Borrowing Constraints, Multiple Equilibria and Monetary Policy*

Tiziana Assenza

CeNDEF, University of Amsterdam

Abstract

The appealing feature of Kiyotaki and Moore's Financial Accelerator model (Kiyotaki and Moore, 1997, 2002) is the linkage of asset price changes and borrowing constraints. This framework therefore is the natural vehicle to explore the net worth channel of the monetary transmission mechanism. In the original model, however, all the variables, credit included, are in real terms. In order to assess the impact of monetary policy the model must be reformulated to fit a monetary economy. In the present paper we model a monetary economy with financing constraints adopting the Money In the Utility function (MIU) approach.The occurrence of multiple equilibria is a likely outcome of the dynamics generated by the model. A change in the growth rate of money supply can affect real output through *the impact of inflation on net worth*. In a sense the monetary transmission mechanism we are focusing on consists of a combination of the *inflation tax* effect and the *net worth channel.* Contrary to the traditional view, at least for some parameter restrictions, an increase of the inflation tax can bring about an increase of aggregate output.

---

1 Introduction

The impact of monetary policy economic activity has been and still is a fundamental question in macroeconomics. The literature has proposed several different mechanisms to explain the propagation of monetary shocks. During the '90s a remarkable body of literature has been developed under the name of *financial accelerator* or *net worth channel* to explore the transmission of monetary policy on investment and production through the impact on firms' financial structure. The ambitious scope of this branch of literature consists in understanding the mechanism through which adverse shocks, in particular a contractionary monetary policy, affect the decisions of firms, households, financial intermediaries and investors and cause large and persistent effects on economic activity and inflation.

In the early '90s two alternative frameworks have been proposed to model the Financial Accelerator hypothesis. Bernanke-Gertler (1989, 1990) and co-authors emphasized the role of agency costs in the decision to supply credit while Greenwald-Stiglitz (1988, 1993) focused on the role of bankruptcy costs in limiting investment and production. In the late '90s Kiyotaki and Moore (1997, 2002) put forward a new framework based on the idea that borrowers face a financing constraint because lenders extend credit up to the present value of borrowers' collateralizable wealth, i. e. "land", which proxies real assets. The novel and appealing feature of their model is the linkage of asset price changes and borrowing constraints: booming asset prices relax borrowing constraints and boost economic activity; the upswing, in turn, affects asset prices.

Thanks to this feature this model is the natural vehicle to explore the balance-sheet channel of the monetary transmission mechanism. In the original model, however, all the variables, credit included, are in real terms. Therefore the model as such is not suitable to explore the effects of monetary policy. In order to assess the impact of a such a policy, the model must be reformulated to fit a monetary economy with borrowing constraint. An attempt in this direction has been proposed by Cordoba and Ripoll (2004), who introduce a cash in advance constraint for consumption and investment to analyze the role of collateral constraints as a trasmission mechanism of monetary shocks. The major result of their model is that a monetary shock can generate persistent movements in aggregate output. The amplitude of this movements depends on whether or not debt contracts are indexed.

In our paper we follow a well known but different route. We model a monetary economy with financing constraints adopting the Money In the Utility function (MIU) approach. [1]

As in KM population consists of financially constrained farmers and non

---

[1] The introduction of money in the utility function is a controversial but well known and largely adopted modelling procedure in the monetary literature. A simple argument in favour of such a procedure is that agents allocate time to shopping to purchase consumption goods and that the amount of shopping time needed to purchase a certain level of consumption goods is negatively related to agents' holdings of real money balances. For description of a shopping time monetary economy see Ljungqvist and Sargent (2000), chapter 17.

constrained gatherers. In the present framework, however, preferences are non linear: they are represented by a Cobb-Douglas utility function whose arguments are consumption and money holdings. As in in the original framework, there are two types of goods, a non storable consumption good (fruit) and a collateralizable, durable asset/input (land). The total supply of land is given and constant.

By assumption the farmer and the gatherer have access to different technologies. The farmer is endowed with a linear technology while the gatherer has a well behaved production function. Notice however, that since preferences are non-linear in consumption, we must not assume that part of production is non-tradable and that consumption is limited to bruised fruit as in Kiyotaki and Moore.

In this setting, we study the effects of a monetary injection on the asset price and the allocation of land to the farmers and the gatherers. By assumption, the farmer is less patient than the gatherer.[2] Moreover, in equilibrium the real interest rate is anchored to the rate of time preference of the gatherer. Since the rate of time preference of the gatherer is exogenous, the real interest rate is given and constant. As a consequence, a change of the inflation rate brings about the same change in the nominal interest rate. Finally, in the steady state the inflation rate is pinned down to the rate of growth of money supply. In the end therefore, in the steady state a change of the rate of growth of money supply brings about a change of the same sign and the same magnitude of the nominal interest rate.

In the present context, therefore, by construction an expansionary monetary policy cannot affect real output through the *liquidity effect.* By construction in fact the liquidity effect is not active because, in the end, an increase in the rate of growth of money supply raises the inflation rate and pushes the interest rate up, not down. This does not necessarily mean, however, that superneutrality always holds. A change in the growth rate of money supply can affect real output through *the impact of inflation on net worth*. In a sense the monetary transmission mechanism we are focusing on consists in a combination of the *inflation tax* effect and the net worth channel. Contrary to the traditional view, at least for some parameter restrictions, an increase of the inflation tax can bring about an increase of aggregate output.

The paper is organized as follows. In sections 2 and 3 we discuss the optimization problems of the farmer and the gatherer respectively. In section 4 we present the equilibrium conditions and in section 5 we discuss the dynamics generated by the model. In particular we explore the effects of a monetary shock. In sections 6 and 7 we present the effects on individual and aggregate variables of unexpected and temporary shocks to the productivity parameter and to the rate of growth of money supply respectively.

---

[2] Another way of characterizing the framework put forward by Kiyotaki and Moore, in fact, is in terms of *preference heterogeneity.*

2　The farmer/borrower

Preferences of both the farmer and the gatherer are defined over consumption and real money balances. Adopting a Cobb-Douglas specification of the utility function, preferences of the farmer are represented by:

$$U^F = \sum_{s=0}^{\infty} \left(\beta^F\right)^s \left[\gamma \ln c_{t+s}^F + (1-\gamma) \ln m_{t+s}^F\right] \tag{1}$$

where $0 < \gamma < 1$; c_{t+s}^F is consumption and $m_{t+s}^F = \dfrac{M_{t+s}^F}{P_{t+s}}$ are real money balances. We assume that the production function of the farmer has constant returns to scale: $y_t^F = \alpha K_{t-1}^F$ where y_t^F is output of the farmer, K_{t-1}^F is land he owns in $t-1$ and $\alpha > 0$.[3]

The farmer maximizes utility subject to two constraints: the flow-of-funds (FF) constraint and the financing constraint. Following Ljungqvist and Sargent (2000) we assume that *at the beginning of period t* the farmer decides the amount of money he wants to hold *at the beginning of period $t+1$*. Therefore the farmer's flow of funds constraint in nominal terms is:

$$P_t y_t^F + B_t + M_t^F \geq Q_t \left(K_t^F - K_{t-1}^F\right) + N_t B_{t-1} + P_t c_t^F + M_{t+1}^F$$

where $P_t y_t^F$ is nominal income, M_t^F are money balances at the beginning of period t, B_t is credit obtained in t, M_{t+1}^F are money balances that the farmer wants to hold at the beginning of period $t+1$, Q_t is the price of land and $N_t = 1 + i_t$ is the gross nominal interest rate.

Dividing by P_t we obtain the *FF* constraint in real terms:

$$c_t^F + (1 + \pi_{t+1}) m_{t+1}^F + q_t \left(K_t^F - K_{t-1}^F\right) \leq \alpha K_{t-1}^F + m_t^F + b_t - R b_{t-1}$$

where $\pi_{t+1} := \dfrac{P_{t+1}}{P_t} - 1$ is the inflation rate, $q_t := \dfrac{Q_t}{P_t}$ is the real price of land, $b_t := \dfrac{B_t}{P_t}$ is credit in real terms and $R = 1 + r := \dfrac{1 + i_t}{1 + \pi_{t+1}}$ is the real interest rate.

In order to establish the way in which liquidity changes over time, let's assume that each agent gets a transfer in money in $t+1$ from the public sector proportional to his money holdings in t: $T_{t+1}^i = g_M M_t^i$; $i = F, G$, where the superscript F (G) denotes the farmer (gatherer). The supply of money, therefore, follows the law of motion $M_{t+1}^i = M_t^i + T_{t+1}^i = M_t^i (1 + g_M)$. In words, money holdings grow at the (exogenous) rate g_M. In order to keep the analysis as simple as possible we assume that the rate of change of money supply g_M is uniform across agents. This means that the allocation of money to the agents, i.e. the ratio of money of the farmer to money of the gatherer, is constant

---

[3] Since preferences are not linear in consumption we must not distinguish between tradable output and bruised fruit as KM do.

(which will be denoted by σ in the following). Therefore real money balances of the agent in $t+1$ are $m_{t+1}^i := \dfrac{M_{t+1}^i}{P_{t+1}} = \dfrac{M_t^i}{P_t}\dfrac{1+g_M}{1+\pi_{t+1}} = m_t^i \dfrac{1+g_M}{1+\pi_{t+1}}$.

Substituting this relation into the FF constraint we get:

$$c_t^F + g_M m_t^F \le \alpha K_{t-1}^F - Rb_{t-1} - q_t \left(K_t^F - K_{t-1}^F \right) + b_t \tag{2}$$

The term $g_M m_t^F = \dfrac{M_{t+1}^F - M_t^F}{P_t}$ is the increase in the farmer's money holdings between t and $t+1$ in real terms, i.e. at prices of period t. For lack of a better term we will refer to this magnitude as the increase in real money balances. In the light of this remark, (2) can be interpreted as follows: "resources" of the farmer, of internal or external origin (αK_{t-1}^F and b_t respectively), can be employed to consume (c_t^F), "invest" ($q_t \left(K_t^F - K_{t-1}^F \right)$), reimburse debt ($Rb_{t-1}$) and increase money balances ($g_M m_t^F$). The increase in desired money balances is always equal to the increase of liquidity engineered by the central bank by means of money transfers. In other words, we are ruling out the mismatch between desired and actual increase in money holdings.

The farmer is also financially constrained, the financing constraint can be expressed as:

$$b_t \le \frac{q_{t+1}}{R} K_t^F \tag{3}$$

The farmer maximizes (1) subject to the FF constraint (2) and the financing constraint (3). From the Lagrangian:

$$
\begin{aligned}
\mathcal{L} = & \sum_{s=0}^{\infty} \left(\beta^F \right)^s \left[\gamma \ln c_{t+s}^F + (1-\gamma) \ln m_{t+s}^F \right] + \\
& + \sum_{s=0}^{\infty} \left(\beta^F \right)^s \lambda_{t+s}^F \left[\alpha K_{t-1+s}^F - Rb_{t-1+s} + \right. \\
& \left. - q_{t+s} \left(K_{t+s}^F - K_{t-1+s}^F \right) + b_{t+s} - g_M m_{t+s}^F - c_{t+s}^F \right] + \\
& + \sum_{s=0}^{\infty} \left(\beta^F \right)^s \phi_{t+s} \left(\frac{q_{t+1+s}}{R} K_{t+s}^F - b_{t+s}^F \right)
\end{aligned}
$$

we obtain the following *FOCs*:

(iF) $\dfrac{\partial \mathcal{L}}{\partial c_t^F} = 0 \Longrightarrow \dfrac{\gamma}{c_t^F} = \lambda_t^F$

(iiF) $\dfrac{\partial \mathcal{L}}{\partial c_{t+1}^F} = 0 \Longrightarrow \dfrac{\gamma}{c_{t+1}^F} = \lambda_{t+1}^F$

$(iiiF)$ $\dfrac{\partial \mathcal{L}}{\partial m_t^F} = 0 \Longrightarrow \dfrac{1-\gamma}{m_t^F} = \lambda_t^F g_M$

(ivF) $\dfrac{\partial \mathcal{L}}{\partial m_{t+1}^F} = 0 \Longrightarrow \dfrac{1-\gamma}{m_{t+1}^F} = \lambda_{t+1}^F g_M$

(vF) $\dfrac{\partial \mathcal{L}}{\partial b_t^F} = 0 \Longrightarrow \lambda_t^F - \phi_t - R\lambda_{t+1}^F \beta^F = 0$

$$(viF) \quad \frac{\partial \mathcal{L}}{\partial b_{t+1}^F} = 0 \implies \lambda_{t+1}^F - \phi_{t+1} - R\lambda_{t+2}^F \beta^F = 0$$

From (iF) and (iiF) follows that the FF constraint is binding in each period. From (iF)-(ivF) follows:

$$g_M m_t^F = \frac{1-\gamma}{\gamma} c_t^F \qquad \forall t \tag{4}$$

i.e. the ratio of the increase in money holdings $g_M m_t^F$ to consumption c_t^F is constant and equal to $\dfrac{1-\gamma}{\gamma}$. Notice that, *given the rate of growth of money supply g_M*, the equality above states that the ratio of consumption to real money balances is constant.

From (vF) and (viF) we conclude that the financing constraint is binding if:

$$\frac{\lambda_t^F}{\lambda_{t+1}^F} > \beta^F R \tag{5}$$

Moreover after trivial substitutions we get:

$$\frac{\lambda_t^F}{\lambda_{t+1}^F} = \frac{c_{t+1}^F}{c_t^F} = \frac{m_{t+1}^F}{m_t^F} \tag{6}$$

i.e. consumption and real money balances should grow at the same rate. This condition ensures that the ratio of consumption to real money balances remains constant as implicitly stated in (4).

From (5) and (6) follows:

$$\frac{c_{t+1}^F}{c_t^F} = \frac{m_{t+1}^F}{m_t^F} > \beta^F R$$

Notice that $\dfrac{m_{t+1}^F}{m_t^F} = \dfrac{1+g_M}{1+\pi_{t+1}}$ by construction. Therefore from (6) we conclude that consumption grows at the rate $\dfrac{1+g_M}{1+\pi_{t+1}}$.

In the end, the financing constraint is binding if the following condition holds:

$$\frac{1+g_M}{1+\pi_{t+1}} > \beta^F R$$

In the steady state real money balances are constant, i.e. the rate of growth of money balances is equal to the inflation rate $g_M = \pi$. The inequality above, therefore, boils down to:

$$R < \frac{1}{\beta^F}$$

The same condition holds also in the original KM framework. Since both the flow of funds and the financing constraints are binding, we can write

$$c_t^F + g_M m_t^F = \alpha K_{t-1}^F - \mu_t K_t^F$$

where $\mu_t = q_t - \dfrac{q_{t+1}}{R}$ is the downpayment. In other words, the output produced by the farmer is employed to consume, hold money balances and provide the downpayment. Given output, the higher the rate of growth of nominal money balances, the smaller consumption and/or downpayment, the smaller therefore the investment in landholding.

Substituting the *FOCs* into the flow of funds constraint we get the optimal level of consumption and real money balances:

$$c_t^F = \gamma \left(\alpha K_{t-1}^F - \mu_t K_t^F \right) \tag{7}$$

$$g_M m_t^F = (1 - \gamma) \left(\alpha K_{t-1}^F - \mu_t K_t^F \right) \tag{8}$$

Thanks to the Cobb-Douglas specification of preferences, consumption and the increase in real money balances are a fraction γ and $1 - \gamma$ respectively of the resources available to the farmer, which in turn are equal to output less downpayment.

Since both the FF and the financing constraints are binding, from the very definition of net worth in the present context, we can conclude that net worth is equal to *saving net of the increase in real money balances*:

$$n_t^F = y_t^F - c_t^F - g_M m_t^F - R b_{t-1}^F + q_t K_{t-1}^F = s_t^F - g_M m_t^F \tag{9}$$

In KM $n_t^F = s_t^F$. The increase in real money balances therefore is a negative component of net worth. In the steady state $g_M = \pi$ so that

$$n^F = s^F - \pi m^F$$

An increase in the rate of growth of money (which translates into an increase of inflation) has an *inflation tax effect* on the accumulation of net worth.

Making use of (4), after some algebra we can rewrite the equation above as:

$$n_t^F = \alpha K_{t-1}^F - \frac{1}{\gamma} c_t^F = \alpha K_{t-1}^F - \delta g_M m_t^F$$

where $\delta = \dfrac{1}{1 - \gamma}$. In words: net worth is equal to output minus *a multiple* of the increase in real money balances. Other things being equal the higher the increase in real money balances, the lower net worth.

Finally notice that since the sum of consumption and the increase in real money balances is equal to the resources available to the farmer, net worth is devoted completely to downpayment:

$$n_t^F = y_t^F - \left(c_t^F + g_M m_t^F \right) = y_t^F - \left(y_t^F - \mu_t K_t^F \right) = \mu_t K_t^F \tag{10}$$

The same condition holds in KM. Considering (9) and (10) simultaneously we infer that in a KM economy with money

$$n_t^F = s_t^F - g_M m_t^F = \mu_t K_t^F$$

In the steady state

$$s^F = \mu K^F + \pi m^F$$

Part of the resources of the farmer cannot be employed as downpayment because they must be devoted to pay the inflation tax.

After trivial substitutions from the flow of funds constraint one gets:

$$K_t^F = \frac{1}{\mu_t}\left[\alpha K_{t-1}^F - \delta g_M m_t^F\right] \tag{11}$$

Equation (11) is the law of motion of the land of the farmer. Notice that it differs from the law of motion obtained by KM due to the term that represents the increase of real money balances. In particular there exists a negative relation between the demand for land and the demand for money: the higher the real money balances demanded by the farmer the lower landholding and viceversa. In the steady state $\alpha K^F = \mu K^F + \pi m^F$. An important implication of this equation is that $\alpha - \pi \frac{m^F}{K^F} = \mu$. With the exception of a zero-inflation steady state, therefore,

$$\mu < \alpha \tag{12}$$

.

3   The gatherer/lender

Following the same modelling strategy of the previous section, we assume that preferences of the gatherer are represented by a Cobb Douglas utility function:

$$U^G = \sum_{s=0}^{\infty} \left(\beta^G\right)^s \left[\gamma \ln c_{t+s}^G + (1-\gamma)\ln m_{t+s}^G\right] \tag{13}$$

where the meaning of the symbols is straightforward. Being unconstrained from the financial point of view, the gatherer maximizes utility subject only to the flow of funds constraint.

We assume that the production function of the gatherer has decreasing returns to scale:$y_t^G = G\left(K_{t-1}^G\right)$ with $G' > 0$, $G'' < 0$. We assume moreove that the Inada conditions are fulfilled.

The flow of funds constraint of the gatherer at current prices is:

$$P_t y_t^G + N_t B_{t-1} + M_t^G \geq Q_t\left(K_t^G - K_{t-1}^G\right) + B_t + P_t c_t^G + M_{t+1}^G + \varphi M_t^G$$

where φM_t^G, $0 < \varphi < 1$, are *reserves* that the gatherer keeps as a buffer stock to carry on the lending business "smoothly". The gatherer, in fact, in this context (as in KM) plays the role of the lender. We can think of φ as a policy parameter, possibly established by the central bank in his role of regulator/supervisor of the banking system or as a rough measure of transaction cost due to "financial frictions". At the beginning of period t the nominal money balances available to the gatherer are $M_t^G\left(1-\varphi\right)$.

Following the modelling strategy already adopted for the farmer we can write the flow of funds constraint of the gatherer in real terms as follows:

$$c_t^G + q_t \left(K_t^G - K_{t-1}^G \right) + b_t + (1 + \pi_{t+1}) m_{t+1}^G \leq G \left(K_{t-1}^G \right) + R b_{t-1} + (1 - \varphi) m_t^G$$

Recalling that $\dfrac{m_{t+1}^G}{m_t^G} = \dfrac{1 + g_M}{1 + \pi_{t+1}}$ and rearranging, the flow of funds constraint becomes:

$$c_t^G + q_t \left(K_t^G - K_{t-1}^G \right) + b_t + (g_M + \varphi) m_t^G \leq G \left(K_{t-1}^G \right) + R b_{t-1} \qquad (14)$$

The term $(g_M + \varphi) m_t^G = \dfrac{M_{t+1}^G - (1 - \varphi) M_t^G}{P_t}$ is the increase in the gatherer's money holdings between t and $t+1$ in real terms, i.e. at prices of period t. According to (14) the resources of the gatherer, i.e. the sum of output $G \left(K_{t-1}^G \right)$ and interest payments $R b_{t-1}$, can be employed to invest $(q_t \left(K_t^G - K_{t-1}^G \right))$, increase money holdings $((g_M + \varphi) m_t^G)$, consume (c_t^G) and extend credit to the farmer (b_t).

The maximization problem of the gatherer is:

$$\max \sum_{s=0}^{\infty} \left(\beta^G \right)^s \left[\gamma \ln c_{t+s}^G + (1 - \gamma) \ln m_{t+s}^G \right]$$

$$s.t. \quad c_t^G + q_t \left(K_t^G - K_{t-1}^G \right) + b_t + (g_M + \varphi) m_t^G \leq G \left(K_{t-1}^G \right) + R b_{t-1}$$

From the Lagrangian:

$$\begin{aligned} \mathcal{L} &= \sum_{s=0}^{\infty} \left(\beta^G \right)^s \left[\gamma \ln c_{t+s}^G + (1 - \gamma) \ln m_{t+s}^G \right] + \\ &+ \sum_{s=0}^{\infty} \left(\beta^G \right)^s \lambda_{t+s}^G \left[G \left(K_{t-1+s}^G \right) + R b_{t-1+s} + \right. \\ &\left. - q_{t+s} \left(K_{t+s}^G - K_{t-1+s}^G \right) - b_{t+s} - (g_M + \varphi) m_{t+s}^G - c_{t+s}^G \right] \end{aligned}$$

we obtain the following *FOCs*:

(iG) $\dfrac{\partial \mathcal{L}}{\partial c_t^G} = 0 \Longrightarrow \dfrac{\gamma}{c_t^G} = \lambda_t^G$

(iiG) $\dfrac{\partial \mathcal{L}}{\partial c_{t+1}^G} = 0 \Longrightarrow \dfrac{\gamma}{c_{t+1}^G} = \lambda_{t+1}^G$

$(iiiG)$ $\dfrac{\partial \mathcal{L}}{\partial m_t^G} = 0 \Longrightarrow \dfrac{1 - \gamma}{m_t^G} = (g_M + \varphi) \lambda_t^G$

(ivG) $\dfrac{\partial \mathcal{L}}{\partial m_{t+1}^G} = 0 \Longrightarrow \dfrac{1 - \gamma}{m_{t+1}^G} = (g_M + \varphi) \lambda_{t+1}^G$

(vG) $\dfrac{\partial \mathcal{L}}{\partial b_t^G} = 0 \Longrightarrow -\lambda_t^G + \beta^G R \lambda_{t+1}^G = 0$

(viG) $\dfrac{\partial \mathcal{L}}{\partial b_{t+1}^G} = 0 \Longrightarrow -\lambda_{t+1}^G + R \beta^G \lambda_{t+2}^F = 0$

$(viiG)\ \dfrac{\partial \mathcal{L}}{\partial K_t^G} = 0 \Longrightarrow -\lambda_t^G q_t + \beta^G \lambda_{t+1}^G \left[G'\left(K_t^G\right) + q_{t+1} \right] = 0$

From (iG)-(ivG) follows that the flow of funds constraint of the gatherer is binding in each period. Moreover:

$$(g_M + \varphi)\, m_t^G = \frac{1-\gamma}{\gamma} c_t^G \qquad \forall t \qquad (15)$$

i.e. the ratio of the increase in money holdings $(g_M + \varphi)\, m_t^G$ to consumption c_t^G is constant and equal to $\dfrac{1-\gamma}{\gamma}$. Considering (vG) and (viG) we reach the following conclusion:

$$\frac{\lambda_t^G}{\lambda_{t+1}^G} = \frac{c_{t+1}^G}{c_t^G} = \frac{m_{t+1}^G}{m_t^G} = \beta^G R$$

In words: real money balances and consumption must grow at the same rate $\beta^G R$.

From the assumption $\dfrac{m_{t+1}^G}{m_t^G} = \dfrac{1 + g_M}{1 + \pi_{t+1}}$ follows:

$$\frac{1 + g_M}{1 + \pi_{t+1}} = \beta^G R$$

In the steady state $g_M = \pi$ so that:

$$R = \frac{1}{\beta^G} \qquad (16)$$

i.e. the real interest rate is pinned down to the rate of time preference of the gatherer (as in KM).

Since the rate of time preference of the gatherer is exogenous, the real interest rate is given and constant. As a consequence, a change of the inflation rate brings about the same change in the nominal interest rate. In the steady state the inflation rate is pinned down to the rate of growth of money supply. In the end therefore, in the steady state a change of the rate of growth of money supply brings about a change of the same sign and the same magnitude of the nominal interest rate.

Let's recall now that from the maximization problem of the farmer we have obtained:

$$\frac{1 + g_M}{1 + \pi_{t+1}} > \beta^F R \qquad (17)$$

From (16) and (17) we infer that:

$$\beta^G > \beta^F \qquad (18)$$

Therefore we can conclude that the farmer is the more impatient agent as in KM.

10

Substituting the financing constraint into the flow of funds constraint and recalling that both constraints are binding be set:

$$c_t^G + (g_M + \varphi)m_t^G = G\left(K_{t-1}^G\right) + \mu_t K_t^F \qquad (19)$$

Equation (19) states that the resources of the gatherer, i.e. output and the downpayment received from the farmer, can be employed to consume and increase money balances.

Substituting (vG) into $(viiG)$ we get:

$$-\lambda_t^G q_t + \frac{\lambda_t^G}{R}\left[G'\left(K_t^G\right) + q_{t+1}\right] = 0$$

from which, using (iG), we obtain:

$$G'\left(K_t^G\right) = R\mu_t \qquad (20)$$

Once again, we get the same condition obtained by KM which equates the present value of the marginal productivity of the gatherer to the downpayment.

Finally substituting the $FOCs$ we determine the optimal level of consumption and real money balances for the gatherer:

$$c_t^G = \gamma\left[G\left(K_{t-1}^G\right) + \mu_t K_t^F\right] \qquad (21)$$
$$(g_M + \varphi)m_t^G = (1 - \gamma)\left[G\left(K_{t-1}^G\right) + \mu_t K_t^F\right] \qquad (22)$$

Thanks to the Cobb-Douglas specification of preferences, consumption and the increase of real money balances are a fraction γ and $1 - \gamma$ respectively of the *resources* available to the gatherer, which in turn are equal to the sum of the output and the downpayment of the farmer.

4 Equilibrium

Total consumption in t is equal to the sum of the optimal consumption of the farmer and the gatherer. Recalling (7) and (21) we can write:

$$c_t = c_t^F + c_t^G = \gamma\left[\alpha K_{t-1}^F + G\left(K_{t-1}^G\right)\right]$$

In words, consumption in t is a share γ of total production available in the same period, $y_t = \alpha K_{t-1}^F + G\left(K_{t-1}^G\right)$.

We assume that Government expenditure is $g_t = (1 - \gamma)y_t$ i.e. the public sector plays the role of "buyer of last resort" purchasing all the output not consumed by the private sector. The aggregate resource constraint $c_t + g_t = y_t$ is always satisfied.

Since by construction there are no taxes, g_t represents also the public sector deficit, which we assume is financed by means of money. The change of money supply between beginning-of-period t and beginning-of-period $t+1$, i.e. $M_{t+1} - M_t$, is equal to nominal Government expenditure $P_t g_t$. Hence $g_t = g_M m_t^F +$

$(g_M + \varphi) m_t^G$. Therefore agents end up "saving" a portion $(1 - \gamma) y_t$ of total income in the form of real money balances.

In fact from (4) and (15) follows:

$$c_t^F + c_t^G = \frac{\gamma}{1 - \gamma} \left[g_M m_t^F + (g_M + \varphi) m_t^G \right]$$

from which we get:

$$\left[\alpha K_{t-1}^F + G \left(K_{t-1}^G \right) \right] (1 - \gamma) = g_M m_t^F + (g_M + \varphi) m_t^G$$

or

$$(1 - \gamma) y_t = g_M m_t^F + (g_M + \varphi) m_t^G \tag{23}$$

We assume also that the ratio between the real money balances of the gatherer and the farmer is constant over time: $\dfrac{m_t^G}{m_t^F} := \sigma$. Substituting into (23) we get:

$$(1 - \gamma) y_t = g_M m_t^F + (g_M + \varphi) \sigma m_t^F$$

After trivial algebra :

$$m_t^F = \frac{1 - \gamma}{g_M (1 + \sigma) + \varphi \sigma} \left[\alpha K_{t-1}^F + G \left(K_{t-1}^G \right) \right] \tag{24}$$

5 Dynamics

In order to compare our results with those obtained by KM we recall that in a real KM economy $\mu_t K_t^F = \alpha K_{t-1}^F$, i.e. tradable output is equal to the downpayment, whereas $\mu_t = G' \left(K_t^G \right) / R$, i.e. the downpayment is equal to the discounted value of the marginal productivity of the gatherer's land. Therefore $\dfrac{G' \left(\bar{K} - K_t^F \right)}{R} = \alpha K_{t-1}^F$. The steady state is represented in figure 1.

In a monetary KM economy such as the present one, the dynamic system obtained by the maximization problems of the farmer and the gatherer consists of equations (11), (20), (24) and the definition of the downpayment, which we rewrite for convenience of the reader:

$$\begin{cases} \mu_t K_t^F = \alpha K_{t-1}^F - \delta g_M m_t^F \\ G' \left(K_t^G \right) = R \mu_t \\ m_t^F = \dfrac{1 - \gamma}{g_M (1 + \sigma) + \varphi \sigma} \left[\alpha K_{t-1}^F + G \left(K_{t-1}^G \right) \right] \\ \mu_t = q_t - \dfrac{q_{t+1}}{R} \end{cases}$$

Notice that the RHS of the first equation represents net worth. Substituting the third equation into the RHS of the first one we get:

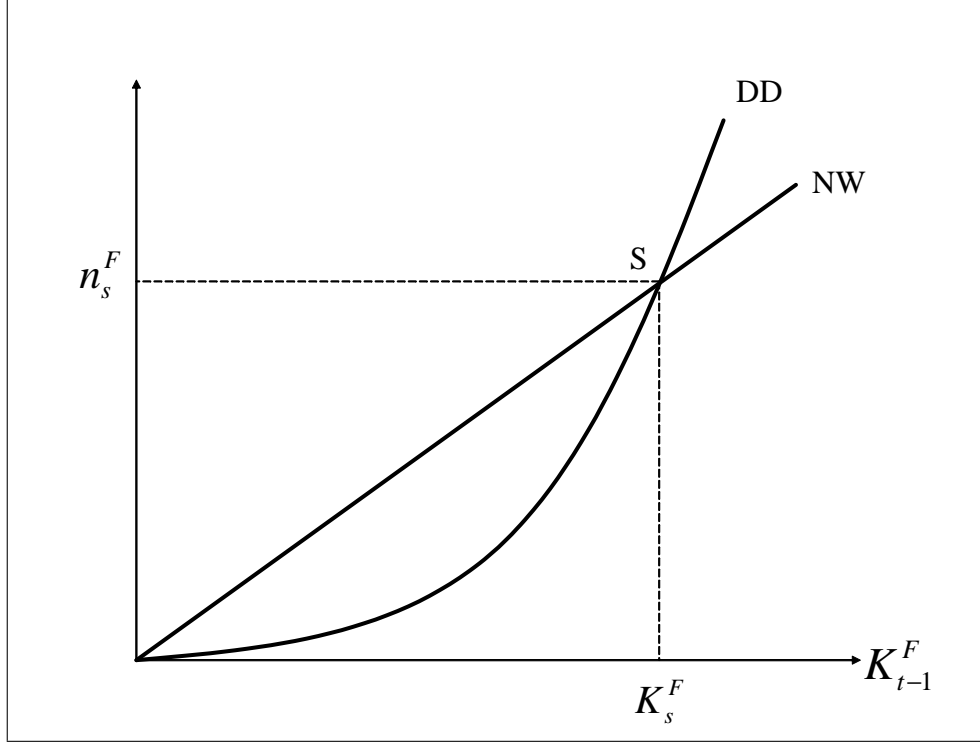$$n_t^F = y_t^F - \delta g_M m_t^F = y_t^F (1 - A) - A y_t^G \tag{25}$$

12

Figure 1: Steady state in KM's framework

where

$$A := \delta g_M \frac{1 - \gamma}{g_M (1 + \sigma) + \varphi \sigma} = \frac{1}{(1 + \sigma) + \frac{\varphi \sigma}{g_M}}$$

is a polynomial of policy parameters: g_M, φ and σ. It is easy to verify that $0 < A < 1$ and that A is increasing with g_M and decreasing with φ and σ.

Therefore, recalling that $\bar{K} = K_t^F + K_t^G$, the RHS of the first equation can be expressed as a function of K_{t-1}^F as follows:

$$n_t^F = \alpha K_{t-1}^F - A \left[G \left(\bar{K} - K_{t-1}^F \right) + \alpha K_{t-1}^F \right] \tag{26}$$

It is easy to see that n_t^F is an increasing convex function of K_{t-1}^F. In fact:

$$\frac{dn_t^F}{dK_{t-1}^F} = \alpha (1 - A) + AG' \left(\bar{K} - K_{t-1}^F \right) = \alpha + A \left[G' \left(\bar{K} - K_{t-1}^F \right) - \alpha \right] > 0$$

$$\frac{d^2 n_t^F}{d \left(K_{t-1}^F \right)^2} = -AG'' \left(\bar{K} - K_{t-1}^F \right) > 0$$

13

since $G'' \left(\bar{K} - K_{t-1}^F \right) < 0$.

Notice now that $n_t^F = 0$ iff $y_t^F (1 - A) = A y_t^G$ i.e. iff:

$$h \left(K_{t-1}^F \right) := \frac{G \left(\bar{K} - K_{t-1}^F \right)}{\alpha K_{t-1}^F} = \frac{1}{A} - 1 = \left(1 + \frac{\varphi}{g_M} \right) \sigma \qquad (27)$$

$h \left(K_{t-1}^F \right)$ is the ratio of output of the gatherer to output of the farmer, $h' < 0$.
Moreover $\lim_{K_{t-1}^F \to 0} h \left(K_{t-1}^F \right) = +\infty$ and $h \left(\bar{K} \right) = \frac{G(0)}{\alpha \bar{K}} = 0$.
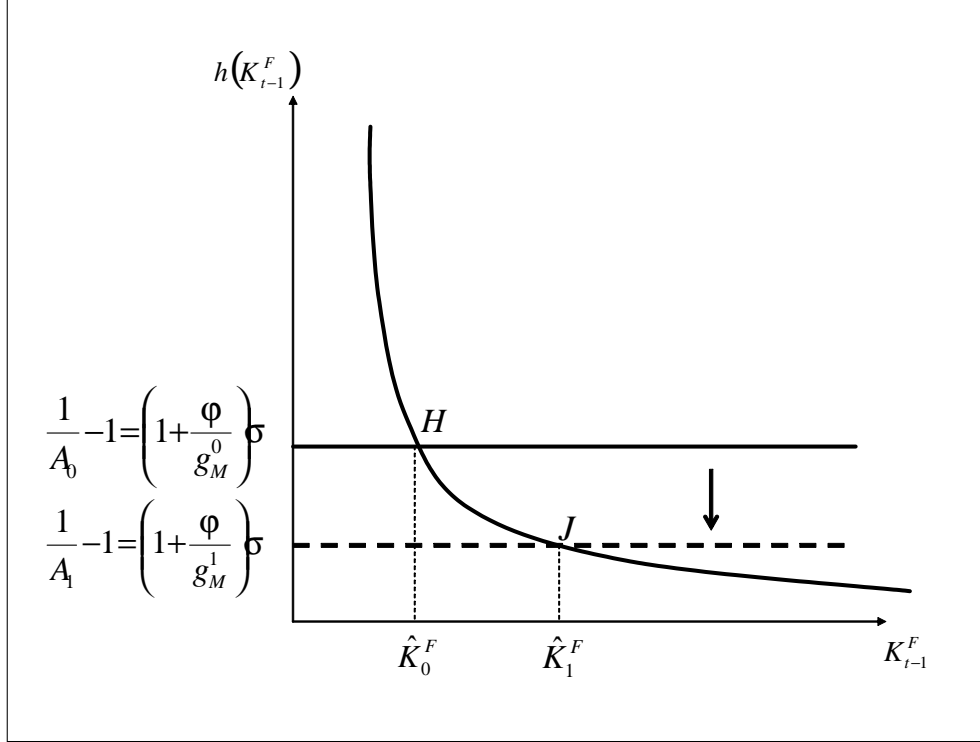

Figure 2: Determination of $\hat{K}^F$

Solving (27) for K_{t-1}^F yields a *threshold level* of K_{t-1}^F, say $\hat{K}^F = h^{-1} \left(\frac{\alpha (1 - A)}{A} \right)$
such that if $K_{t-1}^F < \hat{K}^F$, then $n_t^F < 0$ and the farmer goes bankrupt. We will
deal therefore only with the case $K_{t-1}^F > \hat{K}^F$. In particular this threshold is
a function of the policy parameters φ and g_M. The determination of $\hat{K}^F$ is
represented in fig. 2. It is immediate to conclude that an expansionary policy
move – for instance an increase of g_M – increases the threshold $\hat{K}^F$ (compare
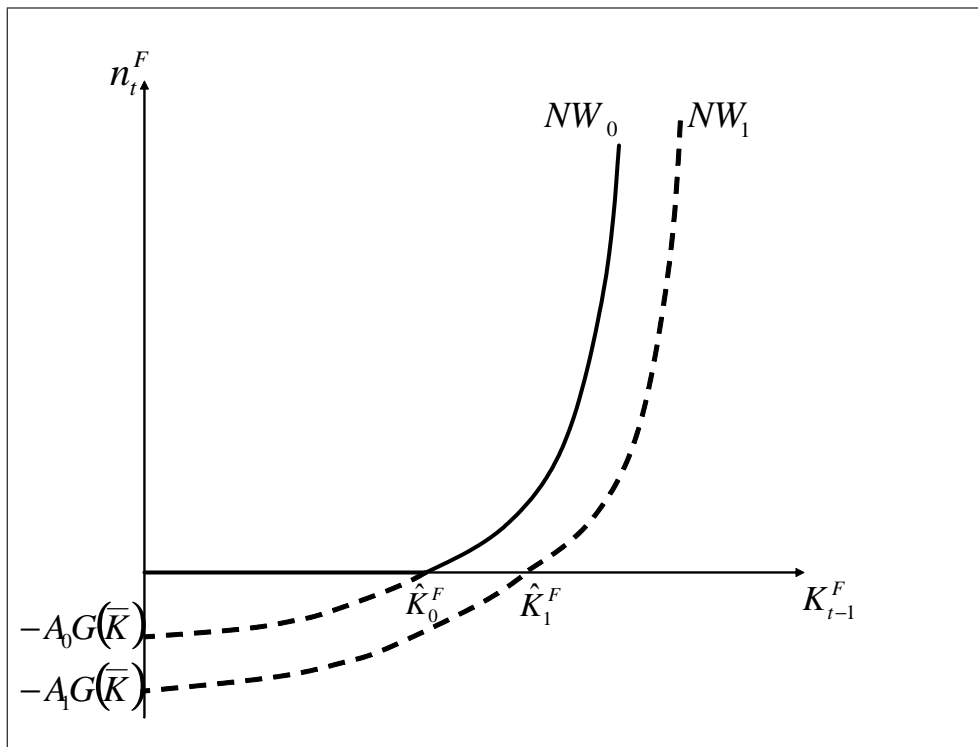points H and J in the figure).

14

Figure 3: The farmer's net worth

In figure 3 net worth is represented as a function of the farmer's landholding according to equation (26). This graph shows the new shape of the Net Worth (NW) curve in a monetary KM economy. There are three differences with respect to the NW curve in a real KM economy represented by the straight line of equation aK_{t-1}^F which represents tradable output.

First, the curve crosses the x-axis at $\hat{K}^F$ instead of the origin. This means that net worth becomes negative (a condition for bankruptcy) when "too much land" is in the hand of the gatherer. We focus on the case of solvent farmers and therefore rule out the dashed portion of the NW curve. Second, a nominal shock can affect the position on the plane of the curve. In particular a monetary injection shifts the curve down. On the contrary, by construction, a nominal shock has no effect on the NW schedule in a real KM economy. Third, in a monetary economy the NW curve undergoes a "convexification". This is due to the fact that also the gatherer's production indirectly affects the farmer's net worth through the money market. An expansionary policy move – for instance an increase of g_M – which makes A go up has also the effect of increasing the slope of the NW curve if $G' > \alpha$ and of reducing it if $G' < \alpha$. The slope remains the same in $G' = \alpha$.
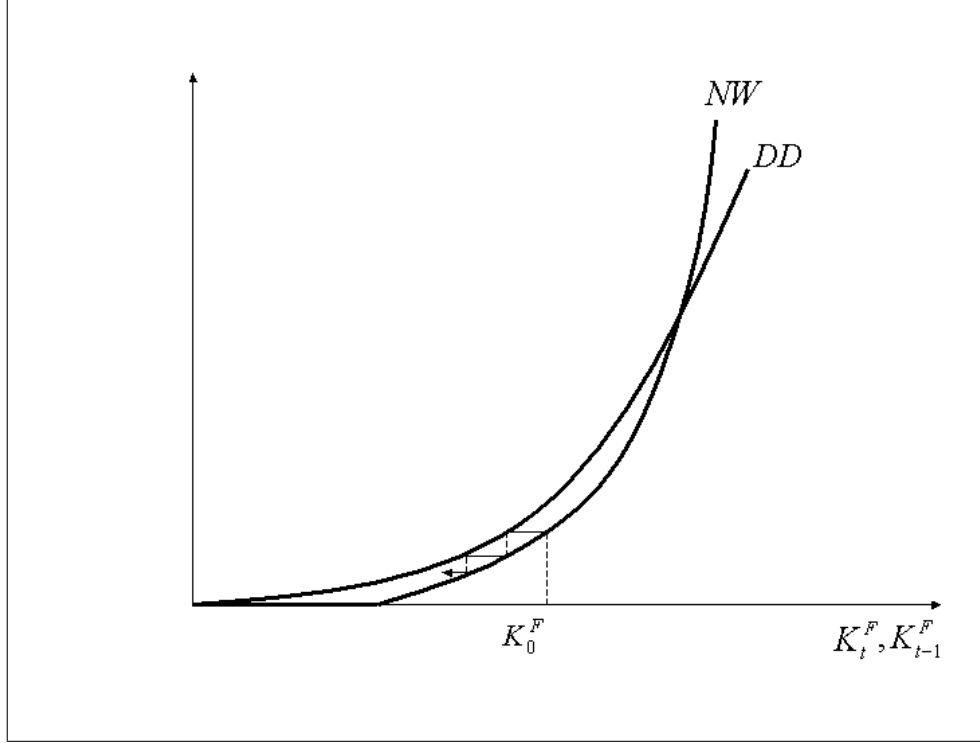
Figure 4: The DD-NW diagram. The case of a unique unstable steady state.

On the other hand the DD curve, which represents total downpayment $\mu_t K_t^F$ is exactly the same as in KM. Therefore we can envisage three scenarios. In the first one there is no intersection between the two curves (not shown). In the second one there is one intersection between the two curves (as shown in figure 4). It is easy to see that the steady state is unstable. Finally the third and most interesting case is represented in figure 5 where the NW curve intersects the DD curve twice. It is easy to reach the conclusion that the equilibrium S_1 is stable while S_0 is unstable.

Substituting the second and the third equations of the dynamic system into the first one we get:

$$
\begin{aligned}
\frac{G'\left(\bar{K} - K_t^F\right)}{R} K_t^F &= \alpha K_{t-1}^F (1 - A) - AG\left(\bar{K} - K_{t-1}^F\right) = \qquad (28) \\
&= \alpha K_{t-1}^F - A\left[G\left(\bar{K} - K_{t-1}^F\right) + \alpha K_{t-1}^F\right]
\end{aligned}
$$

Substituting the fourth equation into the second one and adapting the time index we get:

$$
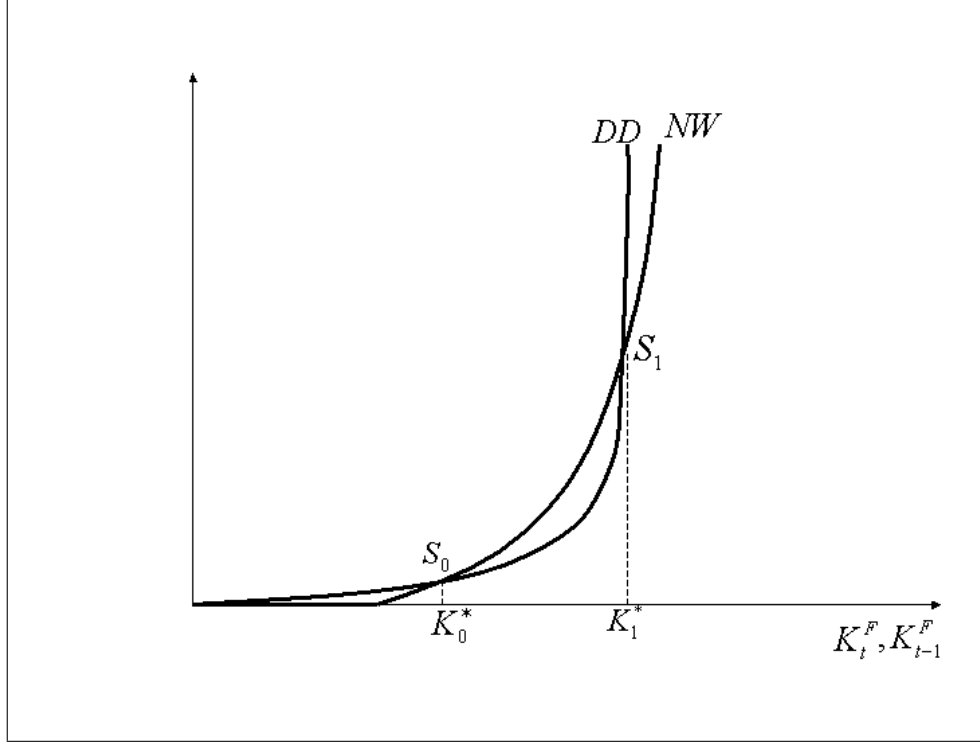q_t = R q_{t-1} - G'\left(\bar{K} - K_{t-1}^F\right) \qquad (29)
$$

16

Figure 5: The DD-NW diagram. Multiple equilibria.

We end up therefore with the system (28) (29) of two non linear difference equations in the state variables K_t^F and q_t. The system is *recursive*. The dynamics of K_t^F in fact, is independent of the dynamics of q_t.

Let's focus on (28) first. It is a first order non-linear difference equation in K_t. In order to study the properties of the steady state, at least qualitatively, we have to sketch the phase diagram. Applying the implicit function differentiation theorem to:

$$ f\left(K_t^F, K_{t-1}^F\right) = G'(\bar{K} - K_t^F)K_t^F - \alpha K_{t-1}^F R\left(1 - A\right) + RAG\left(\bar{K} - K_{t-1}^F\right) = 0 $$

which is another way of writing (28) we obtain:

$$ \frac{dK_t^F}{dK_{t-1}^F} = R\frac{\alpha + A\left[G'\left(\bar{K} - K_{t-1}^F\right) - \alpha\right]}{-G''(\bar{K} - K_t^F)K_t^F + G'(\bar{K} - K_t^F)} $$

Notice now that:

$$ \varepsilon_t = -\frac{G''(\bar{K} - K_t^F)K_t^F}{G'(\bar{K} - K_t^F)} = \frac{\partial G'(\bar{K} - K_t^F)}{\partial K_t^F}\frac{K_t^F}{G'(\bar{K} - K_t^F)} $$

17

is the (non-negative) *elasticity of the marginal productivity of the land of the gatherer with respect to the land of the farmer.* Hence the slope of the phase diagram is:

$$\frac{dK_t^F}{dK_{t-1}^F} = R \frac{\alpha + A\left[G'\left(\bar{K} - K_{t-1}^F\right) - \alpha\right]}{(\varepsilon_t + 1)\, G'(\bar{K} - K_t^F)} > 0 \tag{30}$$

Notice also that when $K_t^F \to \bar{K}$, $G'(0) \to \infty$, thanks to one of Inada's conditions. Moreover elasticity tends to $\varepsilon_{\bar{K}} = \dfrac{-G''(0)\,\bar{K}}{G'(0)} = 0$. Hence:

$$\left. \frac{dK_t^F}{dK_{t-1}^F} \right|_{K_t^F = \bar{K}} = 0$$

It is straightforward to conclude therefore that the phase diagram of (28) is increasing and concave (see fig. 7).

The phase diagram of (28) is reminiscent of the analogous phase diagram derived in the original KM case. At a glance the major qualitative difference is the fact that the origin is not belonging to the phase diagram any more. The introduction of real money balances into the model leads to a "shift" downward of the phase diagram that now cuts the x-axis with a positive intercept $K_{t-1}^F = \hat{K}^F$.

In a sense this is obvious because in the present context the farmer devotes part of his resources (consisting of output and credit) to money holdings. For each level of land inherited from the past, therefore, the amount of new land he can buy is smaller than in the original KM case.

Once again in the present context we can envisage three scenario. The phase diagram may not cross the 45° line at all, may be tangent or may cross the 45° line twice. In the first case there are no steady states (not shown). If the phase diagram is tangent to the 45° line, trajectories starting from initial conditions above (below) the steady state S, converge to the steady state (diverge) (see fig. 6). Finally in the third case there are two steady states (see fig. 7).

We have only one steady state if the 45° line is tangent to the phase diagram. This occurs if $K_{t-1}^F = K_t^F = K_s^F$, where K_s^F is the unique steady state, and simultaneously the slope of the phase diagram is equal to 1. This is the case iff:

$$A = \hat{A} := \frac{\alpha R - G'\left(\bar{K} - K_s^F\right)(\varepsilon_s + 1)}{R\left[\alpha - G'\left(\bar{K} - K_s^F\right)\right]}$$

5.1 Multiple equilibria

Let's assume now that:

$$A < \hat{A} \tag{A1}$$

Due to A1 there are two steady states. In other words A is such that there exist two intersections between the 45° line and the phase diagram.
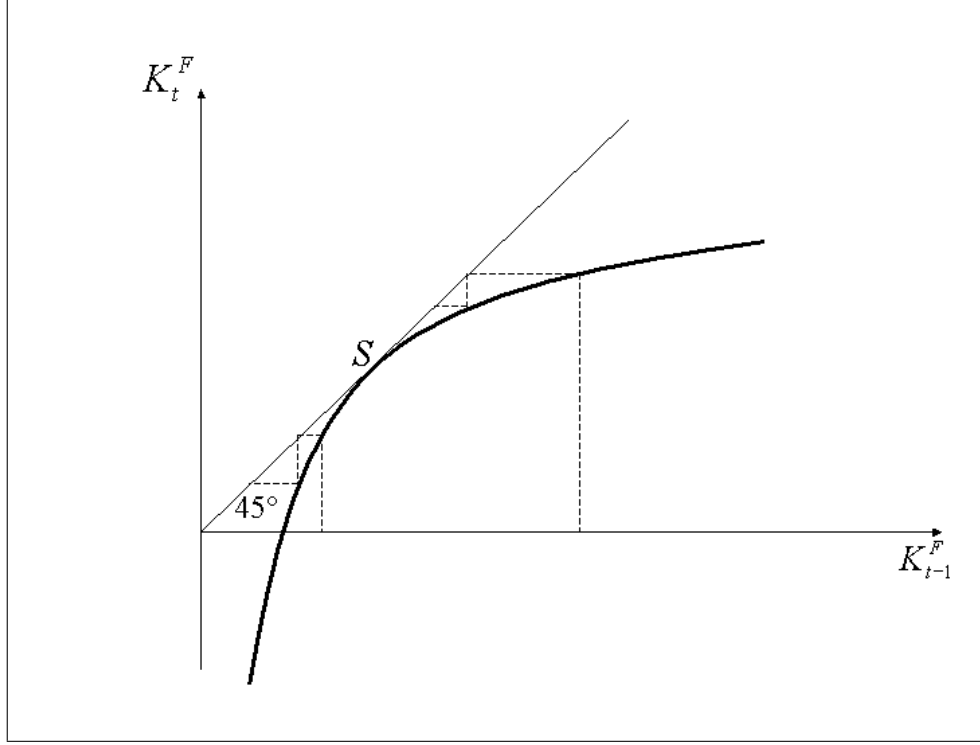
Figure 6: Phase diagram. The case of a unique steady state

Let's go back now to the system (28) (29). In the steady state $K_{t-1}^F = K_t^F = K^F$ and $q_{t-1} = q_t = q$ so that equation (29) becomes:

$$q = \frac{G'\left(\bar{K} - K^F\right)}{R - 1} \tag{31}$$

(31) is the steady state *asset price* equation. It establishes a positive link between the farmer's landholdings and the price of land *in the steady state.*

Notice now that in the steady state $\mu = q\eta$ where $\eta = 1 - \dfrac{1}{R}$. Therefore (28) becomes:

$$q = \frac{\alpha}{\eta}\left[(1 - A) - Ah\left(K^F\right)\right] = \frac{\alpha}{\eta}\left\{1 - A\left[h\left(K^F\right) + 1\right]\right\} \tag{32}$$

where $h\left(K^F\right) = \dfrac{G\left(\bar{K} - K^F\right)}{\alpha K^F}$; $h'\left(K^F\right) < 0$ (see above).

(32) is the farmer's *landholding* equation. In the steady state landholding is increasing with the price of land. In a sense (31) and (32) are the equivalent of isoclines in discrete time. The asset price equation defines the steady state asset
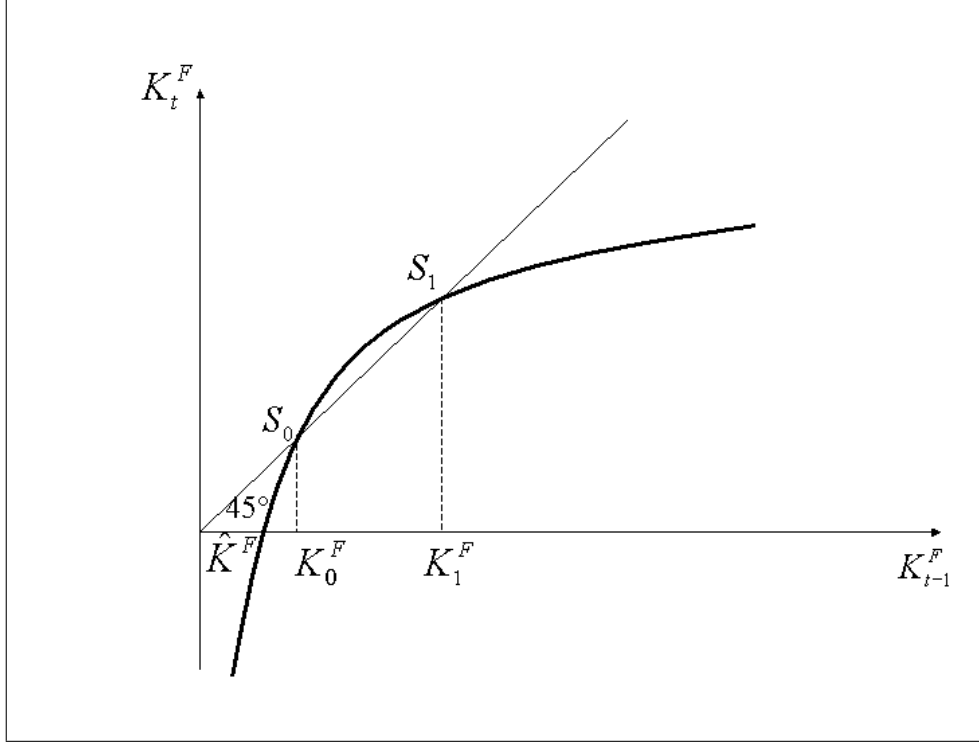
Figure 7: Phase Diagram: the case of multiple equilibria.

price as the sum of the discounted marginal productivities of the gatherer's land (uniform across periods in the steady state) over an infinite horizon. The higher K^F, in the steady state, the lower $K^G = \bar{K} - K^F$, the higher the marginal productivity and therefore the price of land q.

The "story" one can tell to give the economic intuition of (31) is less straightforward. One interpretation is as follows. The higher q, the higher the downpayment $\mu = q\eta$ will be, the smaller should be therefore the amount of resources the farmer devotes to money holdings m^F. In equilibrium, a smaller money holding is brought about by a higher landholding K^F.

In order to study the steady state, let's consider the system (31) and (32) which we rewrite here for convenience of the reader

$$\begin{cases} q = \dfrac{G'\left(\bar{K} - K^F\right)}{R - 1} \\ q = \dfrac{\alpha}{\eta}\left[(1 - A) - Ah\left(K^F\right)\right] \end{cases} \tag{33}$$

(33) is a system of two equations which can be solved for the steady state values of K^F and q. The second equation yields an increasing relationship between q

and K^F. It crosses the x-axis when $K^F = h^{-1} \left(\dfrac{\alpha \left(1 - A\right)}{A} \right) = \hat{K}^F$. If $K^F \to 0$ then $q \to -\infty$, while if $K^F = \bar{K}$ then $q = \dfrac{\alpha}{\eta} \left(1 - A\right)$. Therefore we conclude that, in the domain of interest, the second equation yields an increasing and concave relationship between q and K^F. On the other hand, the first equation yields an increasing and convex relationship between q and K^F on the $\left(K^F, q\right)$ plane. It crosses the y-axis when $q = \dfrac{G'\left(\bar{K}\right)}{R - 1}$. When $K^F \to \bar{K}$ then $q \to \infty$.
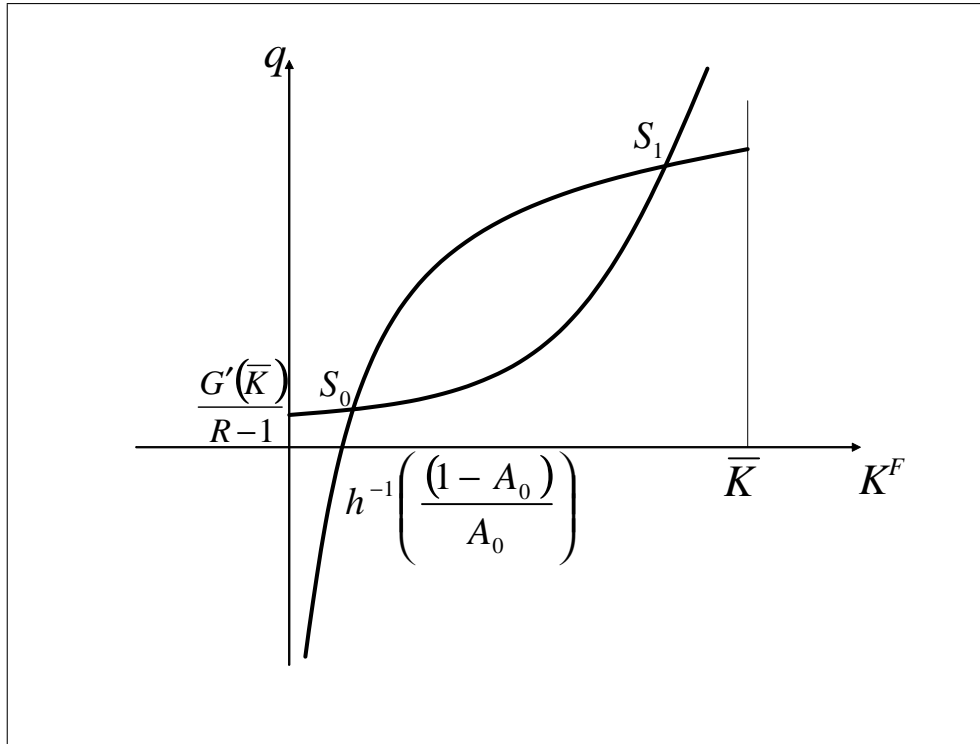

Figure 8: Steady state loci

We can explore different scenarios. In fact, given the properties of the two relations above we can obtain one, two or no steady states depending on the level of the policy parameter (A). Thanks to assumption A1 we focus only on the case of multiple steady states (see fig. 8).[4]

In order to assess the properties of each of the two steady states, we linearize

---

[4]In order to obtain a closed form solution for the steady state we should specify the production function $G\left(.\right)$. We could obtain very complicated closed form solutions without much insight. Therefore we do not proceed on this route.

the system (33) around each steady state and compute the jacobian matrix:

$$J_i = \begin{bmatrix} \frac{\alpha R(1-A) + RAG'\left(\bar{K}-K_i^F\right)}{(\varepsilon_i+1)G'(\bar{K}-K_i^F)} & 0 \\ G''\left(\bar{K}-K_i^F\right) & R \end{bmatrix} \tag{34}$$

with $i = 0, 1$.

J_i is a lower triangular matrix. Therefore the eigenvalues coincide with the elements on the main diagonal. One of the eigenvalues is $R > 1$. The second eigenvalue $\dfrac{\alpha R\left(1 - A\right) + RAG'\left(\bar{K} - K_i^F\right)}{\left(\varepsilon_i + 1\right)G'(\bar{K} - K_i^F)}$ coincides with the slope of the phase diagram of (28) in the steady state. In particular we observe that the slope is greater than one in the lower steady state S_0 $(i = 0)$ while it is smaller than one in the higher steady state S_1 $(i = 1)$. In the latter case therefore the steady state is a saddle point. This means that there is only one trajectory on the $\left(K_t^F, q_t\right)$ phase space which is converging to the steady state S_1, i.e. the saddle path. All the other trajectories diverge.
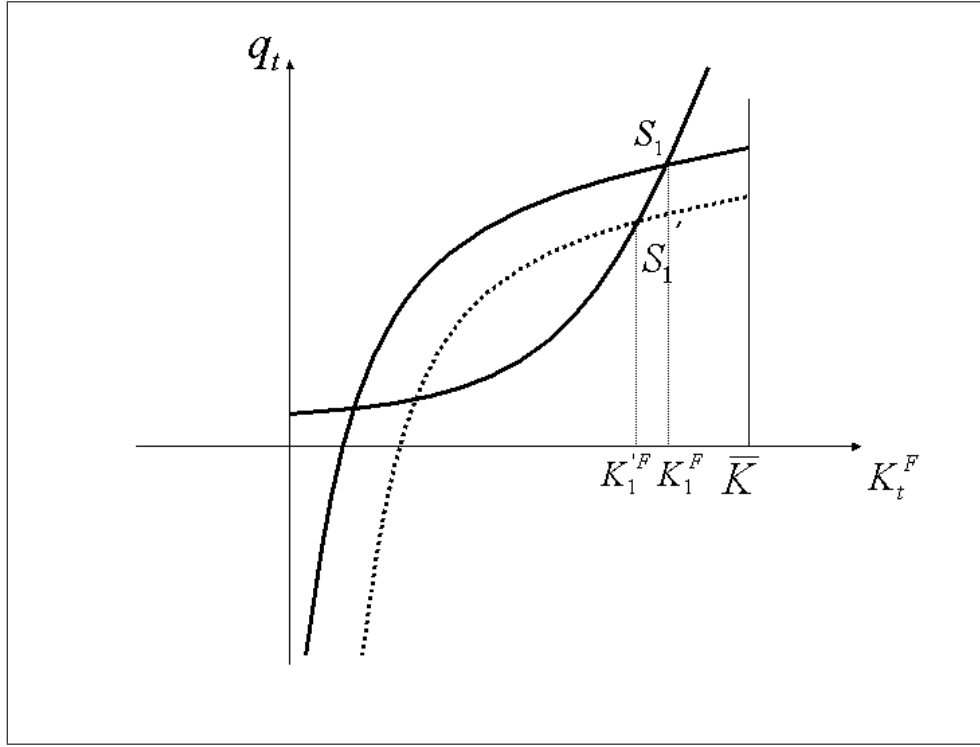

Figure 9: Effects of an expansionary monetary policy when the initial condition is S_1

Notice that A is an increasing function of the rate of growth of money supply.

In fig. 9 the convex solid line represents equation (31), that is independent from A. The concave solid line represents equation (32). Let's assume that the economy lies in the higher steady state (S_1). Suppose that an expansionary monetary policy increases A. The concave curve shifts down: the new situation is represented by the dotted line. Assuming that q_t jumps down to the new saddle path, the economy follows a trajectory which converges to the new saddle point (S_1'). In other words in the end an increase of the rate of growth of the money supply yields a decrease of the farmer's landholding and of the asset price.

5.2   Net worth, money balances and the inflation tax

Notice that "in the long run" $g_M = \pi$, i.e. a change in the rate of growth of money supply translates into a change in the rate of inflation of the same magnitude. Therefore the long run impact of the monetary expansion on the real price of land can be attributed to *an inflation tax effect*.

In order to assess the overall impact of the policy move, we recall that aggregate output is defined as $Y_t = f\left(K_{t-1}^F\right)$ where $f\left(K_{t-1}^F\right) = \alpha K_{t-1}^F + G\left(\bar{K} - K_{t-1}^F\right)$. Therefore aggregate output is a non monotonic function of the farmer's landholding. In particular it is easy to conclude that Y_t is an increasing (resp. decreasing) function of K_{t-1}^F if $\alpha > G'\left(\bar{K} - K^F\right)$ $\left(\alpha < G'\left(\bar{K} - K^F\right)\right)$. In other words Y_t is an increasing (resp. decreasing) function of K_{t-1}^F if $K_{t-1}^F < K_f^F$ $\left(K_{t-1}^F > K_f^F\right)$ where $K_f^F = \bar{K} - G'^{-1}\left(\alpha\right)$ is the farmer's landholding which maximizes aggregate output. In the case of the higher steady state, $Y_1 = f\left(K_1^F\right)$

Proposition 1 *Starting from S_1, so that $K_{t-1}^F = K_t^F = K_1^F$ and $Y_1 = f\left(K_1^F\right)$, an expansionary policy move (i.e. an increase of A due to an increase of the rate of growth of the money supply) has a positive long run effect on aggregate output iff $K_1^F > K_f^F$ and $\underline{K}_1^F < K_1'^F < K_1^F$, where $\underline{K}_1^F$ is such that $f\left(\underline{K}_1^F\right) = f\left(K_1^F\right)$. On the other hand an expansionary policy move has a negative effect on aggregate output iff $K_1^F > K_f^F$ and $K_1'^F < \underline{K}_1^F$ or iff $K_1^F \leq K_f^F \ \forall \ K_1'^F$.*

In figure 10 we represent the effects of an expansionary policy move in the case with $K_1^F > K_f^F$ and $\underline{K}_1^F < K_1'^F < K_1^F$. As claimed in the proposition above an increase of the rate of growth of money supply – i.e. an increase of the inflation tax – has a positive effect on aggregate output. This counterintuitive result is due to the fact that following the decrease in the steady state farmer's landholding from K_1^F to $K_1'^F$, the economy moves along the downword sloping branch of the aggregate output function: The loss of output due to the reduction in farmer's landholding is more than offset by the increase of output due to the increase of the gatherer's landholding.

The opposite holds true if the economy moves along the upward sloping branch as exemplified in fig. 11.
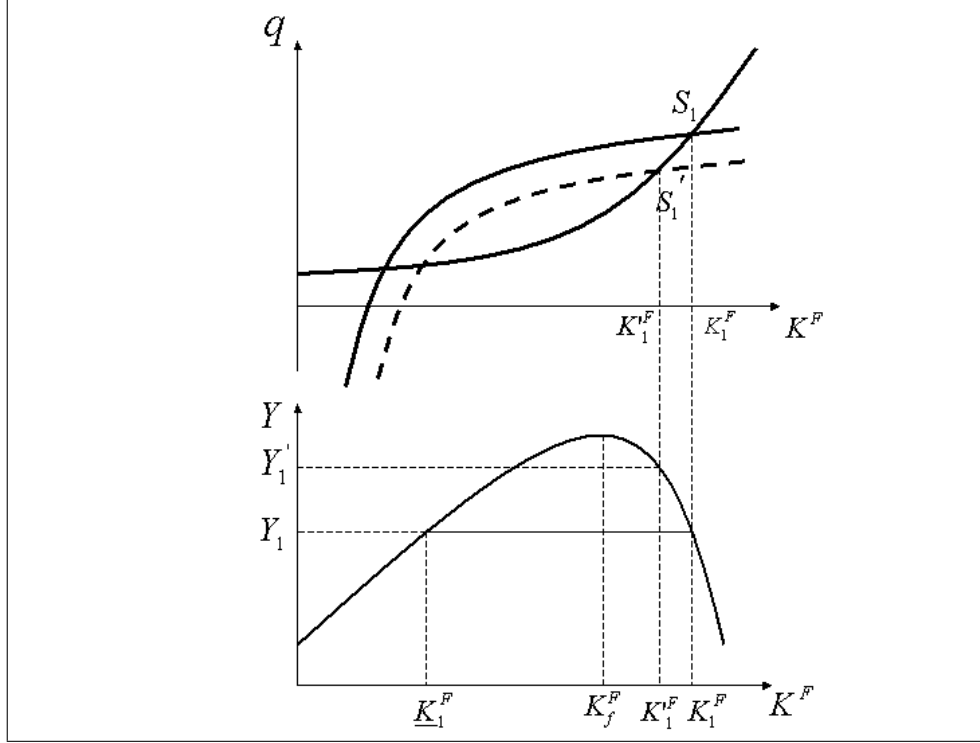
Figure 10: Positive effect of an expansionary policy move on aggregate output.

6   The effects of an unexpected productivity shock

In this section, following the original KM approach we analyze the effects of a small unexpected and temporary shock to technology on output and asset prices by means of a linear approximation around the saddle point.

Suppose that at time 0 the economy is in the saddle point and an unexpected technological shock occurs so that the productivity of the farmer increases from α_0 to α_1. As in KM we assume that the farmer decides whether to supply labour or not before the shock. If the farmer chooses to cultivate land, when the shock occurs it is too late to change his mind. Moreover the shock is temporary, i.e. the parameter α goes back to α_0 immediately after the shock.

In order to study the effects of a shock to productivity, we start from the definition of net worth, i.e. the sum of tradable output $\left(\alpha K_{t-1}^F\right)$ and the *current* value of real assets $\left(q_t K_{t-1}^F\right)$ net of interest payments $\left(R b_{t-1}^F\right)$ and of (a multiple of) the increase in real money balances $\left(\delta g_M m_t^F\right)$:

$$n_t^F = (\alpha + q_t)\, K_{t-1}^F - R b_{t-1} - \delta g_M m_t^F$$

Net worth is employed as downpayment, i.e. $n_t^F = \mu_t K_t^F$. Moreover $R\mu_t =$
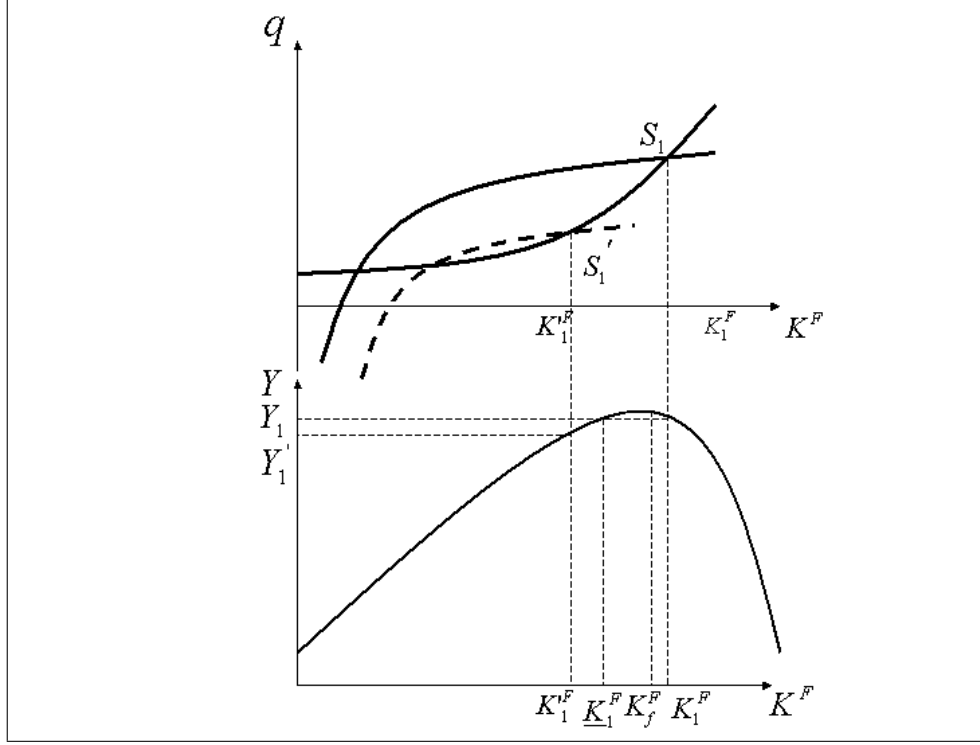
24

Figure 11: Negative effect of an expansionary policy move on aggregate output

$G'\left(\bar{K} - K_t^F\right)$ and $m_t^F = \dfrac{1-\gamma}{g_M\left(1+\sigma\right)+\varphi\sigma}\left[\alpha K_{t-1}^F + G\left(K_{t-1}^G\right)\right]$ .Therefore:

$$\frac{G'\left(\bar{K}-K_t^F\right)}{R}K_t^F = \left[\alpha\left(1-A\right)+q_t\right]K_{t-1}^F - AG\left(\bar{K}-K_{t-1}^F\right) - Rb_{t-1} \qquad (35)$$

At time 0, before the shock, $n_0^F = \alpha_0\left(1-A\right)K_0^F + q_0 K_0^F - Rb_0 - AG\left(\bar{K}-K_0^F\right)$ and $q_0 K_0^F = Rb_0$ – i.e. the current value of the farmer's land is equal to interest payments on debt inherited from the past – so that (35) boils down to:

$$\frac{G'\left(\bar{K}-K_0^F\right)}{R}K_0^F = \left(1-A\right)\alpha_0 K_0^F - AG\left(\bar{K}-K_0^F\right) \qquad (36)$$

Suppose now that in the same period the productivity parameter goes up by $\Delta\alpha = \alpha_1 - \alpha_0$. By assumption, the first round effect of the shock on net worth concerns tradable output and the price of land, *given the (steady state) landholding of the farmer K_0^F*. Immediately after the shock, net worth becomes

$$n_{AS}^F = \left[\left(\alpha_0 + \Delta\alpha\right)\left(1-A\right)+\left(q_t - q_0\right)\right]K_0^F - AG\left(\bar{K}-K_0^F\right) \qquad (37)$$

since interest payments have been predetermined on the basis of the steady state price of land: $q_0 K_0^F = R b_0$. The first round effect of the shock creates a wedge between the current (after shock) value of land $q_t K_0^F$ and interest payments $q_0 K_0^F$. In particular, as will be clear in the following, the current price of land *jumps* from q_0 to q_t creating an (unexpected) *capital gain.*

Substituting (37) into the RHS of (35) we obtain

$$\frac{G'\left(\bar{K} - K_t^F\right)}{R} K_t^F = \left[(\alpha_0 + \Delta\alpha)(1 - A) + (q_t - q_0)\right] K_0^F - AG\left(\bar{K} - K_0^F\right) \quad (38)$$

which describes the impact of the shock on K_t^F.

Assuming that the shock is temporary, in period 1 and all the following periods the situation goes "back to normal", i.e.

$$n_{t+s}^F = \alpha_0(1 - A) K_{t+s-1}^F - AG\left(\bar{K} - K_{t+s-1}^F\right) = \frac{G'\left(\bar{K} - K_{t+s}^F\right)}{R} K_{t+s}^F \quad s = 0, 1, \dots \quad (39)$$

Consider now (38). Let's take a first order approximation of the LHS in K_0^F:

$$\frac{G'\left(\bar{K} - K_t^F\right)}{R} K_t^F \approx \frac{G'\left(\bar{K} - K_0^F\right)}{R} K_0^F + \frac{G'\left(\bar{K} - K_0^F\right)}{R}(\varepsilon_0 + 1)\left(K_t^F - K_0^F\right) \quad (40)$$

where:

$$\varepsilon_0 = \frac{-G''\left(\bar{K} - K_0^F\right) K_0^F}{G'\left(\bar{K} - K_0^F\right)} K_0^F$$

is the elasticity of the marginal productivity of the land of the gatherer with respect to the land of the farmer *evaluated in the steady state* K_0^F.

The rate of change of total downpayment, i.e. of the LHS of (40), relative to the steady state is $\left[\frac{G'\left(\bar{K} - K_t^F\right)}{R} K_t^F \Big/ \frac{G'\left(\bar{K} - K_0^F\right)}{R} K_0^F\right] - 1$. Denoting the rate of change of a variable with respect to the steady state with a hat, the rate of change of the LHS becomes:

$$\widehat{\mu_t K_t^F} = (\varepsilon_0 + 1)\hat{K}_t^F \quad (41)$$

where $\hat{K}_t^F = \dfrac{K_t^F - K_0^F}{K_0^F}$ is the rate of change of the farmer's landholding.

On the other hand, the rate of change of net worth, i.e. of the RHS of (38), is $\dfrac{\left[(\alpha_0 + \Delta\alpha)(1 - A) + (q_t - q_0)\right] K_0^F - AG\left(\bar{K} - K_0^F\right)}{\alpha_0(1 - A) K_0^F - AG\left(\bar{K} - K_0^F\right)} - 1$. But $q_0\left(1 - \dfrac{1}{R}\right) K_0 = \mu_0 K_0 = \dfrac{G'\left(\bar{K} - K_0^F\right)}{R} K_0 = \alpha_0(1 - A) K_0^F - AG\left(\bar{K} - K_0^F\right)$ so that the rate of change of the RHS becomes:

$$\hat{n}_{AS}^F = \hat{\alpha}(1 - A)\frac{\alpha_0}{q_0}\frac{R}{R - 1} + \hat{q}_t\frac{R}{R - 1} \quad (42)$$

26

where $\hat{\alpha} = \dfrac{\Delta\alpha}{a_0}$ and $\hat{q}_t = \dfrac{q_t - q_0}{q_0}$ are the rates of change of the farmer's productivity and of the price of land.

After the productivity shock, the farmer's net worth goes up for two reasons: the direct effect $\hat{\alpha}\,(1-A)\,\dfrac{\alpha_0}{q_0}\dfrac{R}{R-1}$ and the *indirect effect through asset prices* $\hat{q}_t\dfrac{R}{R-1}$. Notice that $0 < A < 1$. Moreover in our context, as stated in (12) $\mu_0 = q_0\dfrac{R-1}{R} < \alpha_0$. Hence the ratio of the productivity to the downpayment $\dfrac{\alpha_0}{q_0}\dfrac{R}{R-1}$,which we will denote with θ_0 in the following, is greater than one. In symbols $\theta_0 := \dfrac{\alpha_0}{q_0}\dfrac{R}{R-1} > 1$.

In the original KM framework the rate of change of the farmer's net worth is $\hat{n}^F_{AS} = \hat{\alpha} + \hat{q}_t\dfrac{R}{R-1}$. From a comparison with (42) it is clear that in the present model the indirect effect is specified exactly as in KM while the direct effect $\hat{\alpha}\,(1-A)\,\theta_0$ is greater than $\hat{\alpha}$ (as in KM) if $A < 1 - \dfrac{1}{\theta_0}$. The smaller the policy parameter A, the higher the direct effect of the productivity shock on the rate of change of net worth after the shock.

Equating (41) and (42) we get

$$(\varepsilon_0 + 1)\,\hat{K}^F_t = \hat{\alpha}\,(1-A)\,\theta_0 + \hat{q}_t\dfrac{R}{R-1} \tag{43}$$

We have to determine now how the asset price q_t changes over time. Following KM, we note that from the definition of the downpayment and (20) follows $q_t = \dfrac{G'\left(\bar{K} - K^F_t\right)}{R} + \dfrac{q_{t+1}}{R}$, $q_{t+1} = \dfrac{G'\left(\bar{K} - K^F_{t+1}\right)}{R} + \dfrac{q_{t+2}}{R}$ and so on. Substituting the second expression in the first one and iterating the procedure we end up with:

$$q_t = \dfrac{G'\left(\bar{K} - K^F_t\right)}{R} + \dfrac{G'\left(\bar{K} - K^F_{t+1}\right)}{R^2} + ... = \sum_{s=0}^{\infty} R^{-s}\dfrac{G'\left(\bar{K} - K^F_{t+s}\right)}{R} \tag{44}$$

i.e. the current price of land is equal to the present value of the stream of future downpayments over an infinite horizon.

Taking a first order approximation of $G'\left(\bar{K} - K^F_{t+s}\right)$ in K^F_0, we can write $G'\left(\bar{K} - K^F_{t+s}\right) \approx G'\left(\bar{K} - K^F_0\right) - G''\left(\bar{K} - K^F_0\right)\left(K^F_{t+s} - K^F_0\right)$. Noting that this approximation holds true for any time period s, equation (44) boils down to:

$$q_t = \sum_{s=0}^{\infty} R^{-s}\dfrac{G'\left(\bar{K} - K^F_0\right) - G''\left(\bar{K} - K^F_0\right)\left(K^F_{t+s} - K^F_0\right)}{R}$$

Recalling that $\sum_{s=0}^{\infty} R^{-s} = \dfrac{R}{R-1}$ and that $\dfrac{G'\left(\bar{K} - K^F_0\right)}{R} = q_0\dfrac{R-1}{R}$, from the

expression above we obtain:

$$\hat{q}_t = \frac{1}{q_0} \frac{-G''\left(\bar{K} - K_0^F\right)}{R} \sum_{s=0}^{\infty} R^{-s} \left(K_{t+s}^F - K_0^F\right)$$

Notice now that $\frac{1}{q_0} \frac{-G''\left(\bar{K} - K_0^F\right)}{R} = \frac{-G''\left(\bar{K} - K_0^F\right)}{G'\left(\bar{K} - K_0^F\right)} \frac{R-1}{R} = \frac{\varepsilon_0}{K_0^F} \frac{R-1}{R}.$

Moreover $\sum_{s=0}^{\infty} R^{-s} K_0^F = K_0^F \frac{R}{R-1}$. Therefore $\hat{q}_t = \varepsilon_0 \frac{R-1}{R} \sum_{s=0}^{\infty} R^{-s} \hat{K}_{t+s}^F.$
In the end we obtain:

$$\hat{q}_t = \varepsilon_0 \frac{R-1}{R} \sum_{s=0}^{\infty} R^{-s} \left(\frac{1}{\varepsilon_0 + 1}\right)^s \hat{K}_t^F$$

Moreover $\sum_{s=0}^{\infty} R^{-s} \left(\frac{1}{\varepsilon_0 + 1}\right)^s = \sum_{s=0}^{\infty} \left(\frac{1}{R\left(\varepsilon_0 + 1\right)}\right)^s = \frac{R\left(\varepsilon_0 + 1\right)}{R\left(\varepsilon_0 + 1\right) - 1}.$ There-
fore

$$\hat{q}_t = \varepsilon_0 \frac{\left(R-1\right)\left(\varepsilon_0 + 1\right)}{R\left(\varepsilon_0 + 1\right) - 1} \hat{K}_t^F \tag{45}$$

Solving (43) and (45) for $\hat{q}_t$ and $\hat{K}_t^F$ yields

$$\hat{q}_t = \varepsilon_0 \left(1 - A\right) \theta_0 \hat{\alpha} \tag{46}$$

$$\hat{K}_t^F = \frac{R\left(\varepsilon_0 + 1\right) - 1}{\left(R-1\right)\left(\varepsilon_0 + 1\right)} \left(1 - A\right) \theta_0 \hat{\alpha} \tag{47}$$

The rate of change of net worth $\hat{n}_{AS}^F = \left(1 - A\right) \theta_0 \left(1 + \frac{R}{R-1} \varepsilon_0\right) \hat{\alpha}$ is larger
than the rate of growth of productivity thanks to the indirect effect of the shock
on the price of land. The rate of change of total downpayment $\left(\varepsilon_0 + 1\right) \hat{K}_t^F$
should match to keep equilibrium on the market for land. Therefore the rate
of change of landholding is a multiple of the rate of change of productivity. In
fact the multiplier $\frac{1}{\varepsilon_0 + 1} \left(1 + \frac{R}{R-1} \varepsilon_0\right)$ is greater than one.

7   The effects of an unexpected monetary shock

In this section we analyze the effects of a small shock to the rate of growth of
money supply on output and asset prices by means of a linear approximation
around the saddle point. Suppose that at time 0 the economy is in a steady
state and an unexpected increase of the rate of growth of money supply makes
A increase from A_0 to A_1. Assume moreover that the shock is temporary, i.e.
the parameter A goes back to A_0 immediately after the shock.

By assumption, the first round effect of the shock on net worth concerns the
price of land, *given the (steady state) landholding of the farmer* K_0^F. Immedi-
ately after the shock, denoting with $\Delta A = A_1 - A_0$, net worth becomes

$$n_{AS}^F = \alpha \left(1 - A_0 - \Delta A\right) K_0^F - A_0 G \left(\bar{K} - K_0^F\right) - \Delta A G \left(\bar{K} - K_0^F\right) + q_t K_0^F - R b_0.$$

Since interest payments have been predetermined on the basis of the steady state price of land $(q_0 K_0^F = R b_0^F)$ we can write:

$$\frac{G' \left(\bar{K} - K_t^F\right)}{R} K_t^F = \alpha \left(1 - A_0 - \Delta A\right) K_0^F - A_0 G \left(\bar{K} - K_0^F\right) - \Delta A G \left(\bar{K} - K_0^F\right) + (q_t - q_0) K_0^F$$

The rate of change of net worth, i.e. of the RHS, is:

$$\hat{n}_{AS}^F = -\left(\theta_0 - 1\right) \hat{A} + \hat{q}_t \frac{R}{R - 1} \tag{48}$$

where $\hat{A} = \dfrac{\Delta A}{A_0}$ and $\theta_0 := \dfrac{\alpha_0}{q_0} \dfrac{R}{R - 1} > 1$.

After the shock, the farmer's net worth changes for two reasons: the direct effect of the rate of growth of money supply $-\left(\theta_0 - 1\right) \hat{A}$ and the *indirect effect* of the rate of growth of money supply on net worth *through asset prices* $\hat{q}_t \dfrac{R}{R - 1}$. This indirect effect is a multiple $\dfrac{R}{R - 1}$ of the asset price increase. The direct effect of the rate of growth of money supply is negative.

Equating (41) and (48) we get:

$$\left(\varepsilon_0 + 1\right) \hat{K}_t^F = -\left(\theta_0 - 1\right) \hat{A} + \hat{q}_t \frac{R}{R - 1} \tag{49}$$

Solving (49) and (45) for $\hat{q}_t$ and $\hat{K}_t^F$ yields:

$$\hat{q}_t = -\left(\theta_0 - 1\right) \hat{A}$$

$$\hat{K}_t^F = -\left(\theta_0 - 1\right) \frac{R\left(\varepsilon_0 + 1\right) - 1}{\left(R - 1\right)\left(\varepsilon_0 + 1\right)} \hat{A}$$

An increase in the rate of change of money supply, has negative effects both on the rate of change of the farmer's landholding and the rate of change of the asset price. The rate of change of net worth in the end is $\hat{n}_{AS}^F = -\left(\theta_0 - 1\right) \hat{A} \dfrac{2R - 1}{R - 1}$.

8 Conclusions

In this paper we have developed a model of a monetary economy with financing constraints. We borrow some of the basic ingredients of Kiyotaki and Moore's financial accelerator framework in order to keep the appealing feature of the intertwined dynamics of asset price changes and borrowing constraints. In order to evaluate the impact of monetary policy, however, we model a monetary economy with financing constraints adopting the Money In the Utility function (MIU) approach.

The basic difference with respect to the original framework is the likely occurrence of multiple equilibria. A change in the growth rate of money supply can affect real output through *the impact of inflation on net worth*. In a sense the

monetary transmission mechanism we are focusing on consists of a combination of the *inflation tax* effect and the *net worth channel.* Contrary to the traditional view, at least for some parameter configurations, an increase of the inflation tax can bring about an increase of aggregate output.

References

[1] Bernanke, B. and Gertler, M . (1989), "Agency Costs, Net Worth and Business Fluctuations", *American Economic Review*, vol. 79, pp. 14-31.

[2] Bernanke, B. and Gertler, M. (1990), "Financial Fragility and Economic Performance",*Quarterly Journal of Economics*, vol. 105, pp. 87-114.

[3] Bernanke, B., Gertler, M. and Gilchrist, S. (1999), "The Financial Accelerator in Quantitative Business Cycle Framework", in Taylor, J. e Woodford, M. (eds), *Handbook of Macroeconomics,* vol 1C, North Holland, Amsterdam.

[4] Cordoba, J. C., Ripoll, M. (2004a) "Collateral Constraints in a Monetary Economy" *Journal of the European Economic Association*, vol. 2, no. 6, pp. 1172-1205.

[5] Cordoba, J. C., Ripoll, M. (2004b) "Credit Cycles Redux" *International Economic Review* vol. 45, no. 4.

[6] Cooley, T. and Hansen, G. (1998) "The Role of Monetary Shocks in Equilibrium Business Cycle Theory: Three Examples," *European Economic Review* vol. 42, pp. 605-617.

[7] Iacoviello, M. (2005a) "House Prices, Borrowing Constraints and Monetary Policy in the Business Cycles" *The American Economic Review*, vol. 95, no. 3, pp. 739-764(26).

[8] Kiyotaki, N. (1998) "Credit and Business Cycles" *The Japanese Economic Review*, vol. 49, no. 1.

[9] Kiyotaki, N. and Moore, J. (1997), "Credit Cycles", *Journal of Political Economy*, vol. 105, pp.211-248.

[10] Kiyotaki, N. and Moore, J. (2002a), "Balance-Sheet Contagion," *American Economic Review*, vol. 92(2), pp. 46-50.

[11] Kiyotaki, N. and Moore, J. (2002b), "Evil is the Root of all Money", *American Economic Review*, vol. 92(2), pp. 62-66.

[12] Ljungqvist L. and Sargent T. (2000) Recursive Macroeconomic Theory, Cambridge (Mass.), MIT Press.