structural changes (e.g. the regime switching models of Ang and Bekaert, 2002).

[3] Stariça (1999) found a high level of dependence between the extreme movements of most of the currencies in the European Union. Hartman *et. al.* (2001) found co-crashes between stock and bond markets as well as some evidence of cross border dependence.

[4] Poon *et. al.* (2004) argue that the application of this approach biases the results towards rejecting the independence of the variables. The degree of such bias will depend on the rate at which $\Pr(T \succ s / S \succ s) \rightarrow 0$ as $s \rightarrow \infty$.

[5] The choice of the threshold is of critical importance and the various methods that have been proposed usually rely on the visual inspection of QQ models, the sample mean excess functions or the Hill-plot, (Gençay, R., and F. Selçuk, 2003). Danielsson and de Vries (1997) suggest a bootstrap method for the threshold selection.

[6] Both the negative and the positive stock return distributions are guaranteed to have finite second moments since the tail index was never found to be greater than 0.50. For $\xi > 0, \mathrm{E}[\mathrm{Y}^{\kappa}]$ is finite for $\kappa < (1/\xi)$.

[7] Gençay, R., and F. Selçuk, (2003) have estimated the *GPD* for both positive and negative extreme daily returns from Hong Kong, Indonesia, Singapore and Taiwan. The tail index estimates they obtain are always greater than the ones presented in this study while the left tail and the right tail returns distributions appear to be symmetric. The difference in our findings can be justified from the different time period that our data span as well as the different stock indices that have been employed.

[8] Poon *et. al.* (2004) apply univariate and bivariate GARCH filters on five daily stock index returns and show that the volatility scaling does not remove completely the tail dependency. In our study the close proximity of the correlation estimates from the MEVT and MVGARCH weakens the argument that there has been a contagion effect among the southeast Asian markets during the most recent crises.

Table 1: Descriptive statistics

Statistic	*Mean*	*St.Dev.*	*Var*	*Kurtosis*	*Skewness*	*Min*	*Max*	*JB*	*Q (16)*
Japan	-1.100e-04	1.423e-02	2.020e-04	7.229	-0.110	-0.161	0.124	1.020e+04 (0)	4.971e+02 (0)
USA	3.390e-04	1.096e-02	1.200e-04	44.813	-2.111	-0.228	0.087	3.953e+05 (0)	3.282e+02 (0)
Hong- Kong	3.670e-04	1.741e-02	3.030e-04	77.913	-3.422	-0.405	0.172	1.194e+06 (0)	9.048e+01 (0)
Taiwan	3.780e-04	2.010e-02	4.040e-04	2.632	-0.060	-0.103	0.128	1.353e+03 (0)	3.621e+03 (0)
Malaysia	2.740e-04	1.593e-02	2.540e-04	36.887	-0.261	-0.242	0.208	2.655e+05 (0)	2.375e+03 (0)
Indonesia	5.670e-04	1.700e-02	2.890e-04	90.762	3.311	-0.225	0.403	1.616e+06 (0)	7.482e+02 (0)
Singapore	2.060e-04	1.412e-02	1.990e-04	52.800	-1.976	-0.292	0.155	5.470e+05 (0)	8.907e+02 (0)
Thailand	5.350e-04	2.149e-02	4.620e-04	11.177	0.560	-0.224	0.216	2.462e+04 (0)	1.029e+03 (0)

Notation: Marginal significance levels in parentheses. Q(16) is the Ljung-Box Q-statistic for H_0 autocorrelation up to order 16. The Jacque-Bera (JB) test statistic for normality follows a χ^2 distribution with 2 degrees of freedom.

Total Period observations (5/1/1987 – 31/12/2004): 4695

Japan: Nikkei 255 Stock Average
USA: S&P 500 Composite
Hong Kong: Hang Seng Price Index
Taiwan: Stock Exchange Weighted Price Index
Malaysia: KLCI Composite Price Index
Indonesia: Jakarta Stock Exchange Composite Price Index
Singapore: Straits Times (New) Price Index
Thailand: SET 100 Basic Industries Index

Source: Datastream

Table 2: EVT Parameters

Parameter	***ξ (Tail Index)***				***σ (scale parameter)***				***u (threshold)***			
	Left tail	*Right tail*	*Left tail*	*Right tail*	*Left tail*	*Right tail*	*Left tail*	*Right tail*	*Left tail*	*Right tail*	*Left tail*	*Right tail*
In-sample Period	*5/1/87 - 5/3/01*		*2/11/90 - 31/12/04*		*5/1/87 - 5/3/01*		*2/11/90 - 31/12/04*		*5/1/87 - 5/3/01*		*2/11/90 - 31/12/04*	
Japan	0.069* (0.046)	0.150 (0.065)	-0.037* (0.047)	0.071* (0.058)	0.009	0.009	0.009	0.009	-0.016	0.016	-0.017	0.017
USA	0.245 (0.063)	0.126* (0.064)	0.055* (0.052)	0.082* (0.059)	0.006	0.006	0.007	0.006	-0.012	0.012	-0.012	0.012
Hong Kong	0.325 (0.078)	0.243 (0.083)	0.128* (0.062)	0.180 (0.061)	0.011	0.010	0.011	0.009	-0.022	0.022	-0.019	0.019
Taiwan	-0.172 (0.047)	0.046* (0.060)	0.009* (0.063)	0.030* (0.053)	0.019	0.013	0.013	0.013	-0.025	0.025	-0.021	0.021
Malaysia	0.265 (0.075)	0.411 (0.092)	0.206 (0.073)	0.338 (0.084)	0.012	0.009	0.011	0.010	-0.020	0.020	-0.018	0.018
Indonesia	0.275 (0.095)	0.427 (0.109)	0.183 (0.074)	0.174 (0.074)	0.012	0.013	0.011	0.011	-0.021	0.021	-0.018	0.018
Singapore	0.355 (0.084)	0.266 (0.079)	0.189 (0.068)	0.231 (0.067)	0.008	0.008	0.008	0.007	-0.017	0.017	-0.015	0.015
Thailand	0.116* (0.071)	0.190 (0.072)	0.151 (0.069)	0.283 (0.078)	0.016	0.017	0.013	0.014	-0.026	0.026	-0.025	0.025

Notation: Standard errors in parentheses. An asterisk (*) indicates an insignificantly different from zero estimate. Other notation as in table 1.

Estimation Periods: *Period-1: 5/1/87 - 5/3/01, Period-2: 2/11/90 - 31/12/04*

Table 3: Correlation Estimates

Bivariate Model	EVT (POT)				MVGARCH(1,1)				UNCONDITIONAL	
	Left tail	Right tail	Left tail	Right tail	DCC(1,1) -GJR	CCC(1,1) -GJR	DCC(1,1) -GJR	CCC(1,1) -GJR		
In-sample Period	5/1/87 - 5/3/01		2/11/90 - 31/12/04		5/1/87 - 5/3/01		2/11/90 - 31/12/04		5/1/87 - 5/3/01	2/11/90 - 31/12/04
Japan – USA	0.320	0.279	0.320	0.312	0.252 (0.169)	0.279	0.176 (0.271)	0.292	0.318	0.293
Japan – Hong Kong	0.338	0.286	0.356	0.313	0.880 (0.265)	0.288	0.133 (0.324)	0.349	0.295	0.348
Japan – Taiwan	0.236	0.195	0.236	0.248	0.186 (0.120)	0.115	0.180 (0.182)	0.191	0.136	0.186
Japan – Malaysia	0.328	0.247	0.257	0.222	0.210 (0.223)	0.241	0.147 (0.201)	0.230	0.255	0.195
Japan – Indonesia	0.176	0.176	0.219	0.217	0.096 (0.096)	0.093	0.170 (0.176)	0.167	0.086	0.175
Japan – Singapore	0.361	0.313	0.375	0.309	0.376 (0.274)	0.283	0.365 (0.298)	0.318	0.357	0.329
Japan – Thailand	0.237	0.199	0.218	0.222	0.205 (0.149)	0.134	0.144 (0.135)	0.125	0.149	0.152
USA – Hong Kong	0.313	0.373	0.321	0.390	0.162 (0.373)	0.350	0.212 (0.372)	0.374	0.293	0.367
USA – Taiwan	0.222	0.222	0.232	0.252	0.314 (0.127)	0.131	0.298 (0.185)	0.180	0.142	0.199
USA – Malaysia	0.300	0.271	0.239	0.213	0.283 (0.257)	0.287	0.157 (0.212)	0.229	0.326	0.220
USA – Indonesia	0.221	0.159	0.265	0.216	0.119 (0.119)	0.123	0.161 (0.175)	0.164	0.113	0.180
USA – Singapore	0.418	0.338	0.393	0.306	0.291 (0.347)	0.372	0.207 (0.319)	0.326	0.480	0.335
USA – Thailand	0.261	0.237	0.237	0.235	0.265 (0.212)	0.197	0.176 (0.176)	0.167	0.210	0.188
Hong Kong – Taiwan	0.198	0.195	0.256	0.266	0.326 (0.129)	0.134	0.210 (0.225)	0.233	0.126	0.220
Hong Kong – Malaysia	0.452	0.337	0.423	0.324	0.308 (0.343)	0.378	0.356 (0.313)	0.336	0.370	0.357
Hong Kong – Indonesia	0.288	0.231	0.350	0.262	0.158 (0.158)	0.174	0.236 (0.262)	0.248	0.191	0.304
Hong Kong – Singapore	0.583	0.499	0.598	0.504	0.640 (0.450)	0.496	0.395 (0.506)	0.525	0.507	0.601
Hong Kong – Thailand	0.324	0.321	0.329	0.326	0.279 (0.267)	0.258	0.269 (0.263)	0.247	0.279	0.296
Taiwan – Malaysia	0.199	0.181	0.198	0.163	0.175 (0.113)	0.105	0.152 (0.133)	0.143	0.127	0.142
Taiwan – Indonesia	0.135	0.128	0.210	0.199	0.043 (0.042)	0.070	0.133 (0.135)	0.131	0.035	0.136
Taiwan – Singapore	0.243	0.213	0.286	0.267	0.543 (0.145)	0.140	0.209 (0.223)	0.228	0.163	0.233
Taiwan – Thailand	0.199	0.192	0.232	0.208	0.156 (0.125)	0.115	0.137 (0.115)	0.110	0.138	0.144
Indonesia – Malaysia	0.350	0.243	0.365	0.266	0.125 (0.139)	0.175	0.162 (0.205)	0.248	0.174	0.254
Indonesia – Singapore	0.362	0.314	0.425	0.337	0.053 (0.195)	0.213	0.204 (0.288)	0.300	0.232	0.371
Indonesia – Thailand	0.276	0.248	0.344	0.278	0.161 (0.160)	0.165	0.202 (0.206)	0.204	0.198	0.292
Malaysia – Singapore	0.619	0.500	0.491	0.393	0.501 (0.493)	0.575	0.337 (0.394)	0.445	0.563	0.432
Malaysia – Thailand	0.319	0.293	0.327	0.298	0.250 (0.252)	0.257	0.240 (0.223)	0.231	0.275	0.268
Singapore - Thailand	0.404	0.324	0.396	0.347	0.317 (0.324)	0.315	0.294 (0.295)	0.279	0.351	0.365

Notation: *MEVT*= Multivariate Extreme Value Theory. *POT* = Peaks over Threshold methods for the generation of the extreme observations. *CCC(1,1)* = Constant conditional correlation method. *DCC(1,1)* =Dynamic conditional correlation method. *GJR*= Glosten, L., R. Jaganathan and D. Runkle, (1993).Other notation as in table 1.

Table 4: Correlation K-Means Clustering

Bivariate Model	EVT (POT)				MVGARCH(1,1)				UNCONDITIONAL	
	Left tail	Right tail	Left tail	Right tail	DCC(1,1) -GJR	CCC(1,1) -GJR	DCC(1,1) -GJR	CCC(1,1) -GJR		
In-sample Period	5/1/87 - 5/3/01		2/11/90 - 31/12/04		5/1/87 - 5/3/01		2/11/90 - 31/12/04		5/1/87 - 5/3/01	2/11/90 - 31/12/04
Japan – USA	2	2	2	2	1	2	1	2	2	2
Japan – Hong Kong	2	2	2	2	3	2	1	2	2	2
Japan – Taiwan	1	1	1	1	1	1	1	1	1	1
Japan – Malaysia	2	1	1	1	1	2	1	2	2	1
Japan – Indonesia	1	1	1	1	1	1	1	1	1	1
Japan – Singapore	2	2	2	2	2	2	3	2	2	2
Japan – Thailand	1	1	1	1	1	1	1	1	1	1
USA – Hong Kong	2	2	2	2	1	2	2	3	2	2
USA – Taiwan	1	1	1	1	1	1	3	1	1	1
USA – Malaysia	2	2	1	1	1	2	1	2	2	1
USA – Indonesia	1	1	1	1	1	1	1	1	1	1
USA – Singapore	2	2	2	2	1	2	2	2	3	2
USA – Thailand	1	1	1	1	1	1	1	1	1	1
Hong Kong – Taiwan	1	1	1	1	1	1	2	2	1	1
Hong Kong – Malaysia	2	2	2	2	1	2	3	2	2	2
Hong Kong – Indonesia	2	1	2	1	1	1	2	2	1	2
Hong Kong – Singapore	3	3	3	3	2	3	3	3	3	3
Hong Kong – Thailand	2	2	2	2	1	2	2	2	2	2
Taiwan – Malaysia	1	1	1	1	1	1	1	1	1	1
Taiwan – Indonesia	1	1	1	1	1	1	1	1	1	1
Taiwan – Singapore	1	1	1	1	2	1	2	2	1	1
Taiwan – Thailand	1	1	1	1	1	1	1	1	1	1
Indonesia – Malaysia	2	1	2	1	1	1	1	2	1	1
Indonesia – Singapore	2	2	2	2	1	1	2	2	2	2
Indonesia – Thailand	1	1	2	1	1	1	2	1	1	2
Malaysia – Singapore	3	3	3	2	2	3	3	3	3	2
Malaysia – Thailand	2	2	2	2	1	2	2	2	2	2
Singapore - Thailand	2	2	2	2	1	2	3	2	2	2

Table 4: Correlation K-Means Clustering (*continued*)

K-Means Centers

K-Groups	*EVT (POT)*				*MVGARCH(1,1)*				*UNCONDITIONAL*	
	Left tail	*Right tail*	*Left tail*	*Right tail*	*DCC(1,1) -GJR*	*CCC(1,1) -GJR*	*DCC(1,1) -GJR*	*CCC(1,1) -GJR*		
In-sample Period	*5/1/87 - 5/3/01*		*2/11/90 - 31/12/04*		*5/1/87 - 5/3/01*		*2/11/90 - 31/12/04*		*5/1/87 - 5/3/01*	*2/11/90 - 31/12/04*
G_1 : Low Correlation	0.217	0.204	0.237	0.233	0.206	0.139	0.156	0.158	0.142	0.187
G_2 : Medium Correlation	0.348	0.313	0.363	0.332	0.515	0.301	0.221	0.273	0.305	0.335
G_3 : High Correlation	0.601	0.499	0.544	0.504	0.880	0.535	0.341	0.448	0.517	0.601

Notation: 1,2,3 refer to the classification to Low, Medium and High Correlation.