# Taxation and Agglomeration

**By Pasquale Commendatore [a] and Ingrid Kubin [b]**

[a] *University of Naples 'Federico II', Italy*

[b] *Department of Economics, Vienna University of Economics and Business Administration, Austria*

**Abstract**

Recently, issues of international taxation have also been analysed from a New Economic Geography perspective. These discussions show that adding agglomerative forces can change the results considerably. In the paper, we introduce explicitly taxation and public expenditures into a Footloose Capital Model and investigate the local and global dynamic implications of such a public policy for industry agglomeration. It turns out that agglomeration can be highly sensitive wrt initial conditions and/or parameters and that these dynamic patterns are surprisingly robust wrt to the taxation principle.

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R12 - Size and Spatial Distributions of Regional Economic Activity
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Pasquale Commendatore and Ingrid Kubin[1]

## 1. Introduction

One of the central concerns in international tax policy is the fear that tax competition leads to a loss of industrial capital to competing countries. Recently these issues have also been analysed from a New Economic Geography perspective, which stresses agglomerative processes and circular causation. Small differences between otherwise identical regions may trigger a self-reinforcing process leading to agglomeration of industrial activity in one region. In our paper, we pursue the question whether and how differences in taxation and public expenditure policy can lead to such a development.

In the models of the New Economic Geography, regions are separated by transport costs and agglomeration is brought about by factor mobility due to differences in regional economic incentives. Because of decreasing average costs in production, factor rewards are the higher, the higher the local demand is. A high share in total expenditure thus leads to an even higher share in industrial capital. This indicates obvious points at which public policy, i.e. taxation and public expenditures can affect this mechanism. Transport costs can be reduced by public expenditures for infrastructure (see Martin and Rogers, 1995). Factor

[1] The paper benefits from previous joint work with Martin Currie and from valuable discussions with him. While working on this paper, Ingrid Kubin enjoyed the kind hospitality of the CENDEF at the University of Amsterdam. Contributions from Florian Wagener; Pietro Dindo and other participants in a CENDEF workshop are gratefully acknowledged. The usual caveat applies.

mobility motivated by differences in net factor income is directly affected by tax policy. Most studies of public policy within the New Economic Geography take this aspect on board (e.g. the literature on tax competition: Baldwin and Krugman, 2004, and Borck and Pflüger, 2006 among the others). The provision of public goods can also impact upon the migration decision (see for this aspect Baldwin et al., 2003). However, public policy is also a central factor determining both the level and the composition of local demand. In a framework with decreasing average cost this directly influences gross factor rewards.[2] Income taxes change the disposable income and thus private expenditures; public expenditures are typically different from the private ones as far as their regional and sectoral structure is concerned. Brülhart and Trionfetti (2004) study the former aspect, while the latter is at the core of our analysis. Thus, we ask whether sectoral differences in public policy may lead to industry agglomeration.

In studying this question, we focus on taxation of capital income – since capital is the factor that is by far more mobile than labour[3] – and differentiate between two forms of taxation: Under the *residence* principle, public policy has no direct effect upon mobility incentives, which allows us to study the effects *via* differences in the sectoral structure of demand in insulation. Under the *source* principle, taxation affects mobility incentives as well. For our purpose, we therefore need a model in which the capital owners' country of residence may be different from the country in which the capital income originates. This leads us to adopting a special variant of the Footloose Capital Model. That is, we extend this model by explicitly incorporating private and public demand and we allow for the possibility that public expenditure has a different sectoral structure from the private one.

[2] The provision of public goods can also affect factor rewards in a region *via* its effect upon productivity. This aspect is important but to our knowledge not yet studied in the New Economic Geography models.

[3] For simplification, we assume that labour income is not taxed at all.

The focus of the following study is on the analysis of the dynamic processes involved.[4] Typically, New Economic Geography models are formulated in continuous time; in past works, we have shown that reformulating them in discrete time may change the results considerably. In Currie and Kubin (2006) we analysed Krugman's the quintessential Core Periphery model and in Commendatore et al. (2006) we explored the dynamics of a symmetric Footloose Capital model. Our present paper extends this study to an asymmetric case, naturally brought about by differences in public policy. With taxation according to the residence principle, differences in regional public policy translate into different regional shares in total expenditures, which are independent of capital allocation; with taxation according to the source principle, the regional shares depend – in addition to the public policy parameters – also upon the capital allocation. This fact considerably complicates the analytic structure. Nevertheless, it turns out that the basic implication of our analysis carries over: Differences in regional tax policy make agglomeration more likely; however, it may depend quite sensitively upon initial conditions and parameters which of the regions under consideration ends up with the core of industrial production.

## 2. Assumptions

The Footloose Capital (FC) model involves two countries or regions, each with a monopolistically competitive manufacturing sector and a perfectly competitive agricultural sector. There are two factors of production. Labour is used in both sectors. Capital is used only in manufacturing. Workers are immobile between regions but instantaneously mobile between sectors within a region. A key feature of the FC model is that physical capital is

[4] Policy issues and welfare implications are elaborated in a twin paper, Kubin (2006).

mobile between regions but capital owners are completely immobile and they spend all their earnings in the region in which they live.

Consumers in both regions have the following utility function, which is linearly separable in private and public consumption:

$$(1) \qquad U = \left(C_A\right)^{1-m}\left(C_M\right)^{m} + \tilde{U}\left(C_G\right).$$

$C_A$ is the quantity consumed of a homogeneous agricultural good; $C_M$ denotes a quantity index that is a CES function of the varieties of manufactured goods. The constant elasticity of substitution between the manufactured varieties is denoted by $s > 1$; the lower $s$, the greater the consumers' taste for variety. The exponents of the agricultural good and of the manufacturing composite in the common utility function – $\left(1-m\right)$ and $m$, respectively – indicate the invariant shares of disposable income devoted to the agricultural good and to manufactures; therefore $0 \le m \le 1$. $\tilde{U}\left(C_G\right)$ denotes the utility derived from the supply of a public commodity $C_G$.

In providing the public good, the government uses the agricultural commodity and/or manufactured commodities according to the following production function:

$$(2) \qquad C_G = \left(C_A\right)^{1-n}\left(C_M\right)^{n}.$$

This specification implies that the share of governmental revenue devoted to the agricultural commodity and to the manufacturing composite – $\left(1-n\right)$ and $n$, respectively, with $0 \le n \le 1$ – may differ from the private shares; the elasticity of substitution between the manufactured varieties, however, is assumed to be the same for the private and the public sector. Public expenditures are financed by capital income taxes, the budget is always balanced.

There are $L$ workers, who are immobile between regions and equally distributed between the regions.[5] Each worker provides one unit of labour per period. With labour being the sole agricultural input, one unit of labour yields one unit of the agricultural product. Transportation of the agricultural product between regions is costless. In addition, the economies are endowed with $K$ units of physical capital. Since we are not focusing on distributional issues or on changes in factor endowments, we assume that each worker owns one unit of capital, i.e. we assume $K = L$. Capital is mobile between the regions at the transitions between time periods in response to economic incentives.

Manufacturing involves increasing returns: each manufacturer requires a fixed input of 1 unit of capital to operate and has a constant marginal labour requirement $b$. Transport costs for manufactures take an iceberg form: if 1 unit is shipped between the regions, $1/T$ arrives where $T \geq 1$. 'Trade freeness' is defined as $f \equiv T^{1-s}$ where $0 < f \leq 1$, with $f = 1$ representing no trade cost and with trade cost becoming prohibitive as $f \to 0$. The manufacturing sectors involve Dixit-Stiglitz monopolistic competition. Given the consumers' preference for variety, a firm would always produce a variety different from the varieties produced by other firms. Thus the number of varieties is always the same as the number of firms. Furthermore, since 1 unit of capital is required for each manufacturing firm, the total number of firms / varieties, *n*, is always equal to the total supply of capital:

[5] We assume that neither region has enough labour to satisfy the total demand of both regions for the agricultural good. Thus, both regions always produce the agricultural commodity – the so-called non-full-specialization condition.

(3) $$n = K$$

The number of varieties produced in period *t* in region *r*, where $r = 1,2$, is:

(4) $$n_{1,t} = l_t n = l_t K \qquad n_{2,t} = (1 - l_t) n = (1 - l_t) K$$

where $0 \leq l_t \leq 1$ denotes the share of physical capital used in region 1 in period *t*.

As with most economic geography models, the primary focus of the FC model is the spatial location of manufacturing industry, governed here by the endogenous regional allocation of capital, $l_t$.

In what follows, we complete the model by characterizing the short-run general equilibrium in period *t* contingent on $l_t$, by specifying explicitly the capital migration process, and by analysing the long-run equilibrium given as fixed point of the capital mobility dynamics. We consider two different cases depending on the principle of capital taxation: In Section 3 we assume that taxes are collected according to the residence principle. In Section 4, we analyse the case of taxation according to the source principle, which turns out to be analytically more complex.

## 3. Full Model with Taxation according to the Residence Principle

### *3.1. Short-run General Equilibrium*

With the instantaneous establishment of equilibrium in the agricultural market and no transport costs, the agricultural price is the same in both regions. Since competition results in zero agricultural profits, the equilibrium nominal wage of workers in period *t* is equal to the agricultural product price and is therefore always the same in both regions. We take this wage / agricultural price as the *numeraire*. Since manufacturers in both regions face that same wage in every period, all set the same mill price $p$, using the Dixit-Stiglitz pricing rule. Given that the wage is 1, the local price of every variety is:

(5) $$p = \frac{b s}{s - 1}$$

The effective price paid by consumers for one unit of a variety produced in the other region is $pT$.

Short-run general equilibrium in period $t$ requires that each manufacturer meets the demand for its variety.[6] For a variety produced in region $r$:

(6) $$q_{r,t} = d_{r,t}$$

where $q_{r,t}$ is the output of each manufacturer in region $r$ and $d_{r,t}$ is the demand for that manufacturer's variety. From (5), the short-run equilibrium profit per variety in region $r$ is:

(7) $$p_{r,t} = pq_{r,t} - b q_{r,t} = \frac{pq_{r,t}}{s} = \left[\frac{b}{s - 1}\right] q_{r,t}$$

This profit per variety constitutes the regional rental per unit of capital.

Consumers and governments (as input demanders) face regional manufacturing price indices given by:

(8) $$\begin{aligned} G_{1,t} &= \left[ n_{1,t} p^{1-s} + n_{2,t} p^{1-s} T^{1-s} \right]^{\frac{1}{1-s}} = \left[ l_t + (1 - l_t) f \right]^{\frac{1}{1-s}} K^{\frac{1}{1-s}} p \\ G_{2,t} &= \left[ n_{1,t} p^{1-s} T^{1-s} + n_{2,t} p^{1-s} \right]^{\frac{1}{1-s}} = \left[ l_t f + (1 - l_t) \right]^{\frac{1}{1-s}} K^{\frac{1}{1-s}} p \end{aligned}$$

[6] As a result of Walras' Law, equilibrium in all product markets implies equilibrium in the regional labour markets.

Consumption and public input demand per variety in each region is:

$$
(9) \qquad
\begin{aligned}
d_{1,t} &= \left[ M_{1,t} G_{1,t}^{s-1} + M_{2,t} G_{2,t}^{s-1} f \right] p^{-s} \\
d_{2,t} &= \left[ M_{1,t} G_{1,t}^{s-1} f + M_{2,t} G_{2,t}^{s-1} \right] p^{-s}
\end{aligned}
$$

$M_{r,t}$ denotes private and public expenditures for manufactured goods in region $r$; $M_t$ defines the world expenditures for manufactures $M_t = M_{1,t} + M_{2,t}$ and $s_{E,t} = \dfrac{M_{1,t}}{M_t}$ its regional split. From (6), (8) and (9)

$$
(10) \qquad
\begin{aligned}
q_{1,t} = d_{1,t} &= \left[ \frac{s_{E,t}}{l_t + (1 - l_t) f} + \frac{(1 - s_{E,t}) f}{l_t f + (1 - l_t)} \right] \frac{M_t}{s} \frac{1}{K} \frac{s-1}{b} \\
q_{2,t} = d_{2,t} &= \left[ \frac{s_{E,t} f}{l_t + (1 - l_t) f} + \frac{1 - s_{E,t}}{l_t f + (1 - l_t)} \right] \frac{M_t}{s} \frac{1}{K} \frac{s-1}{b}
\end{aligned}
$$

Therefore – see (7) – short-run equilibrium profit per variety in region $r$ is:

$$
(11) \qquad
\begin{aligned}
p_{1,t} &= \left[ \frac{s_{E,t}}{l_t + (1 - l_t) f} + \frac{(1 - s_{E,t}) f}{l_t f + (1 - l_t)} \right] \frac{M_t}{s} \frac{1}{K} \\
p_{2,t} &= \left[ \frac{s_{E,t} f}{l_t + (1 - l_t) f} + \frac{1 - s_{E,t}}{l_t f + (1 - l_t)} \right] \frac{M_t}{s} \frac{1}{K}
\end{aligned}
$$

For future reference, note that regional and world profit incomes, $\Pi_{r,t}$ and $\Pi_t$ respectively, are given by

$$
(12) \qquad \Pi_{1,t} = l_t K p_{1,t} \qquad \Pi_{2,t} = (1 - l_t) K p_{2,t} \qquad \Pi_t = \Pi_{1t} + \Pi_{2t} = \frac{M_t}{s}
$$

(for the latter use equ. (11)) and world income $Y_t$ by

$$
(13) \qquad Y_t = L + \frac{1}{s} M_t .
$$

Crucial for the subsequent dynamic analysis is the relative profitability of capital $R(l_t)$ given by:

$$(14) \qquad R(l_t) = \frac{p_{1,t}}{p_{2,t}} = \frac{s_{E,t}\left[l_t f + (1-l_t)\right] + \left(1-s_{E,t}\right) f \left[l_t + (1-l_t) f\right]}{s_{E,t} f \left[l_t f + (1-l_t)\right] + \left(1-s_{E,t}\right)\left[l_t + (1-l_t) f\right]}.$$

We now turn to the specificities of taxation according to the residence principle, under which the tax rate prevailing in the region, in which the capital owner resides, is applied to any income she receives irrespective of the location in which the income originates. It follows that the tax burden on capital owners living in region *r* is identical to the tax revenues for government *r,* denoted by $TR_{r,t}$:

$$(15) \qquad TR_{r,t} = t_r \frac{\Pi_t}{2}$$

$0 \le t_r \le 1$ denotes the regional tax rate on profit incomes. If it is different between the regions, tax revenues and tax burdens differ as well. Regional expenditures for manufactured goods are therefore given as

$$(16) \qquad M_{r,t} = m\left(\frac{L}{2} + \frac{\Pi_t}{2} - TR_{r,t}\right) + n\, TR_{r,t}$$

Public policy affects expenditures for manufactured goods in as far private and public expenditure shares differ; i.e. $m \ne n$ . Those effects differ between regions, if local tax rates are different; i.e. $t_1 \ne t_2$ .

Observing (12) and (15), world expenditures for manufactures are

$$(17) \qquad M_t = m\left(L + \Pi_t\right) + (n-m)\left(TR_{1,t} + TR_{2,t}\right) = m\left(L + \frac{M_t}{s}\right) + (n-m)\frac{t_1 + t_2}{2}\frac{M_t}{s} .$$

Therefore,

(18)
$$M_t = \overline{M} = \frac{m}{s - m + \frac{t_1 + t_2}{2}(m - n)} s L .$$

Its regional split is

(19)
$$s_{E,t} = \overline{s_E} = \frac{1}{2}\left(1 + \frac{n-m}{s}\frac{t_1 - t_2}{2}\right).$$

With the residence principle, world income (see equation (13)), total expenditures for manufactures and its regional split are constant, i.e. independent of the regional allocation of capital. $\overline{s_E}$, the expenditure share for manufactured goods in Region 1, will be one of our central parameters. It summarizes the effects of public expenditure and tax policy. If both regions have the same tax rate, i.e. if $t_1 = t_2$, or if public expenditure behaviour is not different from the private one, i.e. if $m = n$, then $\overline{s_E} = \frac{1}{2}$ as in the symmetric Footloose Capital Model without a public sector. If regional tax policy differs, i.e. $t_1 \neq t_2$, and if this difference matters for expenditure behaviour, i.e. if $m \neq n$, then $\overline{s_E} \neq 0.5$. Equ. (19) shows that the high tax region ends up with the higher expenditure share for manufactured goods, if governments spend more for manufactured goods than private consumer do; i.e. if $n > m$. For future reference we define

(20)
$$\Delta\overline{s_E} = \overline{s_E} - \frac{1}{2} = \frac{1}{2}\frac{n-m}{s}\frac{t_1 - t_2}{2}$$

where $\Delta\overline{s_E}$ can be interpreted as measuring the degree of asymmetry brought about by the combined effect of public tax and expenditure policy. Given the constraints on the parameters, $0 \leq m, n, t_1, t_2 \leq 1 < s$, it follows that $\left|\Delta\overline{s_E}\right| \leq \frac{1}{4s} := \left|\Delta\overline{s_E^{\max}}\right| < \frac{1}{4}$ holds.

Finally, expressions (11), (18) and (19) determine short-run equilibrium regional profits per variety. The relative profitability of capital $R(l_t)$ is given by:

(21)
$$R(l_t) = \frac{\overline{s_E}\left[l_t f + (1-l_t)\right] + \left(1-\overline{s_E}\right) f\left[l_t + (1-l_t) f\right]}{\overline{s_E} f\left[l_t f + (1-l_t)\right] + \left(1-\overline{s_E}\right)\left[l_t + (1-l_t) f\right]} = \frac{\left(1+f^2\right)(1-l_t) + 2 f l_t + 2\Delta\overline{s_E}\left(1-f^2\right)(1-l_t)}{\left(1+f^2\right) l_t + 2 f (1-l_t) - 2\Delta\overline{s_E}\left(1-f^2\right) l_t}.$$

For $\Delta\overline{s_E} = 0$ the analysis is equivalent to that of the symmetric Footloose Capital Model without a public sector.

### *3.2. Capital Movements and the Complete Dynamical Model*

In a Footloose Capital model, the representative capitalist does not move herself, but allocates the physical capital she owns between the regions. In doing so, she is interested in her utility or in her *real net* income.[7] Since all income is taxed and spent in the home region of the capitalist, the relevant tax rate and price index for calculating *real net* income are the ones at home, irrespective of the regional capital allocation. Therefore, in this case orientation at *real net* income is equivalent to an orientation at *nominal gross* income.

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[7] Assuming that the representative capitalist takes the level of the public good provided at home as given, the indirect utility function, which follows from the specification in equation (1), is linear in real income.

In specifying the dynamic process we recur to ideas from the replicator dynamics widely used in evolutionary economics and evolutionary game theory (see e.g., Weibull, 1997). Taking into account the constraint $0 \leq l_{t+1} \leq 1$, the piecewise smooth one-dimensional map whereby $l_{t+1}$ is determined by $l_t$ is:

(22) $$l_{t+1} = Z(l_t) = \begin{cases} 0 & \text{if} \quad F(l_t) < 0 \\ F(l_t) & \text{if} \quad 0 \leq F(l_t) \leq 1 \\ 1 & \text{if} \quad F(l_t) > 1 \end{cases}$$

where $l_t$ is in [0,1] implies that $l_{t+1}$ is in [0,1] and where

(23) $$F(l_t) = l_t + g l_t (1 - l_t) \frac{R(l_t) - 1}{l_t R(l_t) + (1 - l_t)}$$

with $g > 0$. We refer to $g$ as the 'speed' with which the representative capitalist alters his regional allocation of capital in response to the ratio in regional profitability, $R(l_t)$.[8] The map $F(l_t)$ can also be written as:

[8] In the literature on the replicator dynamics, it is common to assume that the change in the share of agents adopting one strategy depends on the "fitness" of the strategy under consideration in comparison to the average "fitness" of all available strategies. In our context the strategies correspond to "using the capital in region 1 or using it in region 2"; and it is straightforward to interpret the resp. profit rates as indicating the "fitness" of those two strategies. Leaving for the moment boundary conditions out of the consideration, this leads to the following standard replicator specification:

$l_{t+1} = \dfrac{p_{1,t}}{l_t p_{1,t} + (1 - l_t) p_{2,t}} l_t$, or $\dfrac{l_{t+1} - l_t}{l_t} = \dfrac{p_{1,t}}{l_t p_{1,t} + (1 - l_t) p_{2,t}} - 1$. In addition, we allow for an adjustment speed $g$ and assume the following law of motion: $\dfrac{l_{t+1} - l_t}{l_t} = g\left(\dfrac{p_{1,t}}{l_t p_{1,t} + (1 - l_t) p_{2,t}} - 1\right)$, which can be transformed into equation (23). Note that this specification is a good approximation to the discrete-time counterpart of the process assumed by Puga (1998) in his core-periphery model, namely to

(24) $$F\left(l_t\right)=l_t+4g l_t\left(1-l_t\right)\frac{\frac{1+f}{1-f}\left(\overline{s_E}-\frac{1}{2}\right)-\left(l_t-\frac{1}{2}\right)}{\left(\frac{1+f}{1-f}\right)^2-4\left(l_t-\frac{1}{2}\right)^2},$$

which nicely reveals the following symmetry property with respect to $\overline{s_E}=0.5$ and $l_t=0.5$: If for $\overline{s_E}$, $l_t$ is mapped onto $l_{t+1}$, then for $\left(1-\overline{s_E}\right)$, $\left(1-l_t\right)$ is mapped onto $\left(1-l_{t+1}\right)$. Recalling the definition of $\overline{s_E}$ (see equ. (19)) this means that swapping private and public expenditure shares or swapping regional tax rates engenders swapping of regional shares in capital allocation. The same holds also for iterates of higher order.

Fixed points for the dynamical system, which correspond to long-run equilibria, are defined by $Z\left(l\right)=l$. Core-periphery equilibria, i.e. $l_0^{CP}=0$ or $l_1^{CP}=1$, are boundary fixed points of the dynamic system. A central question of the New Economic Geography concerns critical values for trade freeness (or for any other parameter) at which agglomeration in either region is sustainable. The so-called sustain points give conditions under which "the advantages created by such a concentration, should it somehow come into existence, [are] sufficient to maintain it" (Fujita et al., 1999). Sustain points therefore specify conditions at which the boundary equilibria $l_i^{CP}$ (where $i$ = 0, 1) become (at least locally) stable. These critical values are defined by $F'\left(l_i^{CP}\right)=1$, with the latter indicating the derivative of the first return map (23). It can be reduced to $R\left(l_i^{CP}\right)=1$ and solved for

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$F\left(l_t\right)=l_t+l_t\left(1-l_t\right)K\,V\ln\left(R\left(l_t\right)\right)$ with $K\,V=g$. In particular, the fixed points and the derivatives at the interior fixed point are the same for both specifications.

(25) $$f^{S(0)} = \frac{\overline{s_E}}{1-\overline{s_E}} \qquad\qquad f^{S(1)} = \frac{1-\overline{s_E}}{\overline{s_E}},$$

where $f^{S(i)}$ indicates the sustain point for $l_i^{CP}$. For $\Delta\overline{s_E} = 0.5$ it holds that $f^{S(0)} = f^{S(1)} = 1$; for $\Delta\overline{s_E} > 0.5$ this condition changes to $f^{S(1)} < 1 < f^{S(0)}$ (and for $\Delta\overline{s_E} < 0.5$ it holds that $f^{S(0)} < 1 < f^{S(1)}$).

In addition to the boundary fixed points, an interior fixed point is given by

(26) $$l^* = \frac{1}{2} + \frac{1+f}{1-f}\left(\overline{s_E} - \frac{1}{2}\right) = \frac{1}{2} + \frac{1+f}{1-f}\Delta\overline{s_E}.$$

A second central question of the New Economic Geography concerns critical values for the trade freeness (or for any other parameter) at which an (interior) equilibrium without spatial concentration "breaks up". This so-called break point gives conditions under which "small differences among locations [will] snowball into larger differences over time, so that the symmetry between identical locations will spontaneously break" (Fujita et al., 1999). I.e. it gives conditions under which an interior fixed point $l^*$ becomes (at least locally) unstable and the dynamics is attracted to one of the boundary equilibria. Analytically, the break point is defined by $F'(l^*) = 1$. In our model, the break point arises when the interior fixed point coincides with one of the boundary fixed points and it is equal to the corresponding sustain point. At that value of the trade freeness a transcritical bifurcation occurs (see Wiggins, 1990). Two curves of fixed points (concerning the interior fixed point and one of the boundary fixed points) cross each other (with the interior fixed point leaving the admissible interval) and they exchange stability.

In our model, interior fixed points can also lose (local) stability because the derivative of the map in equ. (22) evaluated at the fixed point crosses the critical value of $-1$. If by varying one parameter (or more parameters simultaneously) the inequality $F'(l^*) < -1$ holds, an attracting period two-cycle emerges through a Flip bifurcation. The condition $F'(l^*) = -1$ allows to determine critical parameters values.

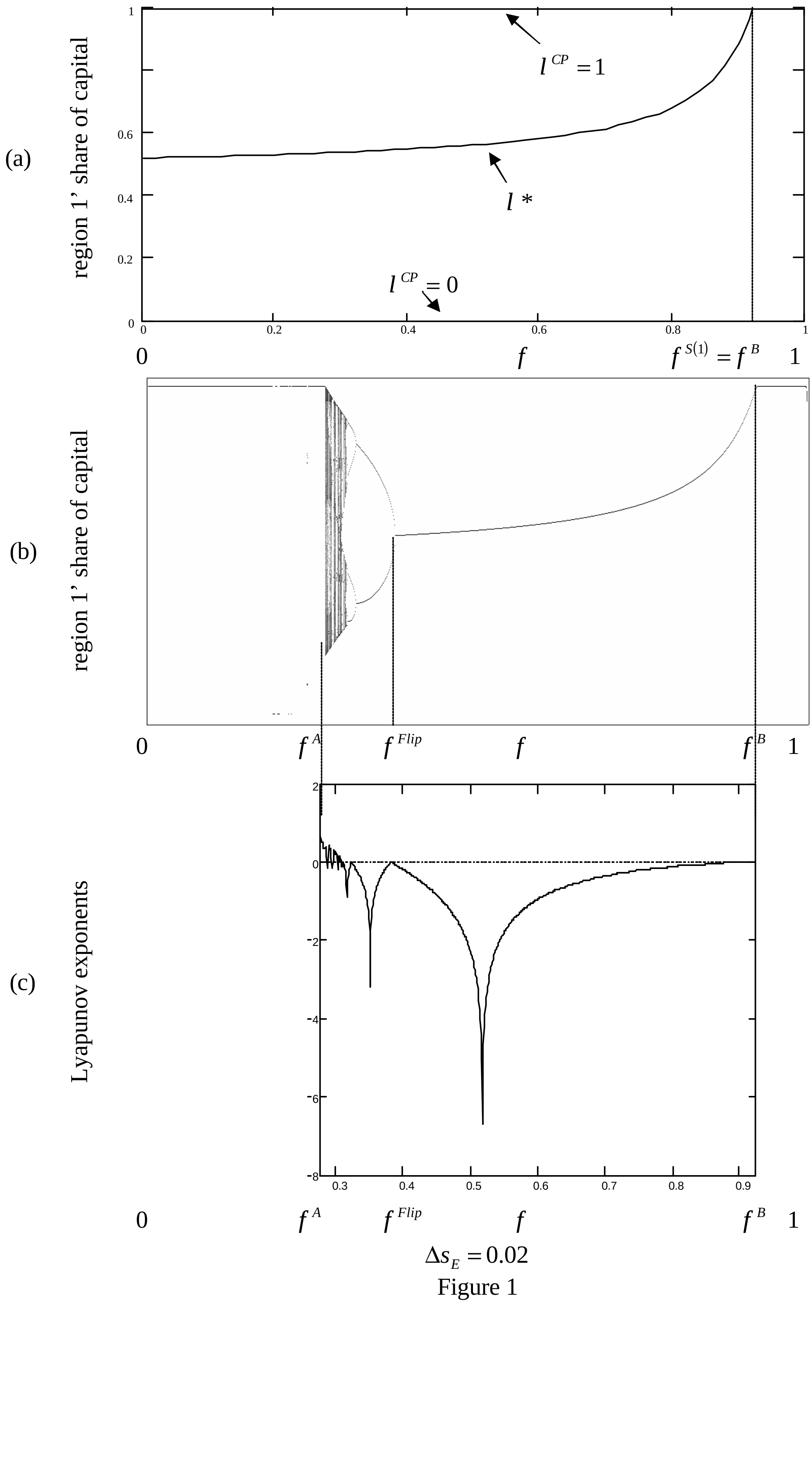
Considering a change in trade freeness, which is the central parameter for the New Economic Geography, a Flip bifurcation occurs if

$$\text{(27)} \qquad \frac{f}{(1+f)^2} = \left(\frac{g-2}{g}\right)\overline{s_E}\left(1-\overline{s_E}\right) = \left(\frac{g-2}{g}\right)\left(\frac{1}{4}-\left(\Delta\overline{s_E}\right)^2\right).$$

For $g > 2$, this equation implicitly defines a unique bifurcation value $0 < f^{Flip} < 1$.[9] Note the term $\overline{s_E}\left(1-\overline{s_E}\right)$: $f^{Flip}$ does not change if $\overline{s_E}$ is replaced by $\left(1-\overline{s_E}\right)$, or – using equ. (19) – if $n$ and $m$ or $t_1$ and $t_2$ are swapped. Note as well that $f^{Flip}$ is declining in $\left(\Delta\overline{s_E}\right)^2$.

Figure 1 summarizes the (local) properties of the fixed points as depending upon the trade freeness $f$. Figure 1a corresponds to the so-called tomahawk diagram representing the fixed points; Figure 1b is a bifurcation diagram and Figure 1c reports the corresponding Lyapunov coefficients. Since we have chosen $\Delta\overline{s_E} > 0$, the boundary equilibrium $l_0^{CP} = 0$ is (locally) unstable for all values of $f$. In a highly open economy (i.e. low transport costs or high trade freeness), in particular for $1 > f > f^{S(1)} = f^B$ no interior fixed point exists

[9] This can be easily verified considering that $\frac{f}{(1+f)^2}$ is strictly increasing in $f$ for $0 \le f \le 1$ with $0 \le \frac{f}{(1+f)^2} \le 0.25$ and that $0 \le \left(\frac{g-2}{g}\right)\overline{s_E}\left(1-\overline{s_E}\right) < 0.25$ holds for $g \ge 2$ and $0 \le \overline{s_E} \le 1$ (which implies that $0 \le \overline{s_E}\left(1-\overline{s_E}\right) \le 0.25$).


(a)
region 1' share of capital
$l^{CP}=1$
$l*$
$l^{CP}=0$
0
0.2
0.4
0.6
1
0
0.2
0.4
0.6
0.8
1
0
$f$
$f^{S(1)}=f^{B}$
1
(b)
region 1' share of capital
0
$f^{A}$
$f^{Flip}$
$f$
$f^{B}$
1
(c)
Lyapunov exponents
2
0
-2
-4
-6
-8
0.3
0.4
0.5
0.6
0.7
0.8
0.9
0
$f^{A}$
$f^{Flip}$
$f$
$f^{B}$
1
$\Delta s_E = 0.02$
Figure 1

within the (0,1) interval, and the boundary equilibrium $l_1^{CP} = 1$ is (locally) stable. As $f$ crosses the sustain point $f^{S(1)}$, a transcritical bifurcation occurs: $l_1^{CP} = 1$ loses its stability; the interior fixed point enters the (0,1) interval and becomes locally stable. As $f$ crosses $f^{Flip}$, the interior fixed point loses stability via a Flip bifurcation. Attracting periodic solutions appear, first a period two cycle, then – at lower values of $f$ – the time path exhibits more complex and even chaotic patterns with an ever increasing volatility of the regional share in capital. Once $f^A$ is reached, the share of capital assumes the value of one (or zero), i.e. one of the boundaries is hit and agglomeration *via* volatility occurs. Given the mobility hypothesis as specified in (22) and (23), the share of capital does no longer change once a boundary value is assumed. The core-periphery fixed points, though locally unstable, act as a so-called "snap-back repeller". In that situation, it might become highly sensitive to initial conditions which region ultimately ends up with the core.

The fact that locally unstable fixed points are globally attracting shows that it might be worthwhile to study the global properties of the dynamic process as well. However, before doing this, we explicitly address the question what are the impacts of public policy on the (local) dynamics.

A Flip bifurcation occurs if the expenditure and public policy parameters $n$, $m$, $t_1$ and $t_2$, satisfy the following condition (using equ. (19) and (27)):

$$(28) \qquad \Delta \overline{s_E} = \frac{(n-m)(t_1 - t_2)}{4s} = \pm\sqrt{\frac{1}{4} - \frac{f}{(1+f)^2}\frac{g}{g-2}}\ .$$

For $\Delta \overline{s_E}$, or for each of the parameters $n$, $m$, $t_1$ and $t_2$, there exist two Flip bifurcation values that are symmetric with respect to $\Delta \overline{s_E} = 0$, i.e. wrt $n = m$ and $t_1 = t_2$, respectively.

This symmetry in connection with the symmetry of the map $Z(l_t)$ as defined above carries over to the bifurcation diagrams as shown in Figure 2 for $s = 1.5$, which implies $\left|\Delta \overline{s_E^{\max}}\right| = 0.167$, and three different values of the trade freeness: a) $f = 0.35$, b) $f = 0.315$ and c) $f = 0.29$.

Let us now take a closer look at the comparative dynamics' impact of public policy as depending upon the trade freeness. Without public policy $\Delta \overline{s_E} = 0$; public policy introduces an asymmetry with $0 < \left|\Delta \overline{s_E}\right| \leq \left|\Delta \overline{s_E^{\max}}\right|$ (see equ. (19)). Recalling that – as noted above – $f^{Flip}$ is declining in $\Delta \overline{s_E}$ it holds that:

(29) $$f^{Flip}\left(\left|\Delta \overline{s_E}\right| = 0\right) > f^{Flip}\left(\left|\Delta \overline{s_E}\right| > 0\right) > f^{Flip}\left(\left|\Delta \overline{s_E^{\max}}\right|\right).$$

This has the following implications:

First, for $f > f^{Flip}\left(\left|\Delta \overline{s_E}\right| = 0\right)$ the fixed point is stable without and with any public policy – no public policy can destabilizes the fixed point.

Second, $f^{Flip}\left(\left|\Delta \overline{s_E}\right| = 0\right) > f > f^{Flip}\left(\left|\Delta \overline{s_E^{\max}}\right|\right)$ the fixed point without public policy is unstable – in Figure 2 the dynamics exhibits a period two cycle – but it is stable for the extreme values of public policy: Strong public policy exerts a stabilizing influence. Intermediate values of public policy, however, can add to the complexity of the dynamics (see Figure 2a and 2b).

Third, for $f^{Flip}\left(\left|\Delta \overline{s_E}\right| = 0\right) > f^{Flip}\left(\left|\Delta \overline{s_E^{\max}}\right|\right) > f$ the fixed point is unstable for all values of the public policy parameters; public policy adds to the complexity of the dynamics;

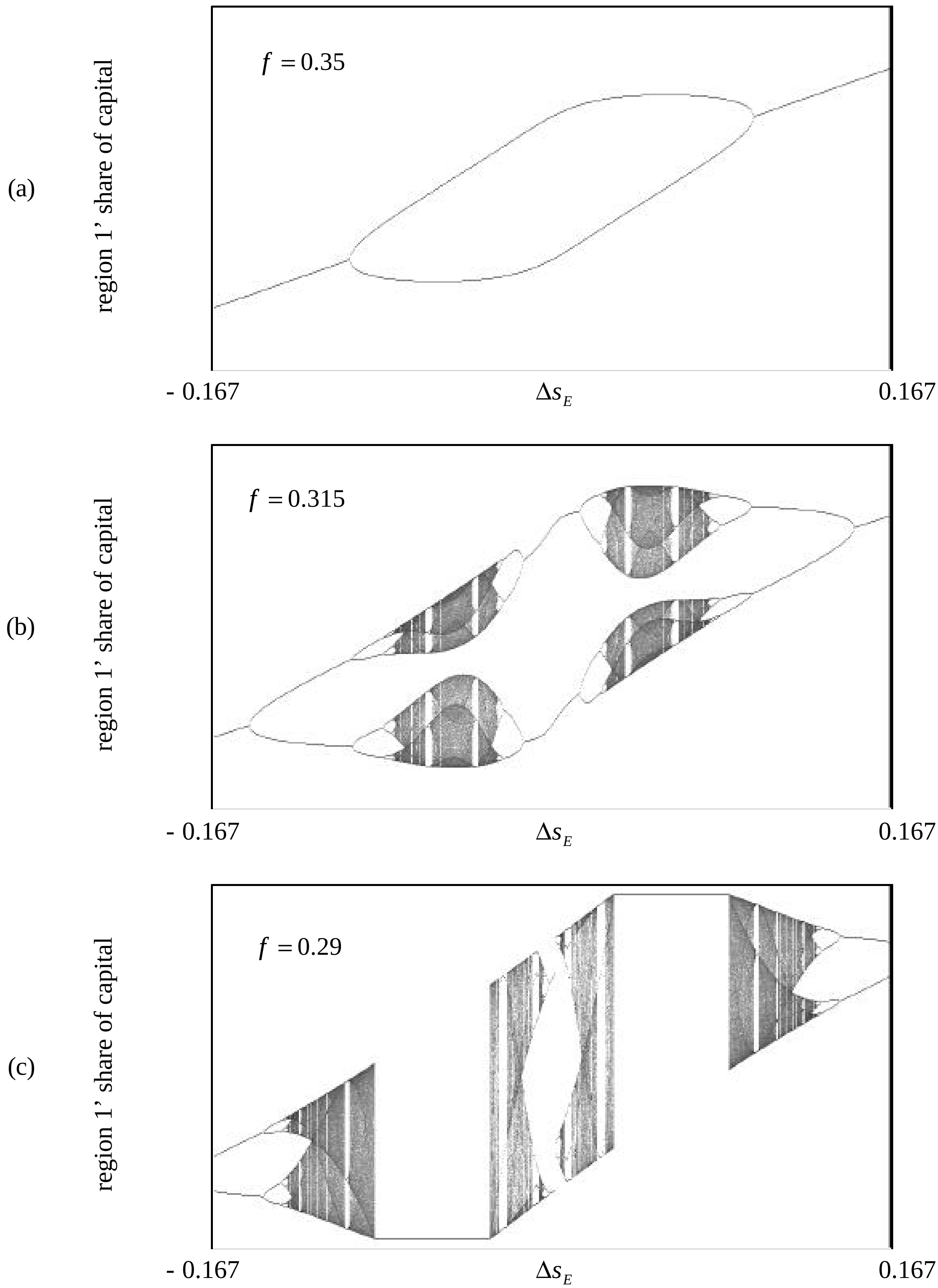

(a)
region 1’ share of capital
$f = 0.35$
- 0.167
$\Delta s_E$
0.167
(b)
region 1’ share of capital
$f = 0.315$
- 0.167
$\Delta s_E$
0.167
(c)
region 1’ share of capital
$f = 0.29$
- 0.167
$\Delta s_E$
0.167

Figure 2

intermediate values of public policy can even lead to full agglomeration in one region (see Figure 2c).

Figure 3, which is drawn for an adjustment speed of $g = 10$ and for $0 \le \Delta\overline{s_E} \le \Delta\overline{s_E^{\max}} = 0.25$, summarizes the properties of the *global* dynamics; Figure 3a delimitates parameter regions with different long run behaviour, Figures 3b to 3d illustrate these different types of behaviour as depending upon the initial condition $0 \le l_0 \le 1$. A black (white) tile indicates that the long run behaviour settles on $l_1^{CP} = 1$ $\left( l_0^{CP} = 0 \right)$; a grey tile indicates that the long run dynamics stays within the boundaries, i.e. that it settles either on an interior fixed point or on an interior cyclical/complex attractor.

Since $0 \le \Delta\overline{s_E}$ the boundary equilibrium $l_0^{CP} = 0$ is locally unstable for all parameter values. In Region N the trade freeness is beyond the sustain point, i.e. $f \ge f^{S(1)}$: No interior fixed point exists and the boundary fixed point $l_1^{CP} = 1$ is locally and globally stable. For the other parameter combinations, both boundary equilibria are locally unstable and an interior fixed point exists, which is locally stable as long as $f \ge f^{Flip}$ holds. It is especially interesting to analyse under what conditions the global dynamics is attracted to one of the (locally unstable) boundary fixed points. Decisive for that is whether or not the boundary conditions impinge upon the dynamics, i.e. whether or not $0 \le F(l_t) \le 1$. Figure 3a shows three regions, the boundaries of which can be determined analytically, and Figure 4 depicts corresponding examples of $F(l_t)$.

In region A (see Figure 4a), no boundary condition is involved – $Max\left(F(l_t)\right) < +1$ and $Min\left(F(l_t)\right) > 0$. Therefore, the two boundary fixed points, which are locally unstable, are also globally unstable – no initial condition will be attracted to them. The interior fixed

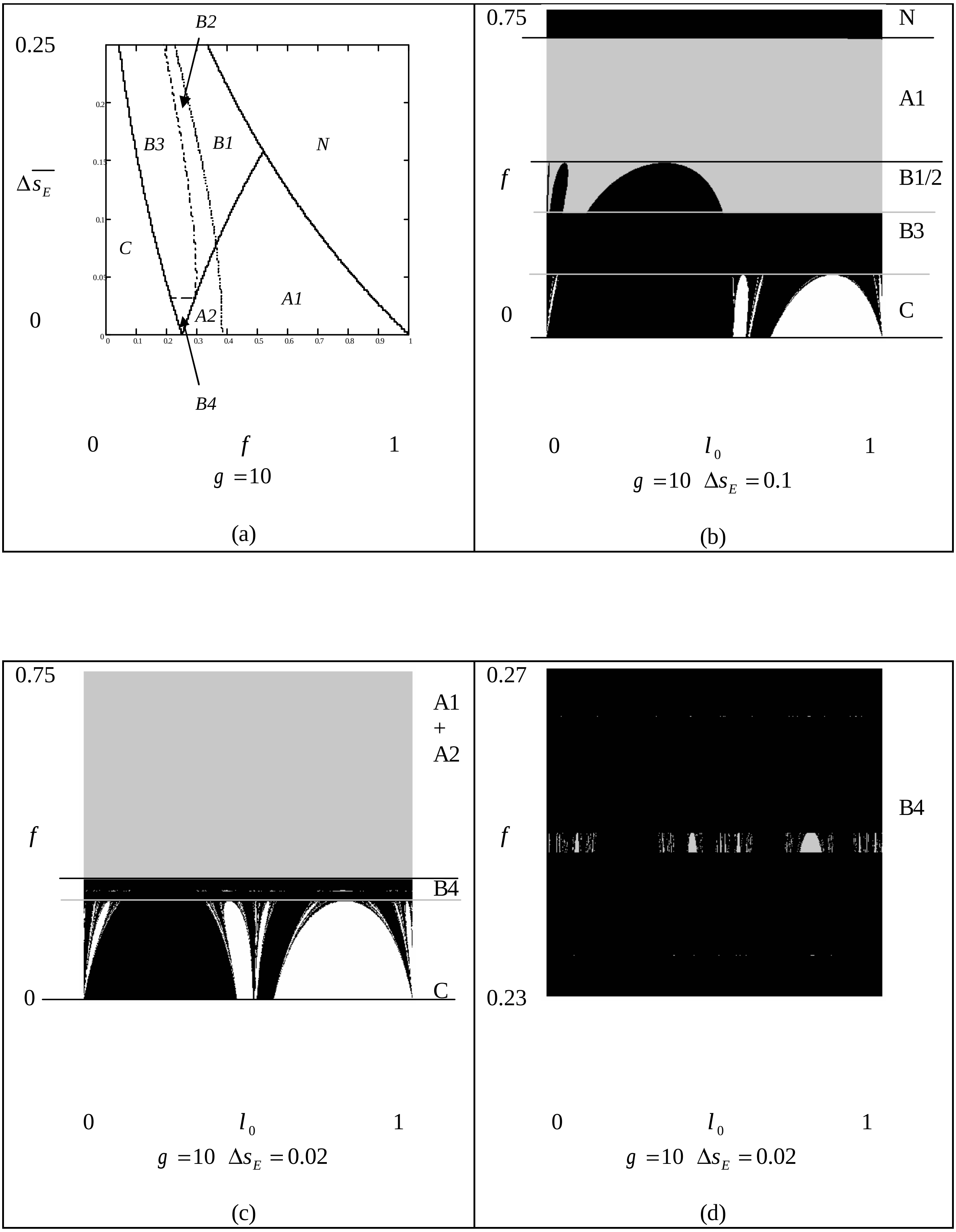
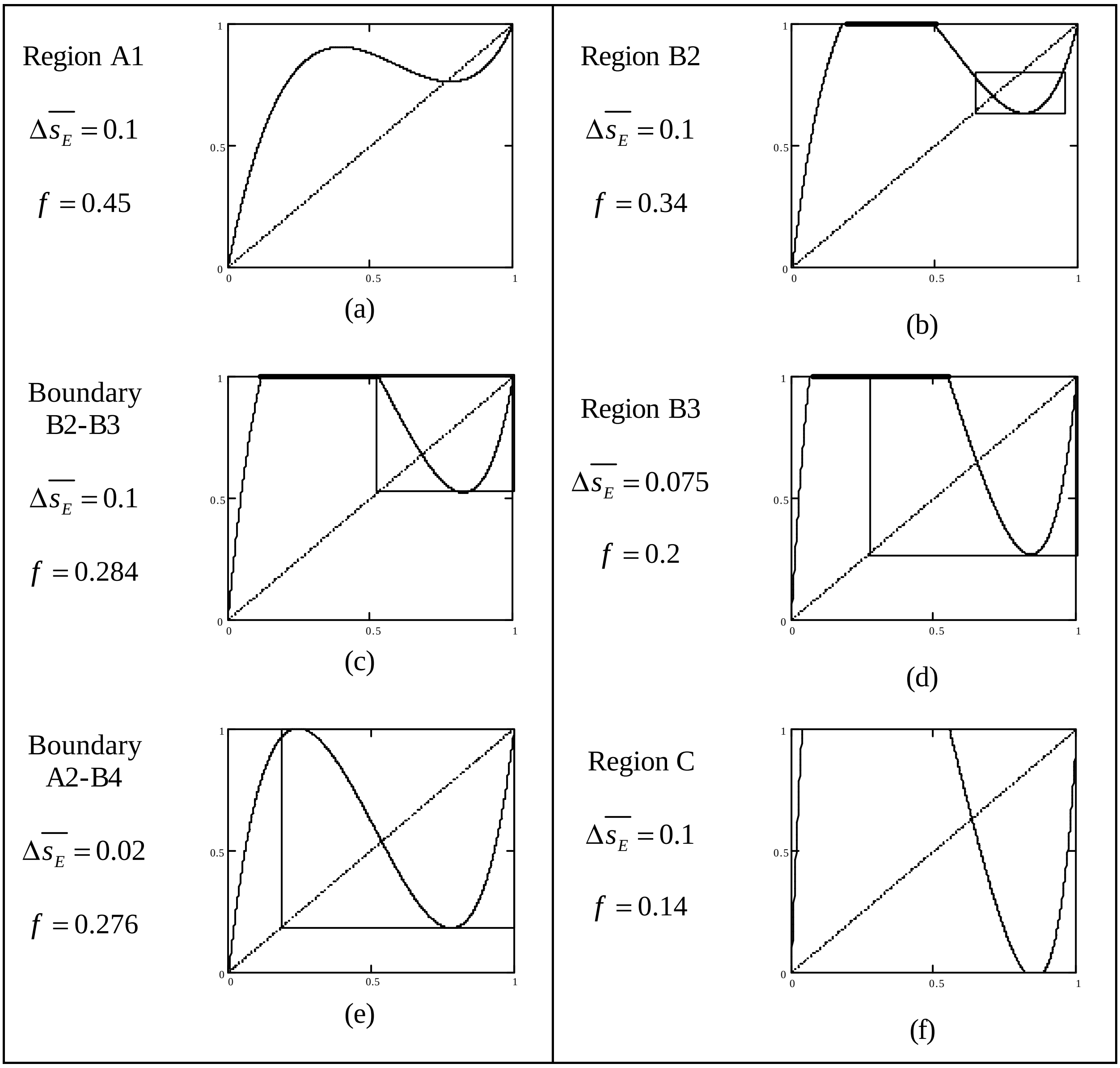

B2
0.25
B3
B1
N
$\Delta\overline{s_E}$
C
A1
A2
0
B4
0
f
1
$g = 10$
(a)
0.75
N
A1
f
B1/2
B3
0
C
0
$l_0$
1
$g = 10 \;\; \Delta s_E = 0.1$
(b)
0.75
A1
+
A2
f
B4
0
C
0
$l_0$
1
$g = 10 \;\; \Delta s_E = 0.02$
(c)
0.27
B4
f
0.23
0
$l_0$
1
$g = 10 \;\; \Delta s_E = 0.02$
(d)

Figure 3


Region A1
$\Delta\overline{s_E} = 0.1$
$f = 0.45$
(a)
Region B2
$\Delta\overline{s_E} = 0.1$
$f = 0.34$
(b)
Boundary B2-B3
$\Delta\overline{s_E} = 0.1$
$f = 0.284$
(c)
Region B3
$\Delta\overline{s_E} = 0.075$
$f = 0.2$
(d)
Boundary A2-B4
$\Delta\overline{s_E} = 0.02$
$f = 0.276$
(e)
Region C
$\Delta\overline{s_E} = 0.1$
$f = 0.14$
(f)

$g = 10$

Figure 4

point, which is stable in region A1, or the periodic orbit in Region A2 born after the Flip bifurcation, are globally stable – almost all initial conditions will be attracted to them (in Region A2 periodic unstable fixed points might exist; those and their pre-images will not converge to the stable attractor).

In Region B only the upper boundary is binding, i.e. $Max\left(F\left(l_t\right)\right)>+1$ and $Min\left(F\left(l_t\right)\right)>0$. Figures 4b to 4d are some examples of the corresponding first return map. In this region the – locally unstable – boundary fixed point $l_1^{CP}=1$ has a basin of attraction: Initial conditions $l_0$, for which $F\left(l_0\right)>1$ – i.e. initial conditions on the bold segment in Figures 4b to 4d – and all pre-images of that range will be attracted to it. In Region B1 and B2 some initial conditions will be attracted to a trapping set, which is constructed by using $Min\left(F\left(l_t\right)\right)$ and its iterates (see Figures 4b and 4c). Therefore, in Region B1 (B2) both the boundary fixed point $l_1^{CP}=1$ and the interior fixed point (the cyclical fixed point born after the Flip Bifurcation) have a basin of attraction. In region B3, the trapping set vanishes and all initial conditions are attracted to the boundary fixed point $l_1^{CP}=1$ (see Figure 4d). Of particular interest is the boundary between Region B2 and B3: The first return map in the trapping set (see Figure 4c) is very similar to the one of the logistic equation (in the limiting case). In analogy, the Li-Yorke theorem can be applied and it can be shown that cycles of any length exist (see Aligood et al., 1996). Therefore, the dynamics is chaotic on that boundary.

In region B4 (see Figure 4e) no trapping set exists: the basin of attraction for the boundary fixed point $l_1^{CP}=1$ and for the interior cyclical fixed point can be complex (this is clearly shown in the enlargement in Figure 3d, which shows in grey the complex basin of attraction for an interior solution). In that region, it is highly sensitive to initial conditions whether both regions coexist or whether agglomeration occurs in Region 1.

In region C, both boundary conditions are binding, i.e. $Max\left(F\left(l_t\right)\right) > +1$ and $Min\left(F\left(l_t\right)\right) < 0$ (see Figure 4e). Now almost all initial conditions will converge to one of the boundary fixed points: Initial conditions $l_0$, for which $F\left(l_0\right) > 1$ and all pre-images of that range will be attracted to $l_1^{CP} = 1$; initial conditions $l_0$, for which $F\left(l_0\right) < 0$ and all pre-images of that range will be attracted to $l_0^{CP} = 0$. The only exceptions are again initial conditions on unstable cycle fixed points and its pre-images. In this Region, the boundaries of the basins of attractions are complex.

Figure 3 also allows to assess the impact of an asymmetry brought about by public tax and expenditure policy upon the global dynamics: In the symmetric case (i.e. for $\Delta\overline{s_E} = 0$) only Region A1, A2 and C are possible. Therefore, either the interior fixed point and the interior cyclical solutions are globally stable (Regions A1 and A2 resp.); or both boundary fixed points have a complex basin of attraction (Region C). In the asymmetric case we analysed (i.e. for $\Delta\overline{s_E} > 0$),[10] the region N and different regions B occur; i.e. regions in which initial conditions exist that are attracted to the boundary fixed point $l_1^{CP} = 1$. In that sense, asymmetry reduces the basin of attraction of the interior solutions (in which both regions coexist) and of the boundary fixed point $l_0^{CP} = 0$ but increases the basin of attraction of the boundary fixed point $l_1^{CP} = 1$.

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[10] For $\Delta\overline{s_E} < 0$, the boundary equilibria would exchange their stability properties symmetrically.

## 4. Taxation according to the source principle

### *4.1. Short-run General Equilibrium*

With taxation according to the source principle capital income is taxed according to the tax rate that prevails in the region, in which the income originates, irrespective of the location, in which the capital owner lives. Also in this case equations (1) to (14) apply. However, regional tax revenues are now given as

$$TR_{1,t} = t_1 \, l_t K p_{1,t} \qquad TR_{2,t} = t_2 \left(1 - l_t\right) K p_{2,t} \tag{30}$$

and regional expenditures for manufactures as

$$\begin{aligned} M_{1,t} &= m\left(\frac{L}{2} + \frac{\Pi_t}{2} - \frac{TR_{1,t} + TR_{2,t}}{2}\right) + n\,TR_{1,t} \\ M_{2,t} &= m\left(\frac{L}{2} + \frac{\Pi_t}{2} - \frac{TR_{1,t} + TR_{2,t}}{2}\right) + n\,TR_{2,t} \end{aligned} \tag{31}$$

Tax burdens are identical between regions; changes in the tax rates affect private spending in both regions in an identical way. However, regional tax burdens are no longer identical to regional tax revenues. Therefore, if regional tax revenues are different and at least some of these revenues is spent for manufactured goods, i.e. $n > 0$,[11] regional expenditures are also different. Taxation according to the source principle relocates expenditures for manufacturers to the region with the higher tax revenue. Regional tax revenues do not only depend upon the tax rates, but also on the allocation of capital between the regions $l_t$, which determines the tax base.

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[11] Note that the crucial point is whether the share in public expenditure devoted to manufactured goods is positive and not whether it is greater than the private share.

World expenditures for manufactured goods are given as

(32)
$$M_t = m\left(L+\Pi_t\right)+\left(n-m\right)\left(TR_{1,t}+TR_{2,t}\right)=$$
$$= m\left(L+l_t K p_{1,t}+(1-l_t)K p_{2,t}\right)+\left(n-m\right)\left(l_t K p_{1,t} t_1+(1-l_t)K p_{2,t} t_2\right)$$

Note that they are no longer constant but depend upon the allocation of capital. The regional split of expenditures for manufacturers is:

$$s_{E,t} = \frac{1}{2}+\frac{1}{2}n\left(\frac{TR_{1,t}-TR_{2,t}}{M_t}\right) \qquad 1-s_{E,t} = \frac{1}{2}-\frac{1}{2}n\left(\frac{TR_{1,t}-TR_{2,t}}{M_t}\right)$$

(33)
$$s_{E,t}M_t = \frac{1}{2}M_t+\frac{1}{2}nK\left(l_t p_{1,t} t_1-(1-l_t)p_{2,t} t_2\right)$$

$$\left(1-s_{E,t}\right)M_t = \frac{1}{2}M_t-\frac{1}{2}nK\left(l_t p_{1,t} t_1-(1-l_t)p_{2,t} t_2\right)$$

Also here it is clearly visible that the region with the higher tax revenue gets the higher share in expenditures (if at least some of the tax revenue is spent for manufactured goods).

Given the regional allocation of capital $l_t$, equations (11), (32) and (33) allow to determine

(34)
$$s_{E,t} = \frac{s+\dfrac{n}{l_t f+(1-l_t)}\left(l_t t_1 f-(1-l_t)t_2\right)}{2s-n l_t t_1\left(\dfrac{1}{l_t+(1-l_t)f}-\dfrac{f}{l_t f+(1-l_t)}\right)-n(1-l_t)t_2\left(\dfrac{1}{l_t f+(1-l_t)}-\dfrac{f}{l_t+(1-l_t)f}\right)}$$

(35)
$$M_t = Lm\frac{s}{s-m+(m-n)C(l_t)}$$

$C(l_t)$ is a complicated expression which collects all terms depending upon tax policy and upon the regional allocation of capital. Note that the region 1's share of expenditure is no longer constant, but depends upon the capital allocation, i.e. $s_{E,t} = s_E(l_t)$. Finally, expressions (11) and (34) determine short-run equilibrium regional profit rates and their ratio:

$$R(l_t) = \frac{(1+f^2)(1-l_t)+2f l_t - \frac{n}{s} t_2 (1-f^2)(1-l_t)}{(1+f^2) l_t + 2f(1-l_t) - \frac{n}{s} t_1 (1-f^2) l_t}. \tag{36}$$

For $n=0$ or for $t_1 = t_2 = 0$ the analysis is equivalent to that of the symmetric Footloose Capital model without a public sector. Note as well the similarity in structure with the corresponding expression for the residence principle in equation (21).

*4.2. Capital Movement and the Complete Dynamical Model*

In contrast to the previous case, with taxation according to the source principle, it is the ratio of *net* nominal profits that is the relevant economic incentive for capital reallocation:

$$\frac{(1-t_1)p_{1,t}}{(1-t_2)p_{2,t}} = \frac{1-t_1}{1-t_2} R(l_t). \tag{37}$$

The central dynamic equation now is

$$l_{t+1} = Z(l_t) = \begin{cases} 0 & \text{if} \quad F(l_t) < 0 \\ F(l_t) & \text{if} \quad 0 \le F(l_t) \le 1 \\ 1 & \text{if} \quad F(l_t) > 1 \end{cases} \qquad \text{with} \tag{38}$$

$$F(l_t) = l_t + g l_t (1-l_t) \frac{\frac{1-t_1}{1-t_2} R(l_t) - 1}{l_t \frac{1-t_1}{1-t_2} R(l_t) + (1-l_t)}. \tag{39}$$

This map is analytically more complex than the one presented above related to the residence principle case. Nevertheless, there are some common features: It also exhibits a symmetry property, which slightly different from the above one. Swapping the regional tax rates and observing that $R(1-l_t, t_2, t_1) = \frac{1}{R(l_t, t_1, t_2)}$ (see equation ((36)), it holds that $F(1-l_t, t_2, t_1) = 1 - F(l_t, t_1, t_2)$. It also has two critical points, which change systematically with a change in the parameters, in a similar way as for the map above (see the examples in Figure 6). The impact of the boundary conditions is similar as well. Therefore, although less analytic results are obtainable much of the previous analysis carries over to this case.

Again, two boundary fixed point capital allocations – $l_0^{CP} = 0$ and $l_1^{CP} = 1$ – exist and change (local) stability at the sustain points.[12] The latter are defined by $F'\left(l_i^{CP}\right) = 1$, which indicates the derivative of the map given in equ. (39), or equivalently by $R\left(l_i^{CP}\right) = \frac{1-t_2}{1-t_1}$.

For $l_1^{CP} = 1$ this condition solves for

(40) $$f^{S(1)} = \frac{1}{1+\frac{n}{s}t_1}\left[\frac{1-t_1}{1-t_2} - \sqrt{\left(\frac{n}{s}t_1\right)^2 + \left(\frac{1-t_1}{1-t_2}\right)^2 - 1}\right].$$

with $0 < f^{S(1)} < 1$.

For $l_0^{CP} = 0$ the sustain points are given by

[12] In the following, we assume $t_1 < t_2$; for $t_2 < t_1$, the boundary equilibria swap their stability properties.

(41) $$f_{1,2}^{S(0)} = \frac{1}{1+\frac{n}{s}t_2}\left[\frac{1-t_2}{1-t_1} \mp \sqrt{\left(\frac{n}{s}t_2\right)^2 + \left(\frac{1-t_2}{1-t_1}\right)^2 - 1}\right]$$

where $f_{1,2}^{S(0)}$ are distinct and real for $\left(\frac{n}{s}t_2\right)^2 + \left(\frac{1-t_2}{1-t_1}\right)^2 > 1$.

In addition, an interior fixed point is defined by $\frac{1-t_1}{1-t_2}R(l_t) = 1$, which solves for (observing equations (14) and (34))

(42) $$l^* = \frac{1}{2} + \frac{1}{2}\frac{(t_1 - t_2)\left(\frac{1+f}{1-f}\right)\left(\frac{n}{s} - \left(\frac{1+f}{1-f}\right)\right)}{2 - t_1 - t_2 + (2t_1t_2 - t_1 - t_2)\frac{n}{s}\left(\frac{1+f}{1-f}\right)}.$$

Note that for $t_1 = t_2$ the fixed point $l^* = \frac{1}{2}$ corresponds to that in the symmetric Footloose Capital model without a public sector. At break points, the interior fixed point changes stability. Analytically, the break point $f^B$ is defined by $F'(l^*) = 1$ or, equivalently by $R'(l^*) = 0$. In our model, the break point and the sustain point for $l_1^{CP} = 1$ coincide, $f^B = f^{S(1)}$. As above, at $f^B$ a so-called transcritical bifurcation occurs. Two curves of fixed points (concerning the interior fixed point and the boundary fixed point $l_1^{CP} = 1$) cross each other (with the interior fixed point leaving the admissible interval) and they exchange stability. Note that – in contrast to the previous analysis – transcritical bifurcations also occur at $f = f_i^{S(0)}$. However, in this case the interior fixed point is (locally) unstable for $f_1^{S(0)} < f < f_2^{S(0)}$. At $f^{Flip} < f^B = f^{S(1)}$ the interior fixed point looses stability *via* a Flip bifurcation. However, in contrast to the simpler case analysed above, no simple analytic expression is available for the bifurcation value.

Figure 5a shows the fixed points as depending upon the trade freeness and the bifurcation diagram in Figure 5b illustrates their (local) dynamic properties; Figure 5c shows the corresponding Lyapunov coefficients.

In our numeric example $f^{Flip} < f^{B} = f^{S(1)} < f_1^{S(0)} < f_2^{S(0)}$, which delimits various ranges for the trade freeness $f$:

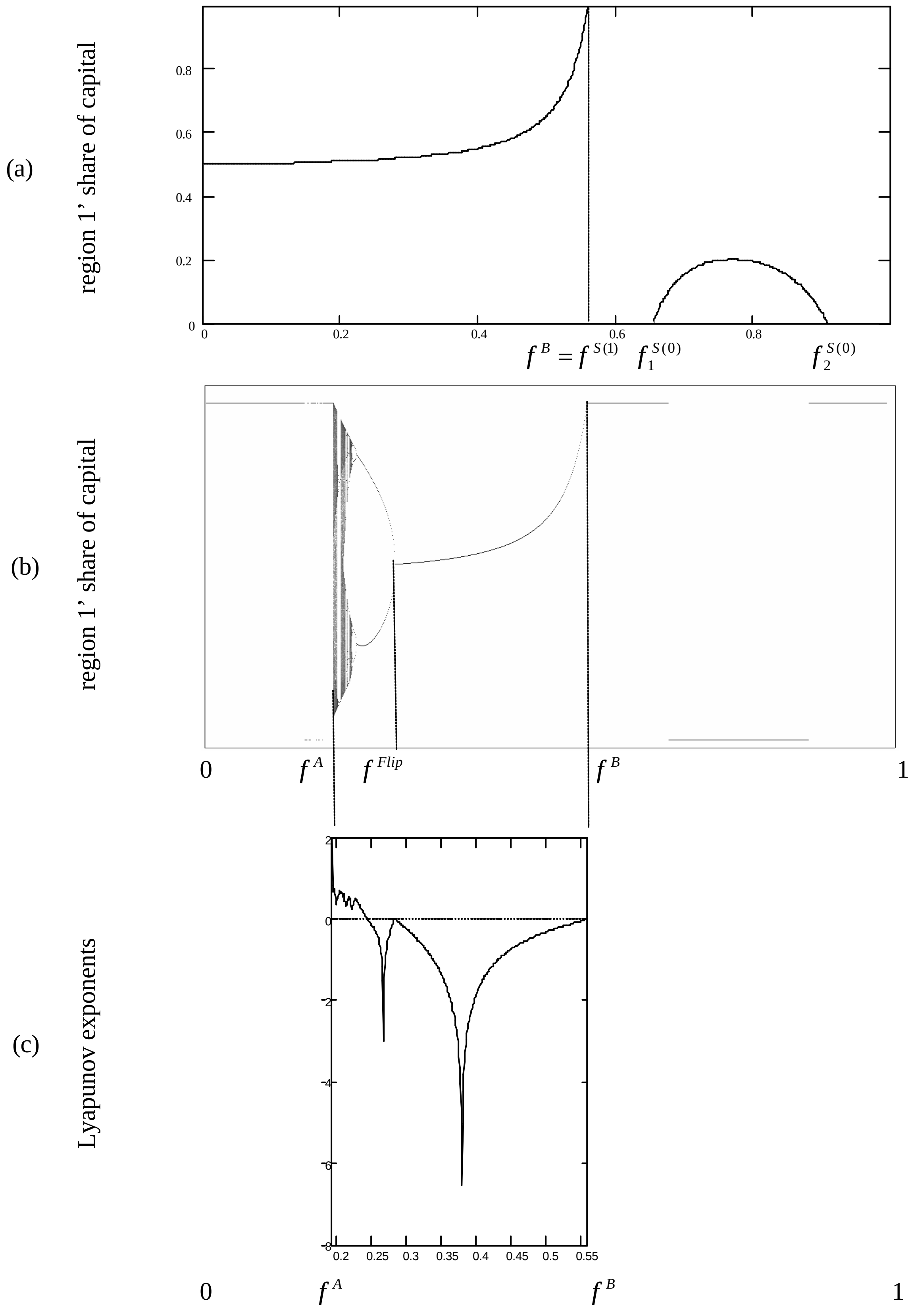
For $f_2^{S(0)} < f$ and for $f^{S(1)} < f < f_1^{S(0)}$, no interior fixed point exists within the interval (0,1). The boundary fixed point $l_0^{CP} = 0$ is (locally) unstable, and $l_1^{CP} = 1$ is (locally) stable.

For $f_1^{S(0)} < f < f_2^{S(0)}$, both boundary fixed point are (locally) stable. In addition, a (locally) unstable interior fixed point exists, which delimits the basin of attraction for the two boundary fixed points. This property is illustrated in the bifurcation diagram for which an initial value $l_0 = 0.1$ has been chosen.

For $f^{Flip} < f < f^{B}$, a (locally) stable interior fixed point exists. Both boundary fixed points are (locally) unstable.

For $f < f^{Flip}$, the interior fixed point has lost stability via a Flip bifurcation and an attracting cyclical solution exists. Both boundary fixed points continue to be (locally) unstable. Note the complex dynamics for low values of $f$ and again the phenomenon, which we call agglomeration through volatility, that occurs for $f < f^{A}$.

Also the impact of differences in regional public policies on the (local) dynamic behaviour is similar to the above analysis of the residence principle (see Figure 2). In numerical experiments it turns out that, also for the source principle case, it depends upon the degree




$t_1 = 0.5 \;\; t_2 = 0.51, \;\; l_0 = 0.1$

Figure 5

of trade freeness: For high values of trade freeness, strong differences in public policy can be stabilizing; for lower values public policy can act as an additional source of fluctuations.

Finally, Figure 6, which is the equivalent of Figures 3 and 4, illustrates the properties of the *global* dynamics. Due to the higher degree of analytic complexity, the boundaries in region A, B and C can only be determined numerically. However, the basic pattern carries over: As Figure 6a shows, the behaviour in the various regions is the same irrespective of whether taxation follows the residence or the source principle. The only difference is that region N is now split into two sub-regions delimitated by the sustain points: In Region N0 both boundary fixed points have a basin of attraction; in Region N1, all initial conditions are attracted to $l_1^{CP} = 1$. For further illustration, Figures 6b to 6d are examples of maps corresponding to specific parameter values: in particular, travelling in Figure 6a horizontally along $t_2 = 0.55$ from right to left, we have chosen four different values for the trade freeness: (i) $f = 0.25$; (ii) $f = 0.228$; (iii) $f = 0.217$; and (iv) $f = 0.5$.

## 5. Conclusions

With the reduction of barriers to commodity trade and factor mobility questions of international tax policy have become increasingly debated: Countries fear to lose industrial capital to competing neighbours. We studied this question in a New Economic Geography framework, in particular in a Footloose Capital model, which we extended to allow for a public sector. Capital income is taxed according to the residence or according to the source principle and tax revenue are spend for providing a public commodity. In doing so, public policy changes the sectoral split of total (public plus private) expenditures, which is the key magnitude for determining regional factor reward and thus factor mobility. We explicitly

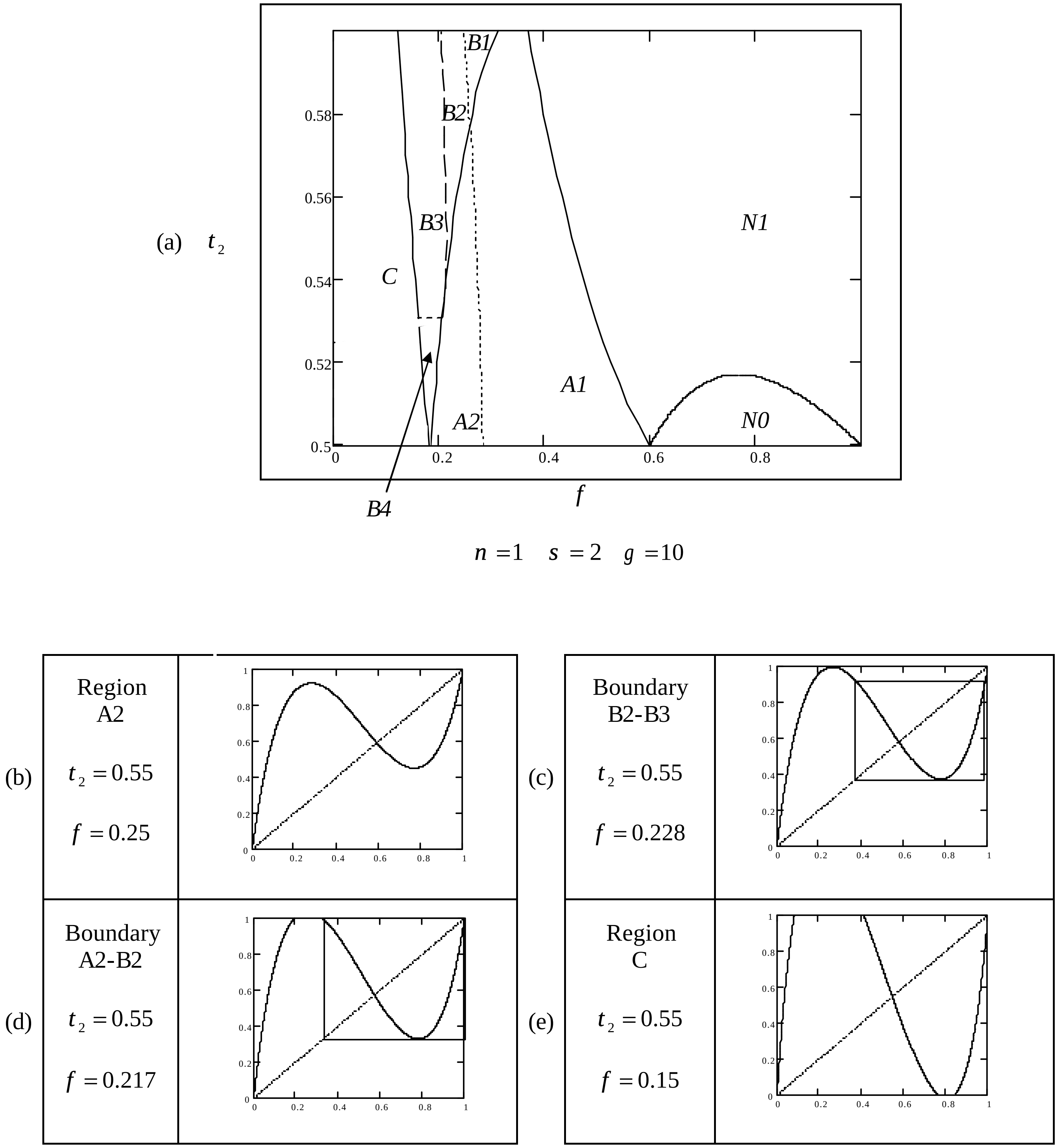

(a) $t_2$
0.58
0.56
0.54
0.52
0.5
0
0.2
0.4
0.6
0.8
B1
B2
B3
B4
C
A1
A2
N1
N0
$f$
$n = 1 \quad s = 2 \quad g = 10$
(b)
Region A2
$t_2 = 0.55$
$f = 0.25$
(c)
Boundary B2-B3
$t_2 = 0.55$
$f = 0.228$
(d)
Boundary A2-B2
$t_2 = 0.55$
$f = 0.217$
(e)
Region C
$t_2 = 0.55$
$f = 0.15$

Figure 6

modeled this process – along the lines of a replicator dynamics – in discrete time and studied under what conditions agglomeration is possible. In addition to its local properties, we paid special attention to the global features. Three fixed points can be differentiated: two boundary fixed points (industrial capital is agglomerated in region 1 or in region 2) and one interior fixed point, which is asymmetric, since we allow for differences in regional tax rates (otherwise the parameters are identical between the regions). We focused our analysis on parameter combinations for which the interior fixed point – as long as it exists – exhibits a larger share of industrial capital in region 1 than in region 2, i.e. we focus on a public policy which favours region 1. The local analysis reveals that the interior fixed point loses stability via a Flip bifurcation after which periodic and chaotic attractors emerge, and that both boundary fixed points are (locally) unstable for some (or the entire) parameter range. However, the global analysis shows that – although being locally unstable – for each of the boundary fixed points a basin of attraction exists (the boundaries of which may be highly complex). Although public policy favours region 1 in the above mentioned sense, agglomeration in either region is a possible outcome. It may be highly sensitive to initial conditions and/or parameters, which region ends with the agglomeration of industrial capital. Comparing the results to the one obtained for the symmetric case without public policy, we noticed that: (i) the basin of attraction for agglomeration in Region 2 – the region which has at the interior fixed point the lower share of industrial capital – shrinks; (ii) the one of the interior fixed point with industry coexisting in both regions also contracts; and (iii) the one of agglomeration in Region 1 increases. Public policy, which favours Region 1 at the fixed point, favours it also from a dynamic perspective. Surprisingly enough, it turns out that those dynamic properties are quite robust and do not change when taxation according to the residence principle is replaced by one according to the source principle.

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