Microfoundations for Switching Behavior in Heterogeneous Agent Models: An Experiment*

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Abstract

We run a laboratory experiment to study how people switch between several profitable alternatives, framed as mutual funds, in order to provide a microfoundation for so-called heterogeneous agent models. The participants in our experiment have to choose repeatedly between two, three or four experimental funds. The time series of fund returns are exogenously generated prior to the experiment and participants are paid for each period according to the return of the fund they choose. For most cases participants' decisions can be successfully described by a discrete choice switching model, often applied in heterogeneous agent models, provided that a predisposition towards one of the funds is included. The estimated intensity of choice parameter of the discrete choice model depends on the structure of the fund returns. In particular, it increases with correlation between past and future returns. This suggests people do not myopically chase past returns, but are more likely to do so when past returns are more predictive of future returns, a feature that is absent in the standard heterogeneous agent models.

Keywords: Heterogeneous agent models, discrete choice, switching, experiments.

JEL Classification: C25, C91, D83.

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1 Introduction

The recent financial crisis has increased interest, both from academics and policy-makers, in agent-based models as a viable alternative to the classic rational choice model, in particular because these agent-based models have been relatively successful in describing bubbles and crashes and other stylized facts of financial markets, such as excess volatility, volatility clustering and fat tails, see Lux (2009).

One important class of agent-based models is that of so-called *heterogeneous agent models*.[1] These models assume that there is a large population of traders, with each trader behaving according to one particular trading or forecasting heuristic from a small set of available heuristics (for most applications the number of considered heuristics is only two or three). Past performance of these heuristics then determines the fraction of the population of traders that uses each of the heuristics. This choice process is typically modeled by the so-called *discrete choice model*. Since both the evolution of asset prices and the performance of the heuristics depend upon the distribution of traders over the heuristics, heterogeneous agent models give rise to low-dimensional, but highly nonlinear, dynamical systems. The interaction between heuristics typically features complex erratic dynamics, in particular when traders are sensitive to performance differentials between heuristics, and may show a striking resemblance with price dynamics observed on actual financial markets. Because heterogeneous agent models only involve a limited number of variables, describing the price and the distribution of traders over heuristics, their properties can still be studied analytically and, in comparison to other, large-scale, agent-based models, the results are typically easier to interpret.

Although heterogeneous agent models have been quite successful in explaining stylized facts of financial markets, they also exhibit an important drawback. The results obtained crucially depend both upon the set of heuristics considered, and on the way the choice between the alternative heuristics is modeled. Indeed, there are many degrees of freedom and ideally heterogeneous agent modeling is disciplined by empirical evidence on which heuristics are used by human decision makers, and how human decision makers switch between those heuristics.

Extensive research on so-called 'Learning-to-Forecast' laboratory experiments already provides substantial insight into the type of forecasting heuristics that are used by human subjects.[2] In the current paper we present results from a laboratory experiment that was designed to understand how human decision makers choose between different alternatives, and therefore complements the earlier Learning-to-Forecast experiments. By focusing on the question of *how* people use the past performance in switching, we hope to provide further microfoundations for

[1] Examples include Brock and Hommes (1998), Brock, Hommes, and Wagener (2005), Chiarella, Dieci, and He (2007), Manzan and Westerhoff (2007), Anufriev and Dindo (2010), Gaunersdorfer, Hommes, and Wagener (2008), Goldbaum and Panchenko (2010), Anufriev and Tuinstra (2013). Hommes (2006, 2013) and LeBaron (2006) review the literature.

[2] See Hommes, Sonnemans, Tuinstra, and van de Velden (2005) and Heemeijer, Hommes, Sonnemans, and Tuinstra (2009). The review in Hommes (2011) provides additional references.

the growing literature on heterogeneous agent models.

In particular, we are interested in finding a specification of the discrete choice model that is able to explain participants' decisions satisfactorily. In addition we will try to infer the appropriate value of the so-called *Intensity of Choice* (IoC) parameter of the discrete choice model. This parameter measures how responsive traders are with respect to differences in past performance of the heuristics and it plays a pivotal role in the dynamic properties of heterogeneous agent models. In particular, low values of the IoC are associated with stable dynamics, whereas high values typically lead to complicated phenomena and endogenous fluctuations in stock prices, volumes and trading positions. Learning the relevant range of values of the IoC is therefore crucial for a proper application of heterogeneous agent models. However, significant estimates of this parameter are difficult to obtain from empirical studies because, due to a lack of direct access to the strategies used, switching behavior can only be inferred indirectly.

Our laboratory experiment is designed such that it facilitates estimating the parameters of the discrete choice models. The advantage of running a laboratory experiment is that it provides a controlled environment where we can observe the choices of the participants directly. We study the switching behavior of human subjects between a small number of alternatives. The participants observe the past performance (framed as "financial returns") of several (either two, three or four) investment alternatives ("funds") and, in every period, are asked to choose one of the alternatives and are paid on the basis of the performance of the chosen alternative in that period. As in actual financial markets, participants do not know the data generating process of the returns, and their choice does not influence the return of the different alternatives.[3] We use different data generating processes that result in differences in the autocorrelation structure (and therefore predictability) of the generated time series. Participants have to choose between funds for 40 consecutive periods for one set of funds, and then for another 40 periods for a different set of the funds. This allows us to study the effect of experience on choice behavior.

We find that participants often switch between funds. Given the information provided in the experiment, it is not surprising that switching is, to a large extent, driven by past performance of the funds. Answering the question of how exactly switching is driven by past performances, we find that a simple discrete choice model with a *predisposition effect* provides a good fit to the data, when there are two or three funds and there is no cyclical/periodic pattern in the time series of returns. If such a cyclical pattern does exist, more lags need to be incorporated in the model in order to provide a good description of the data. This suggests that people recognize and exploit cyclical patterns in returns. Moreover, the estimated IoC is not universal, but positively related to the degree of autocorrelation in the time series of returns. From this we conclude that people do not myopically chase past returns, but are more likely to do so when past returns are more predictive of future returns.

Choosing between a limited number of alternatives is a common phenomenon in many

[3] Anufriev, Bao, Sutan, and Tuinstra (2015) discuss a related experiment where subjects do know the data generating process.

aspects of life, including the decision about which route to take when commuting to the workplace, voting for political parties, and financial investment decisions. Thus our results have implications outside the realm of heterogeneous agent models. The framing of this experiment makes the results specifically connected to the literature on mutual fund choice. Many empirical studies in this field investigate the determinants of financial flows between mutual funds. Our experiment provides a stylized representation of the typical decision problem of the individual non-sophisticated investor,[4] although it abstracts from many relevant real-life features (e.g., service quality, advertising, brand effect, income effect, home country bias, etc.). There is substantial empirical evidence that suggests that the flows in and out of mutual funds are strongly driven by the recent past performance of these funds, despite the fact that the funds which performed well in the past do not necessarily generate an above-average return.[5] Moreover, Choi, Laibson, Madrian, and Metrick (2009) find that such "return chasing behavior" also provides a good description of investment behavior in the 401k account by American households. Our experiment thus focuses on, arguably, the most important variable for the investment decision, the funds' past performance, and directly addresses the question of *how* past performance affects that choice.

The remainder of the paper is organized as follows. Section 2 presents the discrete choice framework and discusses some related literature. Section 3 introduces the experimental design and Section 4 provides a first discussion of the experimental results. In Section 5 we estimate the discrete choice model from the experimental data. Section 6 concludes. The Appendix contains the experimental instructions and some estimation results.

2 Switching and the Discrete Choice Framework

Although not the first to explain market fluctuations by the interaction of different heuristics, arguably the two seminal papers in the field of heterogeneous agent models are Brock and Hommes (1997) and Brock and Hommes (1998).[6] Brock and Hommes (1997) consider the classic cobweb model where suppliers can either form price expectations rationally (against positive information costs) or employ (cost free) naive expectations, i.e., predicting that the price will equal the last observed realized price. Total supply, and therefore the market clearing price, will then depend upon a weighted average of the rational and naive forecasts. Modeling the evolution of the fraction of the population of firms using rational expectations by the discrete choice dynamics, using forecasting accuracy as the performance measure, the authors show that

[4] In fact, it is very common for professional financial websites (e.g., Morningstar, Yahoo Finance) to provide information on past performance of all mutual funds, and recommend or rate funds based upon this past performance.

[5] See Patel, Zeckhauser, and Hendrick (1994), Sirri and Tufano (1998), Zheng (1999), Jain and Wu (2000), Ferreira, Keswani, Miguel, and Ramos (2012) among others.

[6] According to Web of Science (Google Scholar) together the two papers have been cited more than 950 (2500) times by August 14, 2015. Important earlier influential contributions include Day and Huang (1990), Chiarella (1992) and Lux (1995).

if the Intensity of Choice (IoC) is large enough, the dynamics converges to a strange attractor with endogenous fluctuations in the market clearing price. Moreover, the naive expectations heuristic survives the competition with the more sophisticated (but costly) rational expectations heuristic.

The same notion of (evolutionary) competition between different heuristics was applied to financial markets in the follow-up paper, Brock and Hommes (1998), where traders can choose between different forecasting strategies representing, for example, fundamentalist traders, chartists and noise traders. Again, if the IoC parameter is large enough, the asset price dynamics exhibit irregular switches between periods of prolonged deviations from the fundamental value followed by a correction in the direction of that fundamental value. This model of endogenous financial market bubbles and crashes has given rise to a huge stream of theoretical, computational and empirical studies on endogenous dynamics in financial markets. The key mechanism in these models, responsible for the non-linearities leading to excess volatility, bubble and crashes, etc., is the discrete choice model which describes the evolution of the fractions of traders using the different heuristics by the multinomial logit model.[7] In its most basic version the discrete choice model assumes that the fraction n_t^h of the population of decision makers using heuristic h in period t is given by

$$n_t^h = \frac{\exp\left[\beta \pi_{h,t-1}\right]}{\sum_{k=1}^{H} \exp\left[\beta \pi_{k,t-1}\right]}. \tag{1}$$

Here $\pi_{h,t-1}$ is the payoff or profit that was generated by heuristic h in period $t-1$ and $\beta > 0$ is the *Intensity of Choice* parameter that was already discussed above. If this parameter is small (e.g., close to zero) the population of decision makers is spread out almost evenly over the different heuristics, whereas a very high value of β implies that decision makers are sensitive to differences in payoffs and the heuristic with the highest payoff attracts the majority of the population of decision makers.

In this paper we will argue on the basis of experimental evidence that the following, more general, version of (1), can be a more realistic choice in modeling the evolution of fractions:

$$n_t^h = \frac{\exp\left[\alpha_h + \beta_{h,1}\pi_{h,t-1} + \ldots + \beta_{h,L}\pi_{h,t-L}\right]}{\sum_{k=1}^{H} \exp\left[\alpha_k + \beta_{k,1}\pi_{k,t-1} + \ldots + \beta_{k,L}\pi_{k,t-L}\right]}. \tag{2}$$

This specification generalizes (1) in three directions. First, it allows for up to L lags of the payoffs to play a role in the determination of the fraction choosing heuristic h. Second, it does not require that decision makers respond to payoffs of the different heuristics in the same manner. That is, we can have $\beta_{k,\ell} \neq \beta_{h,\ell}$ for some lag ℓ and some heuristics h and k with $k \neq h$. Third, it allows for a so-called *predisposition effect*: if $\alpha_h > \alpha_k$ for some h and all $k \neq h$, then traders are, on aggregate, biased towards choosing heuristic h over the other heuristics, even when there is no difference in the past performance of these heuristics.[8]

[7] The discrete choice model has its origin in econometrics, see Manski and McFadden (1981) for an overview and a discussion of the underlying assumptions.

[8] Other generalizations are possible as well, see, e.g., Anufriev and Hommes (2012b) who consider a discrete

A vast majority of the heterogeneous agent models uses equation (1) to model the fraction of the population of traders that selects heuristic h in period t. As discussed above, the dynamic properties of these models depend critically upon the choice of the IoC parameter β and therefore several researchers have attempted to estimate β. We will briefly discuss some of these contributions here.

The usual approach to estimate the intensity of choice parameter is to use market data or survey data. Branch (2004) uses survey data on inflation expectations from Michigan households to estimate the discrete choice model with three forecasting heuristics: naive expectations, adaptive expectations and a more sophisticated multivariate forecasting procedure, respectively. His estimated model gives a good description of the survey data, with the qualification that many individuals exhibit a predisposition effect: a bias toward one of the heuristics. This effect is captured by the α parameters in equation (2). Boswijk, Hommes, and Manzan (2007) estimate model (1) and the parameters of two heuristics on yearly S&P500 data from 1871–2003. The two estimated heuristics correspond to fundamentalist and trend-following behavior, respectively, and the estimated model can explain the price bubble from the late nineties. The estimated IoC is not statistically significant. Empirical evidence of switching (i.e., an estimated IoC significantly higher than zero) between three heuristics on the interlinked stock markets of Hong Kong and Thailand is presented in De Jong, Verschoor, and Zwinkels (2009). An alternative heterogeneous agent model, with a different market clearing mechanism and a different trend-following heuristic, is estimated on monthly S&P500 data from 1970–2012 by Chiarella, He, and Zwinkels (2014). They also find evidence for switching between fundamentalism and trend-following behavior. Goldbaum and Mizrach (2008) make a distinction between passively and actively managed mutual funds and estimate the IoC parameter from data on the flows of money into and out of these mutual funds. Finally, Franke and Westerhoff (2012) propose a *method of simulated moments* approach to compare how well different types of agent-based models perform in describing stylized facts of financial markets[9] and find that inclusion of switching based upon the discrete choice mechanism (1) improves data fitting.

One drawback of all of this empirical work is that neither the specific heuristics nor actual switches from one heuristic to another are observed, and therefore have to be inferred from the data. Recently, Anufriev and Hommes (2012a,b) calibrated a so-called '*Heuristic Switching Model*', which is similar to the multinomial logit model given by (1), on the experimental data obtained from 'Learning-to-Forecast' experiments. The model explains the types of dynamics observed in the different Learning-to-Forecast experiments, i.e., quick convergence in some treatments and persistent deviations from fundamentals in other treatments. However, the experimental data also do not provide *explicit* evidence for switching. In our experiment, on

choice model with memory in the performance measure (corresponding to geometrically declining weights on lagged payoffs) and *asynchronous* updating.

[9]Some agent-based models explain the excess volatility and other stylized facts without modeling switching between alternative heuristics on the basis of past performance. To generate volatility these models either impose Markov regime switching of fundamental value as in Chiarella, He, Huang, and Zheng (2012), or model herding through imitation learning as in Alfarano, Lux, and Wagner (2008).

the other hand, the heuristics are given (and represented by different investment alternatives) and we observe precisely when participants switch from one alternative to another.

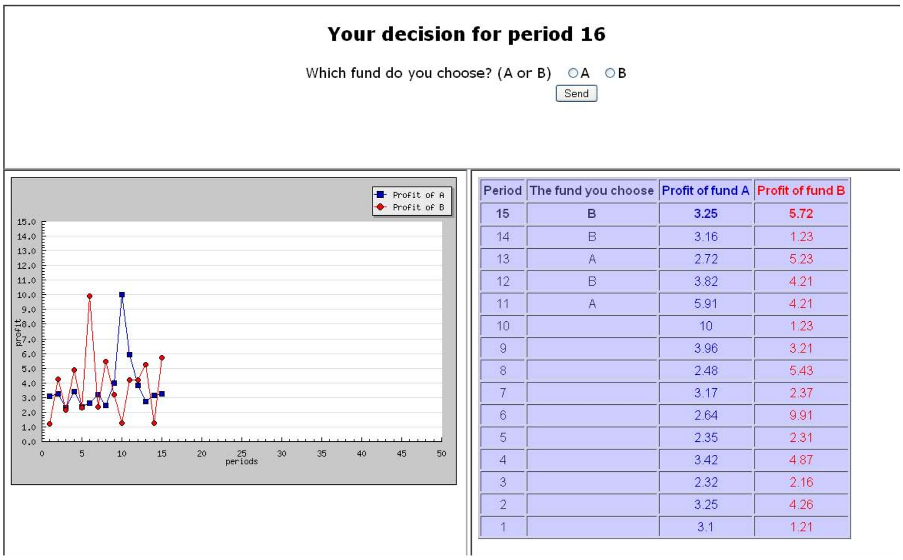
There exist other experimental studies that are somewhat related to our experiment. Anderson and Holt (1997), Drehmann, Oechssler, and Roider (2005) and Alevy, Haigh, and List (2007) study herding behavior in financial markets, and in particular so-called “information cascades”. In this setup participants choose between two assets, one of which generates a positive payoff whereas the other has zero payoff. Participants receive a public signal as well as a private signal about which asset will generate a positive payoff. Here the public signal is the history of choices previously made by other participants facing the same decision problem. An information cascade sets in when participants base their decision purely on the public signal and disregard their private signal, even when the latter suggests a different decision. Interestingly, Anderson and Holt (1997) also estimate a discrete choice model on the probability of choosing a particular asset and report the estimated IoC parameter, which typically lies between 2 and 8. While in their experiment different participants play in different periods and the payoff of the assets are more or less repeated (each asset has equal probability to generate positive and zero payoff), in our experiment the same participant makes decisions for all periods, but the payoffs of the assets are time varying.

Our work is also related to an experimental study on portfolio choice by Bossaerts, Plott, and Zame (2007). They investigate how the fraction of wealth that participants put in a particular financial asset is related to the past performance of that asset. The difference with our experiment is that subjects in their experiment choose the investment fraction of wealth, which is a continuous decision variable, while in our experiment the subjects make discrete choices.

The common denominator of all of the literature referenced above is that almost none of these studies have attempted to estimate the IoC parameter from the choices participants make directly. The aim of the current paper is to fill that void.

3 Experimental Design

The experiment was conducted on June 17-18, 2010, at the CREED laboratory of the University of Amsterdam. A total of 91 students participated in four sessions comprising the six different treatments explained in Section 3.3 below. None of the students participated in more than one session. Each session lasted for about one hour and a half, and the participants' payoffs varied between 18 and 25 euros.



Period	The fund you choose	Profit of fund A	Profit of fund B
15	B	3.25	5.72
14	B	3.16	1.23
13	A	2.72	5.23
12	B	3.82	4.21
11	A	5.91	4.21
10		10	1.23
9		3.96	3.21
8		2.48	5.43
7		3.17	2.37
6		2.64	9.91
5		2.35	2.31
4		3.42	4.87
3		2.32	2.16
2		3.25	4.26
1		3.1	1.21

Figure 1: An example of the experimental screen. Participants submit their decisions by selecting the radio buttons in the upper part of the screen.

3.1 Participants' Task

The experiment is an individual choice experiment where participants have to make an investment decision repeatedly. The participants observe the time series of past returns of either two, three or four investment alternatives, represented as 'funds' A, B, C and D, in two blocks of 50 periods. The type and number of investment alternatives are different in the two blocks. Each block starts with a history of 10 periods shown to the participant, so that (s)he can get acquainted with the investment alternatives. For each of the remaining 40 periods in that block the task of the participant is to choose one of the investment alternatives. After the choice is made the realized returns for that period of all of the available investment alternatives are revealed.

The payoff for a participant in any particular period is equal to the return of the alternative (s)he chooses in that period. This return ranges between 0 points and 16 points, and the total number of points earned during the experiment is translated to euros, with the number of euros given by the number of points earned during the experiment, divided by 20. In addition, participants receive a 'show-up' fee of 5 euro.

Fig. 1 shows an example of the computer screen the participants face during the experiment.[10] A participant sees the past returns of all the funds on a graph as well as in a table, where also the own past choices are displayed. The participant has to choose which fund to invest in, using the radio buttons in the top part of the screen. After the participant has chosen the fund, the computer screen is updated with the new information.

[10]The time series shown in the screen of the figure have not been used in the experiment. This picture was printed on the paper shown to the participants during the instruction stage of the experiment, to explain the different parts of the screen.

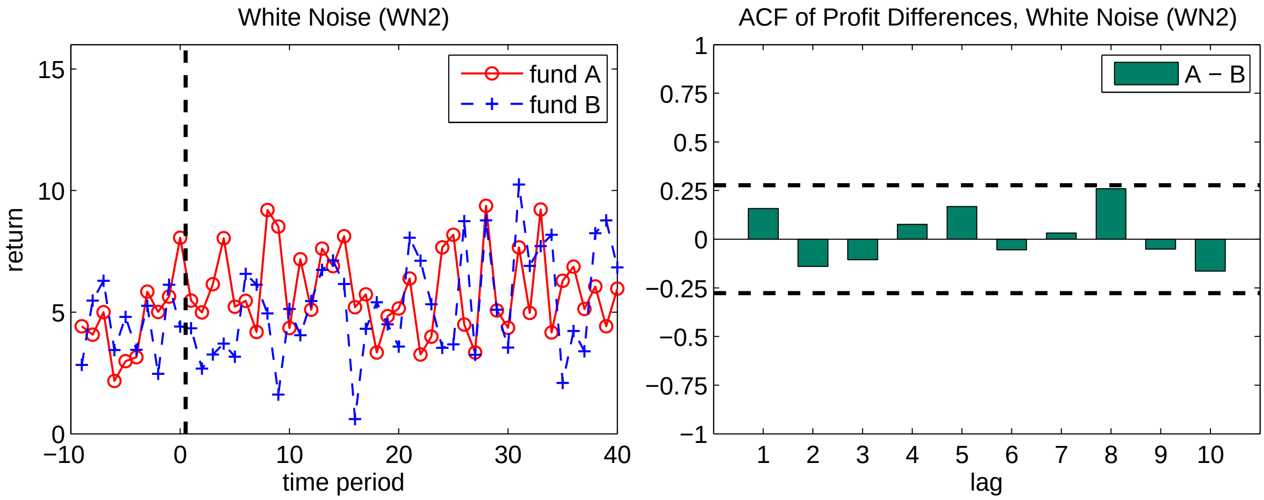


Figure 2: The return time series generated by the white noise model, WN2 (*left panel*) and the autocorrelation of the difference between the returns (*right panel*).

The instructions are provided in Appendix A. Note that the participants did not know the data generating process that underlies the time series they observe, but they knew that a different data generating process is used in the second block of 50 periods.

3.2 Time Series used in the Experiment

Prior to the experiment we prepared seven sets of time series of 50 periods, representing returns (or payoffs) of the different investment alternatives. Letting t denote the period, each time series is defined for $t = -9, \ldots, 40$. The first 10 realizations, corresponding to $t \leq 0$, are shown to the participants at the start of the relevant block. The remaining observations are shown sequentially as decisions are being made, such that for each time period t, $t = 1, \ldots, 40$, in which participants have to make an individual investment decision, they can observe return realizations from periods $\tau = -9, \ldots, t-1$, for all investment alternatives in the relevant set.

The time series were generated using the *data generating processes* (DGPs) discussed below and then grouped in sets of two, three or four time series. The sets used in different blocks of the experiment are illustrated in the left panels of Figs. 2, 3 and 4. At the start of every block the participants could see the part of the time series to the left of the vertical dotted line.[11] The sets of time series can be characterized by their autocorrelation properties. Since we conjecture that for a participant's decisions the *differences* between returns of different alternatives are more important than the absolute value of these returns, we specifically look at the autocorrelation structure of return differences. Taking alternative A as a benchmark, we calculate the return difference as the return of A minus the return of one of the other alternatives. The autocorrelation function is plotted until the 10th lag. The dashed lines

[11]The line was not shown in the experiment.

represent 95% confidence bands.

1. White Noise time series. The first set of time series, denoted WN2, are generated by the following DGP:

$$\pi_{i,t} = 5 + \varepsilon_{i,t}, \quad i = A, B \quad \text{and} \quad t = -9, \ldots, 40\,, \tag{3}$$

where the $\varepsilon_{i,t}$ are independently and identically distributed, with $\varepsilon_{i,t} \sim N(0,1)$. The time series used in the experiment are shown in Fig. 2 together with the autocorrelations of the difference $\pi_{A,t} - \pi_{B,t}$. These two time series are independent from each other and serially uncorrelated.

2. Brock-Hommes time series. Three sets of time series, labeled BH2, BH3 and BH4, are generated from numerical simulations of the two canonical heterogeneous agent models of Brock and Hommes (1997, 1998) described in the previous section. We normalize the generated time series such that returns are always in the range $(0, 16)$, with a mean of around 5.

As explained in the previous section, Brock and Hommes (1997) introduce switching between heterogeneous expectations in the cobweb model. In particular, they study the dynamics of the model with two types of expectations, costly rational expectations and costless naive expectations. Our set BH2 contains the two time series of the realized returns associated with these two forecasting heuristics, for a particular parameter setting. The participants to the experiment experience this as a decision problem where there are two funds available for investment: fund A, which is run by a sophisticated manager that has rational expectations and charges a higher fee, and fund B, which is run by a manager using the naive expectations heuristic. The participant, however, does not know the strategies of the managers (and also does not know that there is a difference in sophistication between the fund managers), but only observes past returns (which also include the fee).

To generate the returns we simulate a version of the Brock and Hommes (1997) model.[12] The resulting set of time series is shown in Fig. 3 (upper left panel). Due to the nonlinear nature of the model, the simulated time series exhibit a quasi-cyclic pattern, with fund A performing slightly worse than fund B most of the time but strongly outperforming fund B at certain periods.[13] The autocorrelation function shown in the right upper panel of Fig. 3 reflects

[12]We used the specification of the Heuristic Switching Model from Anufriev and Hommes (2012b) with intensity of choice parameter $\beta = 3.9$ and other parameters equal to $\eta = 0.5$ and $\delta = 0.2$. The demand, supply and cost parameters (see Brock and Hommes, 1997, p. 1066) are given by $A = 0$, $B = 0.5$, $b = 1.35$ and $C = 1$. To normalize we multiply both time series of profits by 4 and subsequently add 7 to obtain the returns used in the experiment. Note that in the Brock-Hommes model the distribution of the population over the different heuristics is one of the determinants of prices and profits. In the experiment, however, choices of the participants have no effect on the actual payoffs. We leave the experimental study of choices between heuristics, where these choices feed back into the determination of profits, for future research.

[13]For the chosen parameters the model generates a repeating pattern of price fluctuations around the equilibrium, with a constantly growing amplitude, followed by a strong price correction. When the fluctuations are small, the naive forecasting heuristic is more profitable than the precise but expensive rational expectations

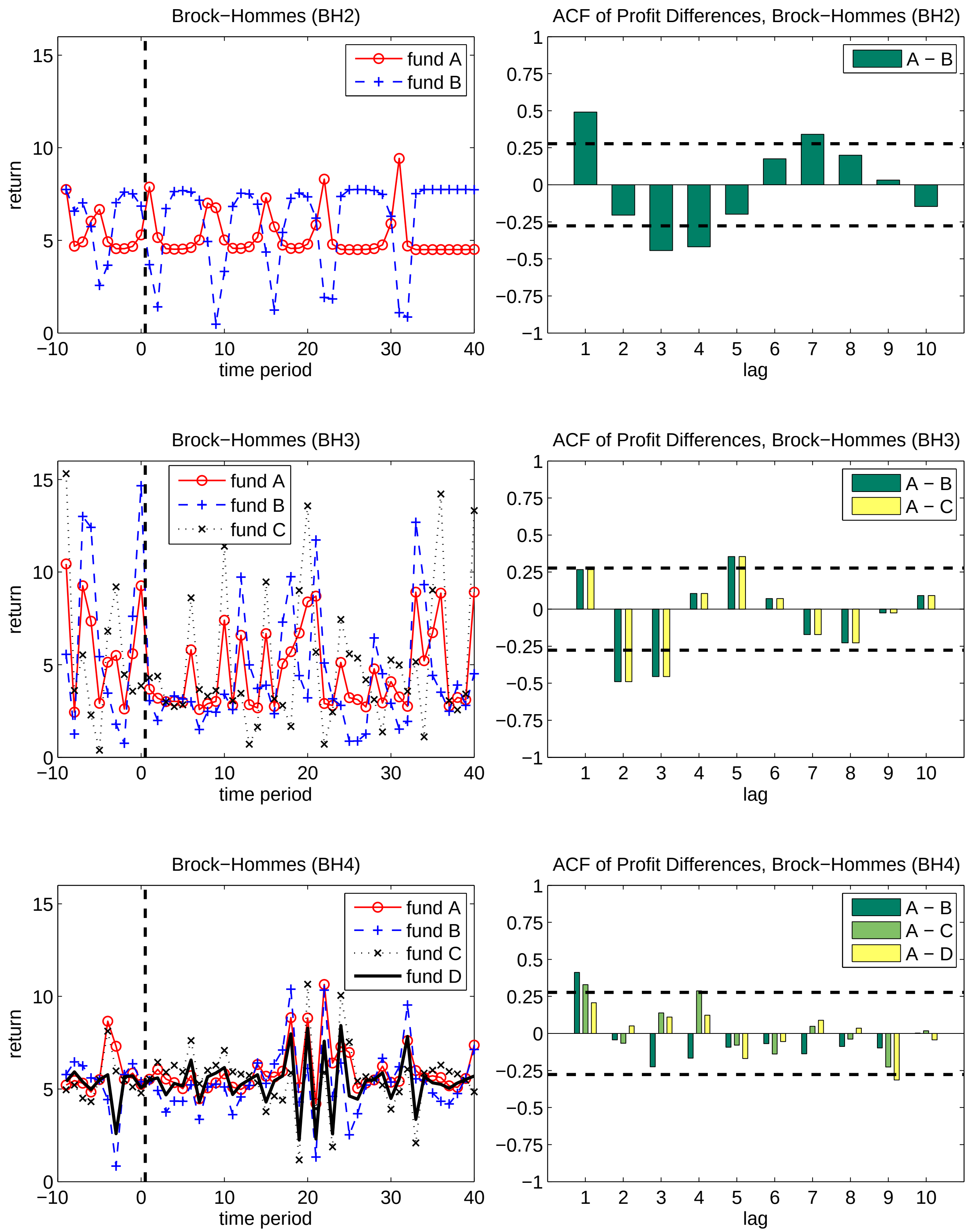


Figure 3: The return time series generated by the Brock-Hommes models (*left panels*) and the autocorrelations of the differences between the returns (*right panels*).

this pattern with significant autocorrelations at lags 1, 3, 4 and 7. Despite the quasi-periodic pattern, the time series are chaotic. In other words, for the participants it is hard to predict the precise moment when fund A performs better than fund B.

The next two sets of time series, BH3 and BH4, are based on Brock and Hommes (1998), who apply the idea of evolutionary switching between heterogeneous forecasting heuristics to a financial market model and study the dynamics for several versions of their model. The three time series in BH3 are generated by the model with three forecasting heuristics, where one of the heuristics corresponds to fundamentalist behavior, and the two others represent biased predictors (optimists and pessimists, respectively, see Brock and Hommes, 1998, p. 1258).[14] The three resulting returns, $\pi_{A,t}$, $\pi_{B,t}$, and $\pi_{C,t}$ are shown in Fig. 3 (middle left panel). The returns of fund A are those generated by the fundamental forecast, whose performance is always in between the performances of the two other funds. Funds B and C are symmetric with respect to A, as their (opposite) biases are the same in absolute value. From time to time a different fund performs best. Fig. 3 (middle right panel) shows the autocorrelations of $\pi_{A,t} - \pi_{B,t}$ and $\pi_{A,t} - \pi_{C,t}$, which are identical due to symmetry in the model. There is no apparent structure in the returns.[15]

The four time series in BH4 are also generated by the asset pricing model from Brock and Hommes (1998) but with four heuristics, representing fundamentalists, trend chasers with an upward bias, trend chasers with a downward bias and strong trend chasers with no bias, respectively (see Brock and Hommes, 1998, pp. $1261-63$).[16] The time series in BH4 are shown in the left bottom panel of Fig. 3. Also for this set of time series the autocorrelation structure in differences is not too strong, although it is significant for the first lag (for $\pi_{A.t} - \pi_{B,t}$ and $\pi_{A,t} - \pi_{C,t}$).

3. Stock Index time series. These sets, labeled SI2, SI3 and SI4, are constructed from four actual stock indices, the *Austrian Trade Index* (ATX), the *Belgium 20 Stock Index* (BFX), the *Dow Jones Index* (DJI) and the *FTSE 100 index* (FTSE), from October 2005 to November

heuristic. The agents in the model gradually switch from rational to the naive heuristic, which destabilizes the cobweb dynamics up to a point that fluctuations become so pronounced that the naive heuristic clearly performs worse than the rational heuristic. This makes agents switch back to the latter, which stabilizes the cobweb dynamics again, and so on. In the experiment we do not show the prices and forecasts, nor do we use non-neutral names for the different funds. Instead, we only show implied returns of the two heuristics – framed as funds – in order to induce our participants to be motivated by the same forces as the agents in the original theoretical Brock and Hommes (1997) model.

[14]The time series were generated by simulating the model with the intensity of choice parameter $\beta = 450$ and the other parameters given by $g_1 = g_2 = g_3 = 0$, $b_1 = 0$, $b_2 = -b_3 = 0.1$ and $R = 1.1$. This gives rise to quasi-cyclical dynamics. In order to inhibit predictability slightly we added an IID shock drawn from $N(0, 0.01)$. Finally, we normalized the resulting returns by multiplying by 75 and adding 3.

[15]At the 95% confidence level the autocorrelation in differences is significant only at lags 2, 3 and 5 (but not at the first lag).

[16]The time series were generated by simulating the model with the intensity of choice parameter $\beta = 95$ and the other parameters given by $b_1 = b_4 = 0$, $b_2 = -b_3 = -0.3$, $g_1 = 0$, $g_2 = 1.1$, $g_3 = 0.9$, $g_4 = 1.21$, $C = 0$ and $R = 1.1$. We added small noise drawn from $N(0, 0.0144)$. Finally, we normalized payoffs by multiplying them by 60 and adding 5.5.

Time series	Fund	Min	Mean	Max	Variance
WN2	A	2.17	5.68	9.37	3.13
	B	0.60	5.16	10.25	4.48
	All	0.60	5.42	10.25	3.83
BH2	A	4.50	5.24	9.42	1.38
	B	0.47	6.01	7.75	5.42
	All	0.47	5.63	9.42	3.52
BH3	A	2.43	4.85	10.44	5.59
	B	0.76	4.64	14.66	12.34
	C	0.39	5.06	15.32	13.29
	All	0.39	4.85	15.32	10.29
BH4	A	4.20	5.93	10.64	1.52
	B	0.84	5.35	10.39	3.05
	C	1.17	5.56	10.66	2.62
	D	2.24	5.36	8.42	1.72
	All	0.84	5.55	10.66	2.25
SI2	A	0.36	5.62	10.72	9.42
	B	0.93	5.18	7.89	5.73
	All	0.36	5.40	10.72	7.55
SI3	A	0.36	5.62	10.72	9.42
	B	0.93	5.18	7.89	5.73
	C	1.96	5.31	7.34	2.44
	All	0.36	5.37	10.72	5.82
SI4	A	0.36	5.62	10.72	9.42
	B	0.93	5.18	7.89	5.73
	C	1.96	5.31	7.34	2.44
	D	2.59	5.44	7.62	2.23
	All	0.36	5.39	10.72	4.91

Table 1: Descriptive statistics of the returns time series used in the experiment.

2009. We use rotation based yearly returns, instead of monthly returns. We normalize the data by dividing the returns by 12 and adding 5.[17] We use the ATX and BFX indices for SI2, the ATX, BFX and DJI indices for SI3, and the ATX, BFX, DJI and FTSE for SI4. These time series are shown in the left panel of Fig. 4. All of these time series are highly correlated as can be seen from the right panels of Fig. 4. Autocorrelations of the difference between funds A and B, between funds A and C and between funds A and D are significant up to lag 4, lag 6 and lag 7, respectively.

The descriptive statistics of all time series (grouped by sets) is provided in Table 1. We normalized the time series from the BH and SI sets to ensure that the maximum, minimum and average returns are similar across time series. This is also necessary in order to be able to compare the estimated intensity of choice parameters across different sets of time series.[18]

[17]Due to using rotation based returns, these time series do not look like the typical monthly returns of mutual funds. We label them here as "stock index" only due to the source of the data. The main purpose of using these time series is to study switching behavior when returns are highly autocorrelated. The results from the white noise time series may have more relevance for the behavior of investors in actual markets for mutual funds.

[18]The intensity of choice in (1) is not a scale-free parameter: multiplying all payoffs, $\pi_{k,t-1}$ for all k, by a constant is equivalent with dividing the estimated intensity of choice parameter by that same constant.

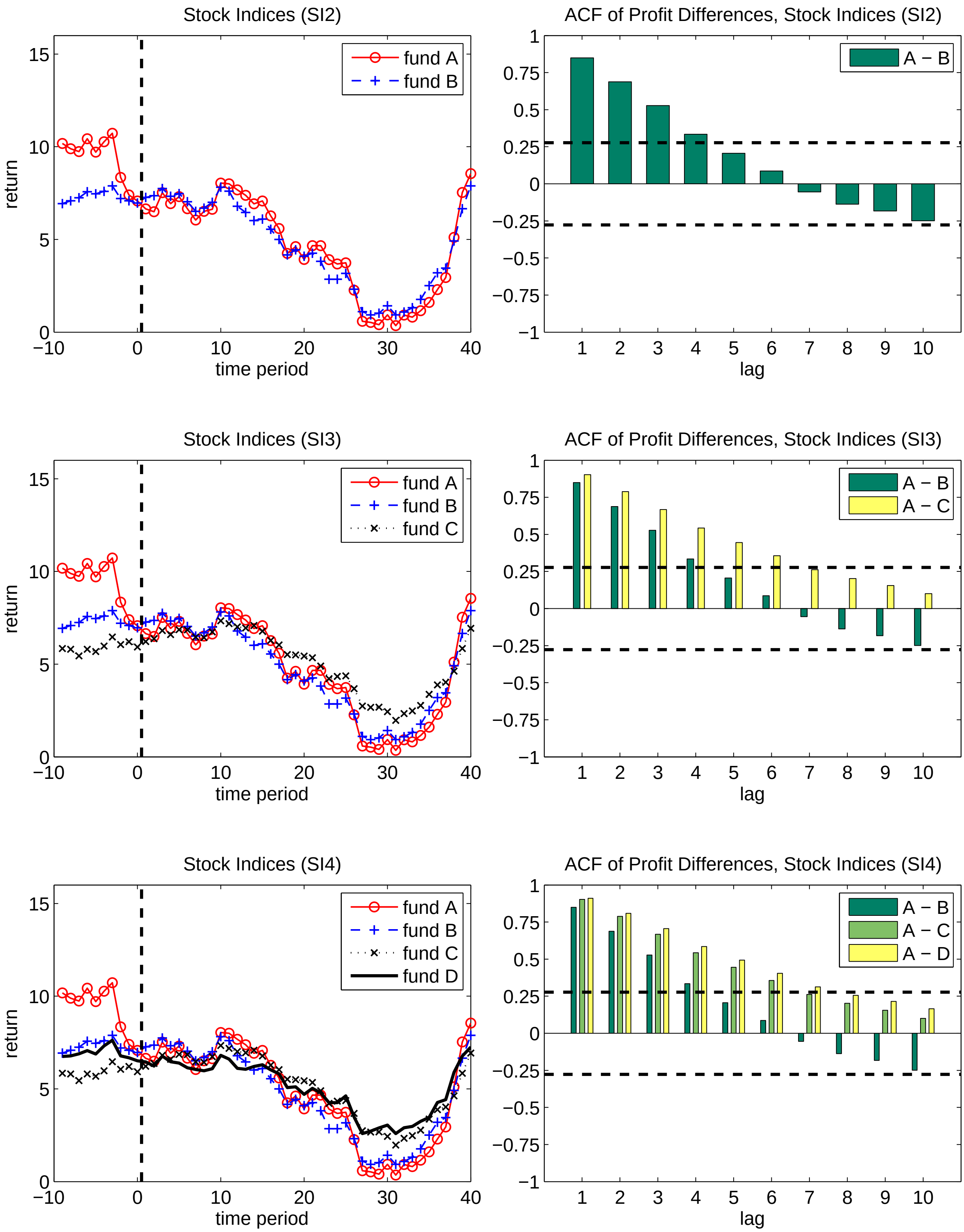


Figure 4: The return time series of the stock indices (*left panels*) and the autocorrelations of the differences between the returns (*right panels*).

Sets of time-series	Treatments	Number of participants		
		First Block	Second Block	Total
WN2	T1 and T6	16	13	29
BH2	T2 and T5	15	14	29
BH3	T3 and T1	14	16	30
BH4	T4	-	14	14
SI2	T4 and T3	14	14	28
SI3	T5	14	-	14
SI4	T6 and T2	13	15	28

Table 2: Structure of the treatments and blocks.

Time series from the different sets differ substantially in their autocorrelation structure. The time series from set WN2 exhibit almost no autocorrelation or periodic pattern. The time series from sets BH2 and BH3 exhibit a strong quasi-periodic pattern. Set BH4 has a high autocorrelation only for the first lag. Finally, the time series in sets SI2, SI3 and SI4 exhibit very strong autocorrelations.

3.3 Treatments

After generating the seven sets of time series, we constructed six treatments. Each treatment consists of two blocks, and in each block a participant is confronted with one of the possible seven sets of time series. Of the $7 \times 6 = 42$ possible combinations of two different sets, we selected six, under the restrictions that the two blocks in a treatment had to consist of sets with both a different number (2, 3 or 4) and a different type (WN, BH or SI) of investment alternatives. Moreover, in order to test the effect of experience with another set of time series, we organized the treatments in such a way that all the sets (except BH4 and SI3) appear exactly once in the first and once in the second block. The participants who have a given set of time series in their first block are denoted "inexperienced" (with this set) and those who have the set in their second block as "experienced".

The six resulting treatments T1 – T6 have the following sequences of sets of time series: WN2 $\rightsquigarrow$ BH3 (T1), BH2 $\rightsquigarrow$ SI4 (T2), BH3 $\rightsquigarrow$ SI2 (T3), BH4 $\rightsquigarrow$ SI2 (T4), SI3 $\rightsquigarrow$ BH2 (T5) and SI4 $\rightsquigarrow$ WN2 (T6). Table 2 is organized by the sets of the time series and shows the design and the number of participants for each of the time series.

3.4 Hypotheses

In the remainder of the paper we will report and analyze the experimental results. The main goal of the analysis is to estimate a discrete choice model, similar to (1), that gives a good description of the aggregate experimental data.

Discrete choice models, as they are used in heterogeneous agent models, are backward looking models, i.e., it is assumed that past returns determine the fractions of the population choosing the different alternatives. This basic assumption amounts to the following hypothesis.

Hypothesis 1. *Estimating model* (1) *or* (2) *on aggregate data gives estimates of* β *or* $\beta_{h,1}$ *(for all* h*) that are positive and significantly different from* 0 *for all sets of time series.*

Confirmation of Hypothesis 1 would suggest that participants exhibit return chasing behavior.

The discrete choice model (1) furthermore assumes that agents do not respond to patterns in past profits. The implication of this is that there should be no significant differences between the discrete choice models that are estimated from the experimental data on the different time series.

Hypothesis 2. *Experimental data from different sets of time series can be explained by exactly the same discrete choice model.*

If Hypothesis 2 is rejected, a universal discrete choice model – and therefore a universal value of the intensity of choice β – does not exist. The responsiveness of participants with respect to past payoff differences may then vary with the economic environment or with, for example, observable patterns in these past returns.

Model (1) assumes that the returns determine the current fractions, but returns from the more distant past do not play a role. Although longer memory can easily be incorporated in the discrete choice model, most heterogeneous agent models rule this out, typically for reasons of tractability. The following hypothesis can be used to test whether this restriction is appropriate.

Hypothesis 3. *A discrete choice model with more profit lags does not provide a substantially better description of the aggregate data than model* (1) *(or* (2) *with* $L = 1$*) does.*

If Hypothesis 3 is confirmed, a simple model with one lag will be a reasonable compromise between tractability and realism.

4 Experimental Results

In this section we present a first, qualitative, discussion of the experimental results, before estimating the discrete choice model in Section 5. In Section 4.1 we investigate the dynamics of individual choices and switches and in Section 4.2 we discuss how well participants performed in the experiment. Section 4.3 is concerned with the question whether the decisions of participants improve over time and finally, in Section 4.4 we investigate to what extent participants condition their decision on payoff information from the previous period.

4.1 Choices and Switches

There are two ways to look at the decisions that participants make over time in our experiment. On the one hand, using participants' choices between alternatives (for the same set of time series and with the same level of experience, i.e., in the same block), we can construct the evolution of the *share* of participants choosing a particular alternative by aggregating over all participants in that block. On the other hand, we can focus on those instances when a participant has *switched*, i.e., when a participant's choice in a given period differed from his or her choice in the previous period, and construct the empirical distribution of the *number of switches*. Note that a relatively stable share of participants choosing one alternative can be consistent both with a small and a large number of switches. Fig. 5 presents the shares and numbers of switches for all the sets of times series that are used in two separate blocks (that is, all sets of time series except BH4 and SI3, see Table 2). The left (middle) column contains the panels with the area charts plotting the evolution of the shares of inexperienced (experienced) subjects choosing a particular alternative. The right column contains the normalized histograms of number of switches for a particular set of time series (separately for inexperienced and experienced subjects).

There are substantial fluctuations in the shares over the course of the experiment, and clearly many participants switch between alternatives regularly during each block. To get a better idea of the frequency with which participants switch between alternatives we count the number of switches for each treatment. Because the number of participants varies with the treatments, we use the number of switches per participant per period as a measure to compare different blocks. The results are shown in Table 3. This normalized fraction of switches lies between 21% (SI2, Experienced) and 55% (BH3, Inexperienced).

A visual comparison between the left and middle panels of Fig. 5 suggests that the effect of experience is limited. Choice behavior of experienced participants seems to be quite similar, for a given set of time series, to that of inexperienced participants. Since by the design of the experiment the choices of individual participants are independent from each other for any given time period, the distribution of choices is multinomial. For every $t = 1, \ldots, 40$ we tested the hypothesis that the distributions for experienced and inexperienced are not statistically different. We could reject this hypothesis only for 5 of the 200 periods.[19]

The right panels of Fig. 5 suggest that the distributions of switches are also similar for inexperienced and experienced participants. This turns out to be correct as well as the Kolmogorov-Smirnov test of the hypothesis that the distribution of switches for inexperienced and experienced subjects are identical is only rejected for the SI2 blocks.[20] In addition, the frequency of switching presented in Table 3 is also quite similar for experienced and inexperienced partici-

[19]Using the Pearson's Chi-squared test we reject the null hypothesis, at the 5% significance level, only for $t = 19$ in WN2, for $t = 3$ in SI2, and for $t = 3$, $t = 15$ and $t = 21$ in BH3. We are never able to reject the null hypothesis in BH2 and SI4. The $p-$values for any time period are given in Appendix B.

[20]The Kolmogorov-Smirnov test statistics are 0.375 for WN2, 0.176 for BH2, 0.5 for SI2, 0.188 for BH3 and 0.185 for SI4. The corresponding $p-$values are 0.207, 0.962, 0.039, 0.930 and 0.952, respectively.

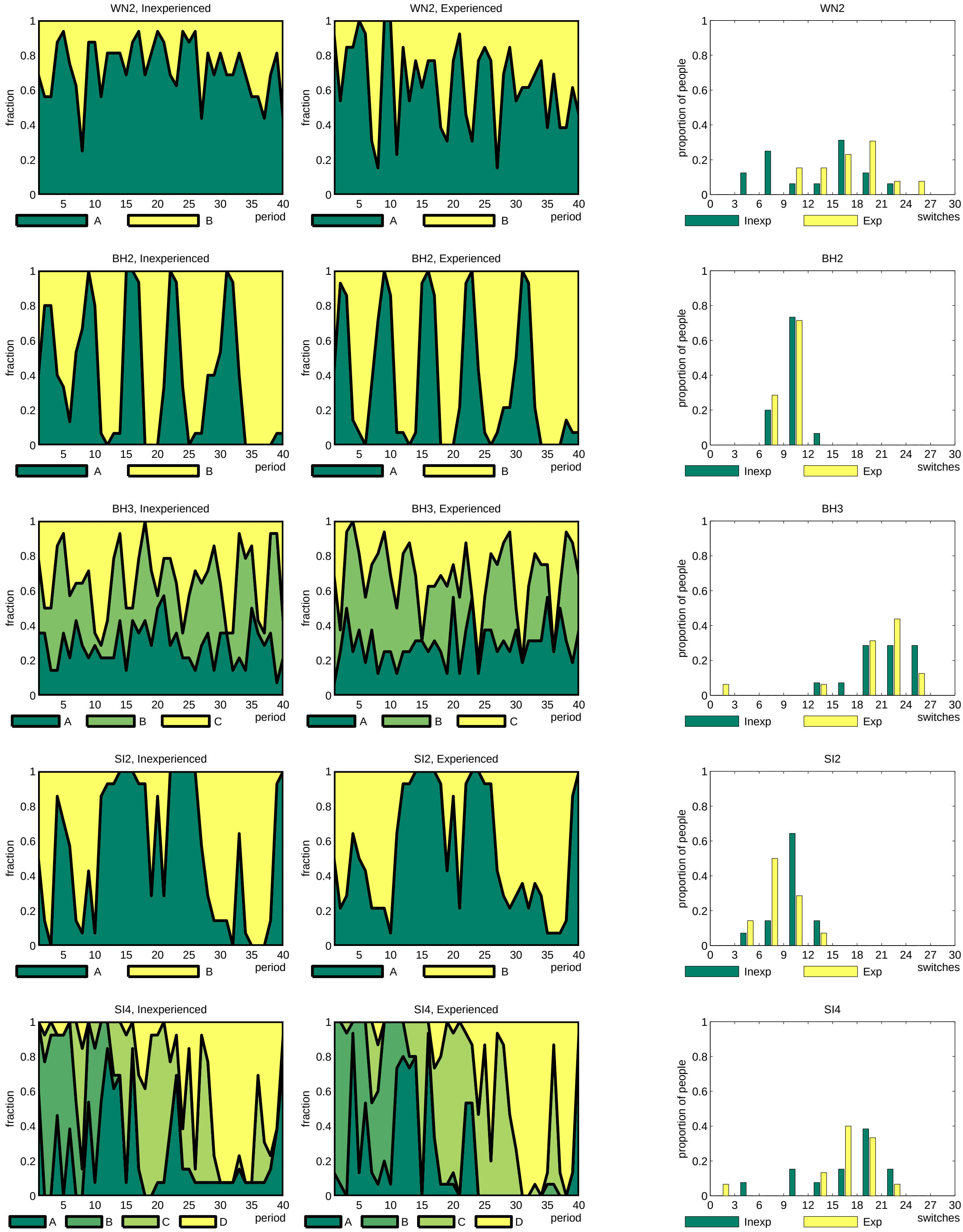


Figure 5: Shares of choices over time for inexperienced subjects (*left panels*) and for experienced subjects (*middle panels*), and distributions of number of switches in a session (*right panels*). From top to bottom: WN2, BH2, BH3, SI2 and SI4.

Time-series	Block, Treatment	Switches	
		Total number	Per participant per period
WN2	1 (Inexperienced), T1	210	32.81%
	2 (Experienced), T6	224	43.08%
BH2	1 (Inexperienced), T2	159	26.50%
	2 (Experienced), T5	145	25.89%
BH3	1 (Inexperienced), T3	308	55.00%
	2 (Experienced), T1	332	51.88%
BH4	2 (Experienced), T4	203	36.25%
SI2	1 (Inexperienced), T4	147	26.25%
	2 (Experienced), T3	119	21.25%
SI3	1 (Inexperienced), T5	169	30.18%
SI4	1 (Inexperienced), T6	218	41.92%
	2 (Experienced), T2	251	41.83%

Table 3: Number of switches for each block in the experiment.

pants faced with the same set of time series.

Whereas, as argued above, experience of the participant seems to have a rather modest effect on his or her choice behavior, there are remarkable differences of choice behavior between different sets of time series. Fig. 6 shows the evolution of fractions of choices using all the data (pooling experienced and inexperienced participants) for the BH2, BH3, SI2 and SI4 sets of time series, as well as for the BH4 and SI3 sets (for which only experienced or only inexperienced participants made choices). Comparing the BH2 and BH3 blocks, for example, we see that for the former there are large swings in the fraction of participants choosing alternative A (such large swings are also observed in the SI2 blocks, although the pattern is different). In the BH3 blocks, however, the fraction of participants choosing a particular alternative is much more stable over time and only subject to relatively modest fluctuations (the same holds for the WN2 blocks, see the top panels of Fig. 5).

It is remarkable that in the WN2 blocks, although expected payoffs for alternatives A and B are the same, participants still choose A with a higher frequency than B. This may be attributed either to a “default choice bias” for alternative A or to the realization of the random white-noise process where alternative A had a higher return than alternative B for 7 out of the first 10 experimental periods (i.e., for $t = 1, \ldots, 10$) or a combination of the two. Note, however, that when the same participants (in treatment T1) are faced with the set of time series from BH3, where alternative B is typically more profitable than alternative A, they do actually choose B more often than A. Also, for the set of time series in SI4, the majority of participants learns to switch from fund A (which is initially the most profitable alternative on average) to fund D (which is the most profitable alternative, on average, for the last couple of periods). These observations suggest that participants do positively respond to past profit differences.

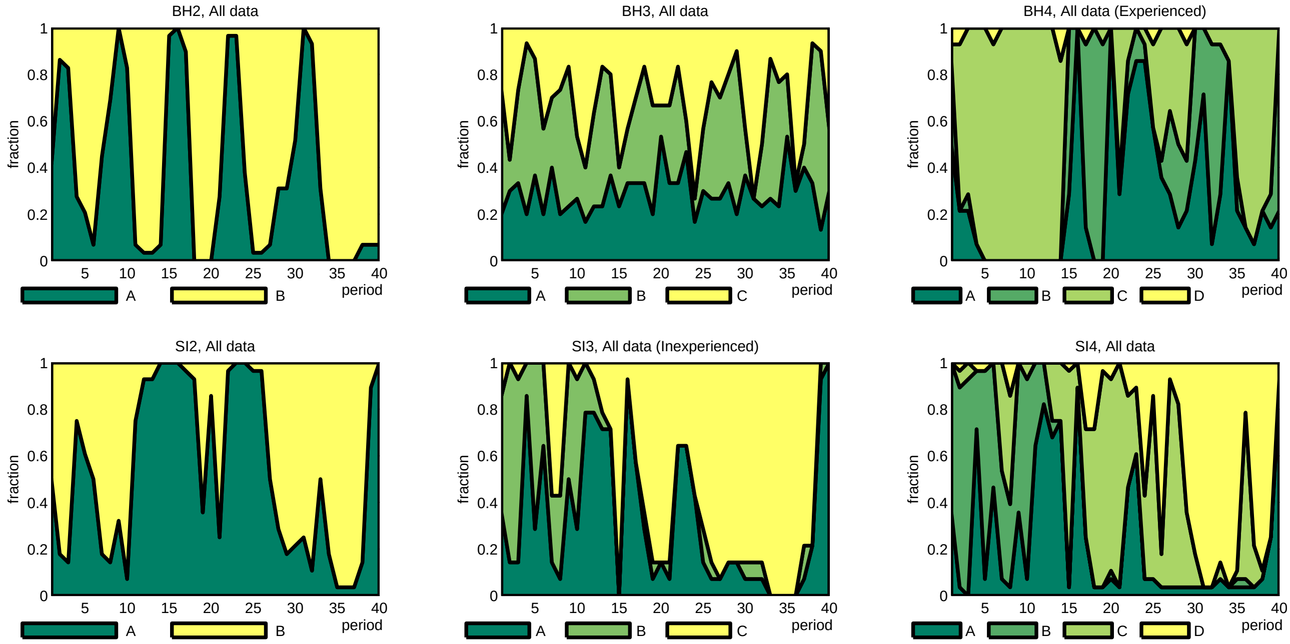


Figure 6: Shares of choices over time for different sets. For WN2, BH3, SI2 and SI4 the data from the two blocks are pooled.

Time series	Treatments	Fraction of Best Choices		
		Inexperienced	Experienced	Total
WN2	T1 and T6	53.13%	51.15%	52.24%
BH2	T2 and T5	81.33%	85.00%	83.10%
BH3	T3 and T1	46.25%	43.13%	44.58%
BH4	T4	-	57.68%	57.68%
SI2	T4 and T3	72.50%	70.89%	71.70%
SI3	T5	65.18%	-	65.18%
SI4	T6 and T2	56.92%	59.67%	58.39%

Table 4: Fraction of best choice in each block.

4.2 Performance and Efficiency

In order to establish whether participants performed well we can evaluate how often they made the optimal decision, that is: chose the alternative that, for that period, indeed generates the highest actual return. Table 4 shows, for each block, the fraction of optimal choices by the participants in that block. First note that choice success does not seem to be related to experience: the fractions of optimal choices are very similar for experienced and inexperienced participants (in fact, for three of the five sets of time series inexperienced participants slightly outperform experienced participants, although none of the five differences is significant at the 5% level, according to a Mann-Whitney-Wilcoxon test).

Second, the fractions in Table 4 are much higher than what they would be under random choice (where they would be around 50%, 33% or 25%, depending upon the number of alternatives). Participants are most successful in choosing the best alternative from time series in

BH2, SI2 and SI3, which exhibit substantial autocorrelation structure. The fraction of optimal choices is lowest with the highly unpredictable time series from WN2 and BH3.[21]

In order to compare efficiency obtained by the participants in the different blocks, which is complicated by the fact that the number of alternatives differs between sets of time series, we define the efficiency measure Eff as

$$\text{Eff} = \frac{\text{Average Experimental Earnings} - \text{Expected Earnings when Choosing Randomly}}{\text{Earnings when Choosing Optimally} - \text{Expected Earnings when Choosing Randomly}} . \quad (4)$$

If this measure is close to 1 it means that participants almost always make optimal decisions, whereas if it is close to 0 participants' performance is not much better than their expected payoff when they choose an alternative randomly (note that Eff can become negative if participants earn less points than they would earn, in expectation, when choosing randomly). Table 5 presents the values of this measure for the different sets of time series (the sixth column of the table). It also presents the maximum and minimum number of points that could be earned, the expected number of points under random choice and the actual average number of points earned. By construction, there is no structure or predictability in the time series from WN2, and indeed efficiency, although still positive, is lowest there. The time series from BH3 show little structure as well and participants find it difficult to make correct choices for those time series. For the other five sets of time series the efficiency level is between 63% and 78%.

4.3 Learning within Blocks

We study whether participants learn during the experiment by comparing their performance in later periods (within the same block) with their performance in earlier periods. As a first indication of learning consider the last two columns of Table 5. These columns give, for each set of time series, the value of the efficiency measure (4) separately for the first 20 periods and the last 20 periods in the blocks. Note that for WN2 efficiency goes down in the last twenty

[21]The fraction of optimal choices with the time series from WN2 is higher than 50%, but this difference is not significant at the 5% significance level, according to a Wilcoxon sign rank test.

Time series	**Max**	**Min**	**Random**	**Experiment**	**Efficiency**		
					Total	**First half**	**Second half**
WN2	271.19	179.65	225.42	231.27	12.80%	32.19%	−5.33%
BH2	285.30	161.08	223.19	269.87	75.16%	66.87%	82.36%
BH3	258.71	105.16	181.94	200.10	23.65%	16.60%	28.64%
BH4	263.59	179.88	223.22	249.50	65.09%	71.31%	59.20%
SI2	197.22	175.81	186.51	193.32	63.55%	58.11%	68.08%
SI3	217.82	170.72	193.03	210.89	72.04%	56.39%	80.42%
SI4	228.81	166.25	195.49	216.38	76.46%	70.76%	80.15%

Table 5: The average payoff, payoffs under different scenarios and efficiency for each type of time series. Data from experienced and inexperienced participants are pooled.

periods and even becomes negative, but since the evolution of the WN2 returns series is, by construction, unpredictable we would not expect any learning to take place in these blocks anyway. In fact, it is surprising that, by chance, participants reach a quite reasonable level of efficiency in the first 20 periods. For five of the other six sets of time series there is a substantial increase in efficiency, with the measure increasing by 10 to 24 percentage points, which suggests that, during the block, participants learn to make better decisions. The only surprising result in this respect is that the efficiency for the BH4 set of time series goes substantially down in the last 20 periods: apparently learning for that set of time series is particularly challenging.

Additional evidence that participants learn for five of the seven sets of time series is provided by Fig. 7. This figure plots the time series of the fraction of participants who choose the best alternative for that period (that is, the alternative that generates the highest profit in that period) for each set of time series. If participants learn to make better decisions this fraction should increase over time. From Fig. 7 it follows that whether there is a learning effect depends upon the type of time series. For the time series from BH2, SI2, SI3 and SI4, and to a lesser extent BH3, we see an overall increase in the fraction of optimal choices (note that these are also the time series with the most salient autocorrelation structure), while there is a negative trend for the time series from WN2 and BH4.

4.4 Return Chasing Behavior

The results that we discussed thus far suggest that participants make choices conditional upon past returns of the different alternatives. To investigate this further Fig. 8 plots the fraction of participants choosing the alternative in the current period that had the highest returns in the *previous* period (note the difference with Fig. 7 which shows the fraction of choices that are optimal in the *current* period). The more participants are chasing past returns, the closer the fractions in Fig. 8 should be to 1. This is most obvious for the time series in BH2, BH4 and SI2, where participants choose the best alternative from the previous period around 80% of the time, and true – but to a lesser extent – for the other sets of time series as well. The only exception is the BH3 block where only around 40% of the time the best alternative from the previous period is chosen.

These observations suggest that a model where participants condition their decision upon the profits of the different alternatives from the previous period might organize the experimental data quite well. In the next section we try to estimate such a model.

5 Explaining the Data by the Discrete Choice Model

In this section we investigate the extent to which the discrete choice models (1) and (2), discussed in Section 2, can explain the switching behavior of the participants in our experiment.

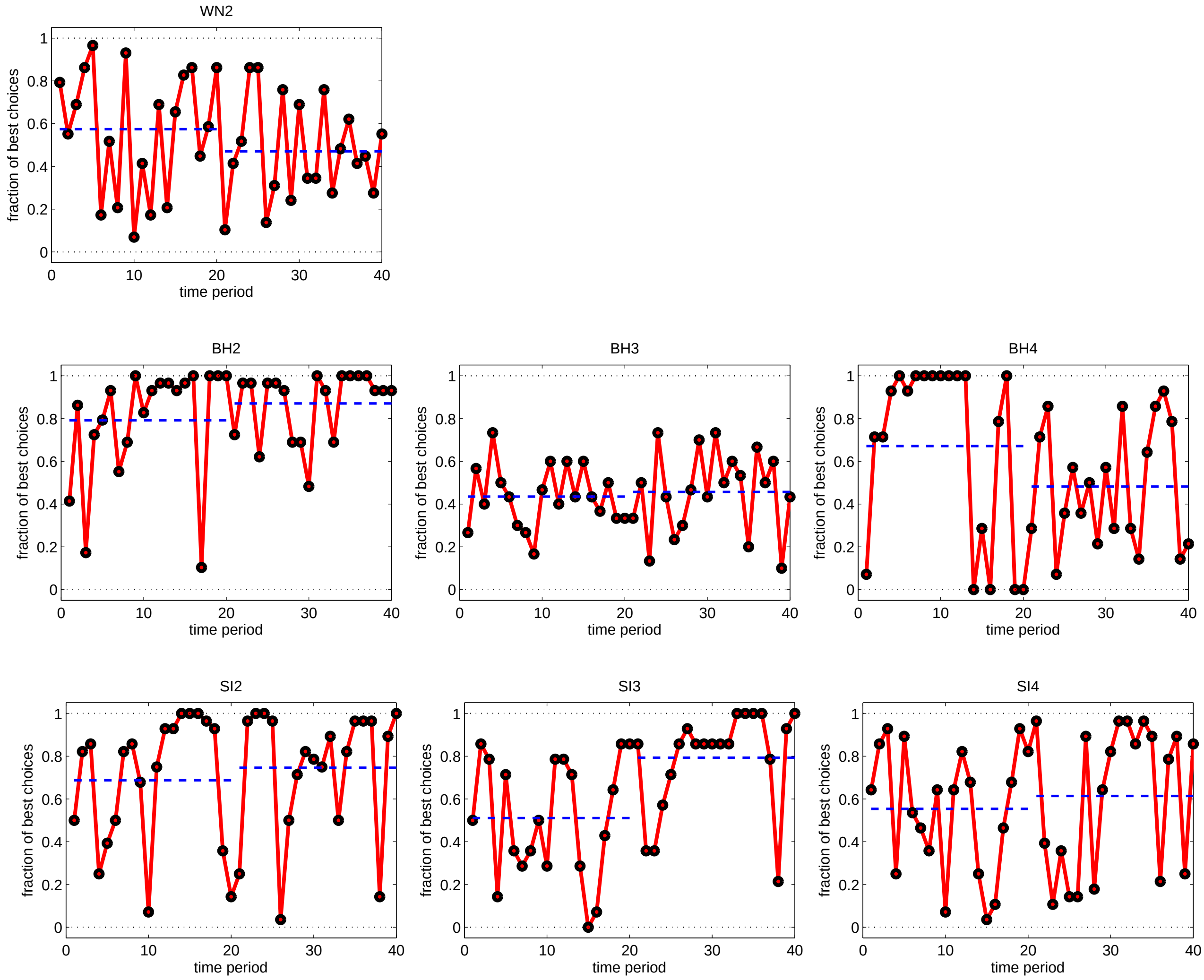


Figure 7: The fraction of participants choosing the best fund. Inexperienced and experienced sessions are pooled together. The horizontal dashed lines show the averages of the fractions over the first 20 and last 20 periods.

We are particularly interested in the Intensity of Choice parameters: β in (1) and $\beta_{h\ell}$ in (2). They measure how sensitive participants' decisions are with respect to differences in returns and play a pivotal role in heterogeneous agent models, where (1) is typically used with β exogenously given.

In Section 5.1 we consider the *binary choice* model, using experimental data from those blocks of the experiment where the set of time series contains only two alternatives (that is, WN2, BH2 and SI2). The details, including our methodology, other estimated models and details of goodness-of-fit tests are reported in Appendix C. In Section 5.2 we briefly discuss the *multiple choice* model, for the experimental data from blocks where there are three or four alternatives (BH3, BH4, SI3 and SI4).

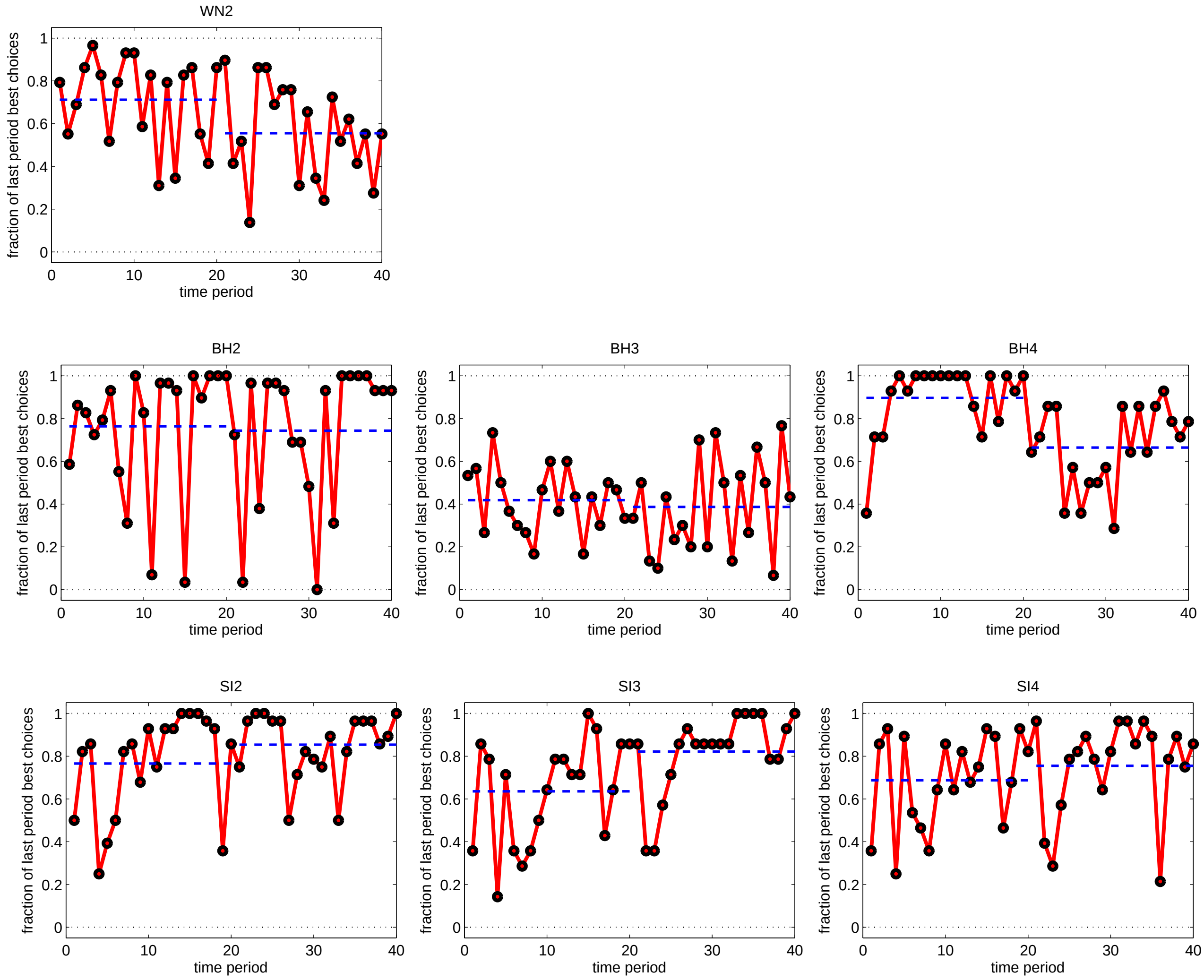


Figure 8: The fraction of participants choosing in period t the fund that had highest returns in period $t-1$. Inexperienced and experienced sessions are pooled together. The horizontal dashed lines show the averages of the fractions over the first 20 and last 20 periods.

5.1 Choices in WN2, BH2 and SI2 and the Binary Choice Model

For each of the three sets of time series WN2, BH2 and SI2 we will try to explain the fraction of participants choosing alternative A as a function of the past returns of alternatives A and B.[22] First let us recall that the analysis in Section 4 shows that: *(i)* there is little evidence of learning *between* blocks, that is, choice behavior of experienced participants is not substantially different from that of inexperienced participants (for the same set of time series) and, *(ii)* there is some evidence that participants learn *within* blocks, because their choices in the last 20 periods of a block are often better than in the first 20 periods (see Section 4.3). Because of these two

[22] Alternatively, we could estimate a version of discrete choice model (1) or (2) on *individual choices* of participants. In that case the dependent variable would be the *probability* that a particular individual chooses alternative A (instead of the *fraction* of participants choosing A). Note however that in heterogeneous agent models the discrete choice approach is used to describe aggregate instead of individual decisions. As our interest in the discrete choice approach originates from its validity and viability as a building block of heterogeneous agent models, we will focus on how well it explains aggregate choices.

Model	Treatment	α	β_A	β_B	log-likelihood	AIC	BIC	McFadden R-squared	LR test
Sym1	WN2	0.000	0.100 (0.030)	0.100	−396.432	794.863	799.226	−0.061	
	BH2	0.000	0.431 (0.034)	0.431	−285.560	573.121	577.484	0.210	
	SI2	0.000	3.747 (0.277)	3.747	−203.347	408.694	413.022	0.475	
PreSym1	WN2	0.693 (0.090)	0.133 (0.032)	0.133	−364.927	**733.854**	**742.580**	0.023	63.009 (0.000)
	BH2	−0.437 (0.114)	0.365 (0.035)	0.365	−278.606	561.212	569.938	0.230	13.908 (0.000)
	SI2	0.626 (0.162)	4.409 (0.370)	4.409	−194.622	**393.245**	**401.901**	0.498	17.449 (0.000)
PreAsym1	WN2	0.532 (0.350)	0.151 (0.050)	0.123 (0.038)	−364.813	735.626	748.715	0.024	0.228 (0.633)
	BH2	−12.973 (1.516)	2.609 (0.299)	0.095 (0.051)	−223.469	**452.938**	**466.027**	0.382	110.274 (0.000)
	SI2	0.925 (0.374)	4.652 (0.471)	4.754 (0.548)	−194.221	394.442	407.426	0.499	0.803 (0.370)

Table 6: Comparison of the three models with one lag over all treatments. Last 20 periods of data are used (pooled over experiences). The last column shows the results of LR test of the corresponding model against the previous (restricted) model.

findings we pool the data from experienced and inexperienced participants for our estimations, and only use data from the last 20 time periods of each block.

Moreover, motivated by the evidence on return chasing behavior in Section 4.4, we will start out with a version of the discrete choice model (2) with one lag. To be specific, we assume the fraction of participants choosing alternative A at time t is given as

$$n_t^A = \frac{\exp\left[\alpha + \beta_A \pi_{A,t-1}\right]}{\exp\left[\alpha + \beta_A \pi_{A,t-1}\right] + \exp\left[\beta_B \pi_{B,t-1}\right]}. \tag{5}$$

Note that only a constant for alternative A is included.[23] As discussed in Section 2, parameter α represents a *predisposition* or *bias* towards one of the alternatives (see also Branch, 2004): if α is positive (negative) the participant tends to choose alternative A (B) even if returns in the previous period were the same.[24] Eq. (5) allows for only one lag in past profits of the two alternatives, but the responses to these past profits may be asymmetric, $\beta_A \neq \beta_B$. Note that the vast majority of heterogeneous agent models assume (5) with $\alpha = 0$ and $\beta_A = \beta_B$.

In finding a parsimonious model that gives a satisfactory description of participants' choices we proceed in the following way. First, using the method of maximum likelihood, we estimate model (5). This is the most general model with one lag; it incorporates a predisposition effect as well as asymmetric responses and we call it **PreAsym1**. Then, we estimate model (5) with the restriction $\beta_A = \beta_B$ imposed. We call this model **PreSym1** as it still includes a predisposition effect, but has symmetric responses to payoffs. Finally, we estimate (5) with the two restrictions $\beta_A = \beta_B$ and $\alpha = 0$ imposed. This canonical discrete choice model (1) is called **Sym1**.

[23]This can be done without loss of generality (also see the discussion in Appendix C.1.1).

[24]Note that this interpretation assumes symmetric responses, $\beta_A = \beta_B$. For asymmetric responses the actual bias towards alternative A will also depend on β_A, β_B, and the average values of π_A and π_B; see footnote 25.

Table 6 presents the three estimated models, arranged from the most parsimonious model (**Sym1**) on top to the most general model (**PreAsym1**) at the bottom, for each set of time series containing two alternatives. We report the values of the parameters, the standard deviations (in parentheses) and the value of the log-likelihood function, which increases when a less parsimonious model for the same treatment is considered. We compute the two standard information criteria (Akaike, AIC and Bayesian, BIC) that penalise the models for additional parameters. The value of the information criterion for the best model from this perspective (that is, with the lowest AIC or BIC value) is shown in bold in Table 6. McFadden's R^2 evaluates the goodness of fit of a model by comparing it with the model where only the constant term is included (the latter model corresponds to a 'pure predisposition' model). Since our models have a nested structure (**Sym1** is nested within **PreSym1**, which is nested within **PreAsym1**) we can use the log-likelihood ratio (LR) test for pairwise comparisons. The last column of Table 6 (this column should be read from down to up) shows, for the corresponding treatment and model, the value of the log-likelihood ratio and the p-value at the 5% significance level (in parentheses) when the model is tested against its restricted version from the row immediately above. For instance, for the WN2 blocks we test *(i)* the hypothesis that $\beta_A = \beta_B$ in the **PreAsym1** model, leading to the **PreSym1** model (the LR is 0.228 and we cannot reject the hypothesis), and *(ii)* the hypothesis that $\alpha = 0$ in the **PreSym1** model leading to the **Sym1** model (the LR is 63.009 and we therefore have to reject this hypothesis). This suggests that the **PreSym1** model is the most parsimonious model that gives a good description of the aggregate data in the WN2 blocks.

It follows from the estimations and test results presented in Table 6 that model **Sym1** – corresponding to the canonical discrete choice model (1) and used in the vast majority of the theoretical and computational contributions to the literature on heterogeneous agent models – does *not* provide a satisfactory description of aggregate choice behavior for *any* of the three sets of time series considered. In fact, a comparison of the three models on the basis of the AIC, the BIC and the LR tests gives very consistent results. The best model for the experimental data from the WN2 and SI2 blocks (**PreSym1**) has symmetric responses to payoffs, but includes a predisposition effect. Furthermore, the best model for the experimental data from the BH2 blocks (**PreAsym1**) has asymmetric responses in addition to a predisposition effect.

The left panels in Fig. 9 compare the share of participants choosing alternative A in the experiment (thick solid line) with the predicted fraction of participants choosing alternative A using the estimated **PreSym1** model from Table 6 (thick dashed line) and, for the BH2 time series, also with the predictions of the **PreAsym1** model (thin solid line). The panels illustrate that the **PreSym1** model describes the data from the WN2 and SI2 blocks quite well. In fact, also for the data from the BH2 blocks model **PreSym1** gives a fairly good description although model **PreAsym1**, with asymmetric responses to payoffs, performs significantly better.

When we consider the estimated models from Table 6 in a bit more detail we can make three additional observations. First, the predisposition effect is significantly different from 0 for all

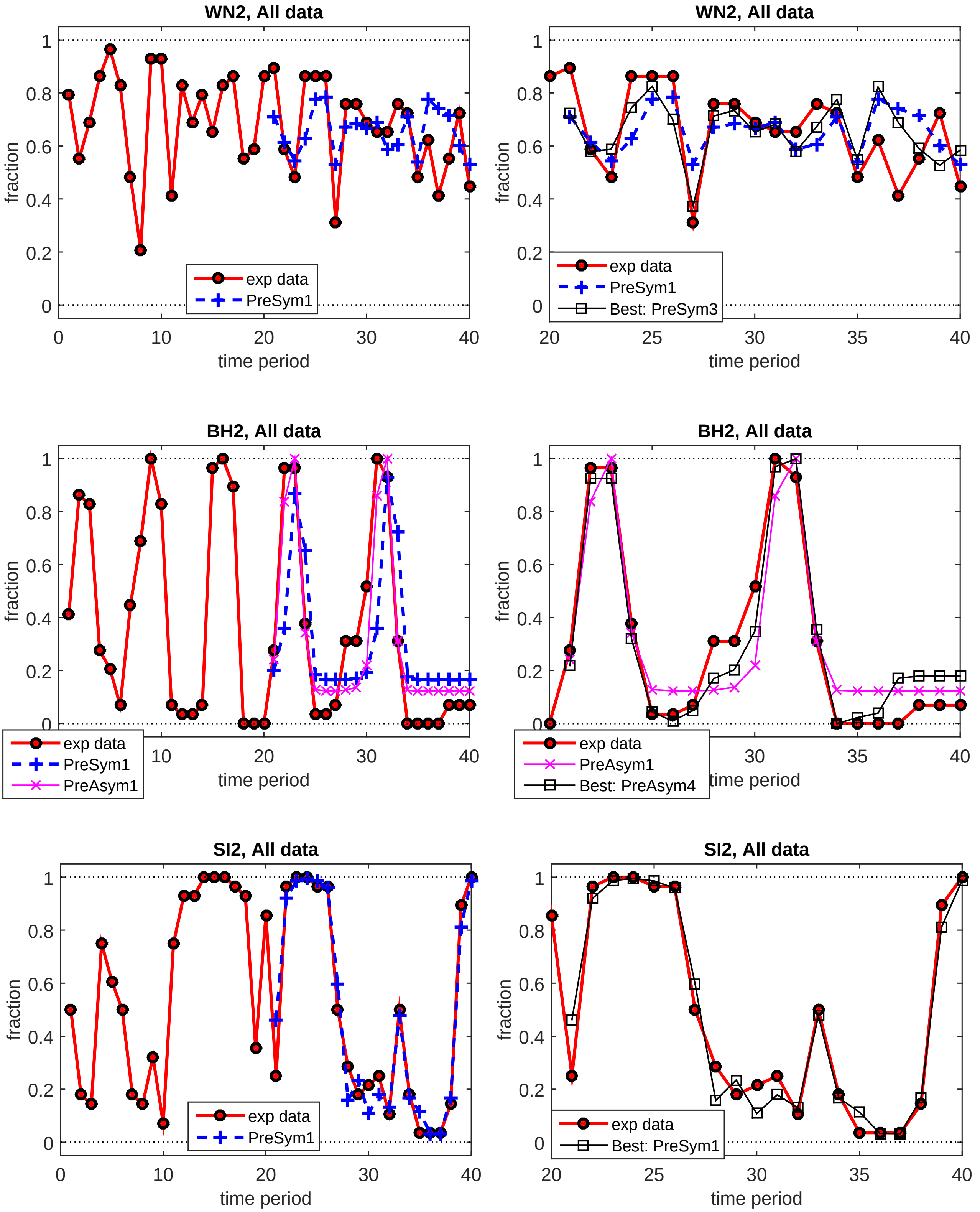


Figure 9: Experimental data (thick solid line with circles) and model predictions. *Left panels:* All 40 periods are shown; predictions are from the one-lag models, **PreSym1** and **PreAsym1** (only for BH2). *Right panels:* the 20 last periods are shown; predictions are from multiple lags model compared with the best prediction from the one-lag model. The best model is shown by the thin line with squares.

three sets of time series, and implies a bias towards alternative A.[25] A possible explanation for this is a simple preference of participants to choose the first presented option. Closer inspection of the different time series may provide sensible alternative explanations as well.[26] In the time series from WN2, for example, by chance A gives a higher payoff than B in 7 out of the first 10 periods ($t = 1, \ldots, 10$). For the SI2 blocks, on the other hand, payoffs for these 10 periods are roughly the same for choices A and B, but in the starting periods ($t = -9, \ldots, 0$) alternative A is dominating alternative B by a substantial margin. These initial differences may explain the existence of the predisposition effect, even allowing for a learning phase of 20 periods.[27]

Second, it is clear that the estimated value of the intensity of choice parameter β is significantly positive at the 5% level for model **PreSym1** in the WN2 and SI2 blocks. This is particularly noteworthy for the WN2 blocks since past returns in those blocks have no predictive power for future returns. Also for the BH2 blocks, the estimated value of β_A in model **PreAsym1** is significantly positive at the 5% level.[28] Our first result therefore is the following.

Result 1. *We confirm Hypothesis 1 and conclude that people do exhibit 'return chasing behavior' in the WN2, BH2 and SI2 blocks.*

Finally, we see that the estimated intensity of choice parameter β differs for different sets of time series. As the payoff for all alternatives in each block is on average about 5 points per period (see Table 1), the differences between the estimated intensities of choice can not be attributed to a scaling effect. We find the estimated β to be quite high for the SI2 blocks, and low for the WN2 blocks. There is an intuitive explanation for this: choosing the alternative that

[25]To see this, one can substitute, for the three sets of time series, the average payoffs for alternatives A and B (as given in Table 1) in the estimated model from Table 6 that gives the best description of the data and compute the predicted fraction of participants choosing A. This gives $n^* \approx 0.68$ for WN2, $n^* \approx 0.53$ for BH2 and $n^* \approx 0.93$ for SI2. Note that although the estimate of α is negative in BH2, the asymmetry in responses to payoffs is large enough to lead to a bias towards A.

[26]It is difficult to explain the predisposition effect by risk aversion. To see this, note from Table 1 that the differences in variances in returns is substantial for the choice sets BH2 and SI2, whereas the difference in means is small. Participants' risk aversion should then be represented by a predisposition towards the alternative with the lower variance. Comparing Table 1 with Table 6 we see that this is the case for the BH2 set of time series but not for the SI2 set, where there is a predisposition towards the alternative with the *higher* variance.

[27]To illustrate that the predisposition effect diminishes somewhat over time we also estimated model **PreSym1** for WN2 on *all* 40 periods. Recomputing, for this estimated model, the predicted fraction for A when payoffs for the alternatives are equal to their mean values, we find $n^* \approx 0.73$ (cf. footnote 25).

[28]In that model, however, β_B is not significantly different from 0. This means that participants respond to the payoffs for alternative A, but not so much to payoffs for alternative B. Considering the time series of these alternatives, shown in upper left panel of Fig. 3, we see that for most periods alternative A has a lower payoff (around 4.50) than alternative B (which in many periods gives a payoff of around 7.50). Moreover, an increase in the returns for alternative A occurs simultaneously with a decrease in the return for alternative B. The estimated **PreAsym1** model from Table 6 suggests that the default option for the participants is to choose alternative B, except if payoffs for alternative A in the previous period were substantially higher than their lower bound of 4.5. That is, the estimated model can be written as

$$n_t^A = \frac{1}{1 + \exp\left(-2.608\left(\pi_{A,t-1} - 4.97\right)\right)}$$

meaning that for $\pi_{A,t-1} = 4.50$ (which is around the value that π_A takes on most often), the fraction of the participants choosing alternative A is approximately 0.22, whereas $n_{A,t}$ will be above 0.50 (0.90) if $\pi_{A,t-1}$ is above 5 (6).

generated a high profit in the previous period makes sense in particular when profits are highly and positively autocorrelated. The results suggest that the intensity of choice β is affected by the structure of the time series of profits, and subjects are successful in adjusting their reaction to past information in the right direction (e.g., their behavior is described by a higher value of β when past profits have more predictive power about future profits). Based on this observation we conclude the following:

Result 2. *We reject Hypothesis 2. The estimated value of the intensity of choice parameter β depends strongly on the predictability of returns. In particular, β is larger when the autocorrelation structure in the returns is stronger (and positive).*

Result 2 suggests that predictability of future payoffs is an important determinant of the estimated intensity of choice parameter β, which is higher the higher the autocorrelation in the time series of payoffs is. It therefore makes sense to consider an extended model that includes more lags of past payoffs, to allow for the possibility that participants try to exploit the structure in the time series. That is, we consider specification (2), which for two alternatives becomes:

$$n_t^A = \frac{\exp\left[\alpha + \sum_{\ell=1}^{L} \beta_{A,\ell}\pi_{A,t-\ell}\right]}{\exp\left[\alpha + \sum_{\ell=1}^{L} \beta_{A,\ell}\pi_{A,t-\ell}\right] + \exp\left[\sum_{\ell=1}^{L} \beta_{B,\ell}\pi_{B,t-\ell}\right]}. \quad (6)$$

Note that we allow for a predisposition effect and for differences between the coefficients on past payoffs (both across time and across alternatives), but we do assume that participants consider the same number of lags L for each alternative.[29] We estimate model (6) on the same data as before (i.e., using the last 20 observations and pooling inexperienced and experienced participants).

Before discussing the estimation results, let us first briefly outline our chosen methodology. We begin with estimating the model with only a constant (that is, with $L = 0$). We then add one lag for both alternatives, estimate this less parsimonious model and test (using the LR test with a significance level of 5%) the restriction $\beta_{A,1} = \beta_{B,1} = 0$, which leads to the first model. If this restriction is rejected we reject the first model (with $L = 0$) and proceed in the same way with the second model. That is, we add a second lag for both alternatives, estimate this new model and test if this model is significantly better than the previous one, and so on. The procedure stops as soon as adding a lag for both alternatives does not significantly improve the performance of the model. We then choose the model which was the last one that provided a significant improvement.[30] Using this procedure we end up with the **PreAsym3** model for the WN2 set of time series (that is, the model includes three lags, $L^* = 3$) and with the **PreAsym4** model for the BH2 set of time series ($L^* = 4$). The estimation results are shown in the bottom

[29]See Appendix C.3 for a formal specification of the model with lags. We also estimated the models which include an arbitrary number $L_A < 10$ of lags for alternative A and an arbitrary number $L_B < 10$ of lags for alternative B. The results are available upon request.

[30]An alternative procedure is to start with, for example, $L = 10$ lags and then sequentially reduce the number of lags as long as the hypothesis that the two coefficients on the highest lag are equal to zero is not rejected. In general this procedure may lead to different results, e.g., with a higher number of lags in the final model.

Model and Lag	α	$\beta_{A,1}$ $\beta_{B,1}$	$\beta_{A,2}$ $\beta_{B,2}$	$\beta_{A,3}$ $\beta_{B,3}$	LL	AIC	BIC	McF R-sq	LR test
Sym3		0.081	−0.098	−0.063	−390.527	787.053	800.142	−0.045	
		(0.033)	(0.033)	(0.035)					
PreSym3	0.704	0.114	−0.109	−0.059	−358.741	725.482	**742.934**	0.040	63.572
	(0.092)	(0.035)	(0.035)	(0.037)					(0.000)
PreAsym3	2.106	0.134	−0.208	−0.140	−355.299	**724.599**	755.140	0.049	6.883
	(0.879)	(0.073)	(0.060)	(0.058)					(0.332)
		0.115	−0.103	0.018					
		(0.039)	(0.045)	(0.051)					

Table 7: Estimation and test results for multi-lag models for the WN2 blocks. Last 20 periods of data are used (pooled over experience). Before generating this table, the **PreAsym3** model was chosen as the most parsimonious among asymmetric models with many lags providing significant improvement at the 5% level, see the main text.

Model and Lag	α	$\beta_{A,1}$ $\beta_{B,1}$	$\beta_{A,2}$ $\beta_{B,2}$	$\beta_{A,3}$ $\beta_{B,3}$	$\beta_{A,4}$ $\beta_{B,4}$	LL	AIC	BIC	McF R-sq	LR test
Sym4		1.207	−0.738	0.239	−0.312	−209.796	427.592	445.044	0.420	
		(0.106)	(0.076)	(0.103)	(0.083)					
PreSym4	−0.250	1.122	−0.673	0.173	−0.310	−209.463	428.925	450.740	0.421	0.667
	(0.313)	(0.146)	(0.108)	(0.138)	(0.087)					(0.414)
PreAsym4	9.223	10.896	0.717	−16.112	3.366	−189.981	**397.962**	**437.229**	0.475	38.963
	(64.671)	(2.032)	(1.677)	(4.976)	(3.366)					(0.000)
		−8.931	12.341	−2.422	−0.259					
		(1.886)	(2.970)	(3.699)	(−0.259)					

Table 8: Estimation and test results for multi-lag models for BH2 treatment. Last 20 periods of data are used. Before generating this table, **PreAsym4** model was chosen as the most parsimonious among asymmetric models with many lags providing significant improvement at the 5% level, see the main text.

parts of Tables 7 and 8 for the WN2 and BH2 blocks, respectively. For the SI2 set of time series, we found $L^* = 1$, implying that the **PreAsym1** model (already presented in Table 6) with one lag also provides the best description of the data when allowing for more lags.

Subsequently we test the hypothesis that coefficients on past payoffs are the same for both alternatives, that is $\beta_{A,\ell} = \beta_{B,\ell}$ for all $\ell \leq L^*$. We only reject this hypothesis for the data from the BH2 blocks. For the resulting symmetric **PreSym** models we test the additional restriction that $\alpha = 0$, which is rejected for the WN2 blocks. Tables 7 and 8 give the estimated coefficients, other statistics and the LR test results for the multi-lag models. From the LR tests and from comparing the BIC's for the different models we select the **PreSym3** model for the data from the WN2 blocks and the **PreAsym4** model for the data from the BH2 blocks. As mentioned before, for the data from the SI2 blocks the **PreSym1** model works best.

Let us compare the estimated multi-lag models from Tables 7 and 8 with the estimated one-lag models in Table 6, for the WN2 and BH2 blocks. Using the BIC criterion it follows that only for the BH2 blocks there is a substantial improvement in considering a discrete choice model with more than one lag in payoffs.[31] For the BH2 blocks, the estimated coefficients of the

[31]Note that for the WN2 blocks, the value of the BIC is only slightly smaller for the **PreSym1** model than for the **PreSym3** model.

PreAsym4 model are remarkably high, with only the first and third coefficient significantly different from zero but with opposite signs. This suggests that participants try to pick up the cyclic behavior in the payoff differences. Although they respond positively to last periods payoff difference, they correct this with a negative response to the payoff difference of two periods before.

The right panels of Fig. 9 compare, for the last 20 periods, the share of participants in the experiment choosing alternative A, the predicted share by the selected model with one lag from Table 6, and the predicted share by the chosen multi-lag model from Tables 7 and 8. The estimated multi-lag model clearly outperforms the one-lag model for the data in the BH2 blocks. For the SI2 blocks the best multi-lag model is in fact the model with one lag, and for the WN2 blocks performance of the two models is similar.

These findings suggest that the structure in the time series is an important determinant of the choice behavior of experimental subjects. When there is either very limited structure (as in the WN2 blocks), or very salient structure (as in the SI2 blocks), participants tend to condition their choice only upon the most recent information and a discrete choice model with one lag seems sufficient to describe the data. When the structure is more complicated (for example, when it has a cyclical pattern as in the case with the time series from the BH2 blocks), participants seem to condition their behavior on a longer history of returns. We therefore have the following

Result 3. *Hypothesis 3 is rejected. In particular for the time series from BH2 the multi-lag model gives a substantially better, and qualitatively different, description of the participants' decisions.*

To summarize, we find that the discrete choice model is, to a certain extent, successful in explaining the experimental data from the WN2, BH2 and SI2 blocks, taking into account the following qualifications. First, a predisposition effect in the discrete choice model is warranted, as this effect is significant for all three models that performed best in describing the data (the **PreSym1** model for the WN2 and SI2 blocks and the **PreAsym4** model for the BH2 blocks). Second, although for models with either a little or a lot of structure in the time series a discrete choice model with one lag and symmetric responses performs well, for time series with a nontrivial structure more lags and asymmetric responses may be required. Importantly, the intensity of choice parameter increases with the predictability in the time series. Finally, note that the discrete choice model that is typically used in the literature, model **Sym1**, does not do a very good job in explaining the experimental data.[32]

[32]Some extensions of the discrete choice model (Hommes, Kiseleva, Kuznetsov, and Verbic, 2012; Anufriev and Hommes, 2012b) explicitly allow for more payoff lags in the performance measure, introducing so-called "memory" with geometrically declining weights. That is, the weight of payoff $\pi_{t-\tau}$ is given by $\beta\eta^{\tau-1}$, with $\eta \in (0, 1)$. Inspection of Tables 7 and 8 suggests that such a model would not describe the behavior of participants in this experiment very well, since the estimated coefficients are clearly not geometrically decreasing.

5.2 Choices in BH3, BH4, SI3, SI4 and the Multinomial Logit Model

In this section we briefly discuss estimation results for a *general* discrete choice or multinomial logit model with *one lag*, applied to the experimental data from the blocks with three or four alternatives (that is, the BH3, BH4, SI3 and SI4 blocks). An important feature of the multinomial logit model with more than two alternatives is that it is more flexible than the binary choice model. In the case of three alternatives, for example, the general model assumes the following proportionalities of the fractions of choices for alternatives A, B and C as functions of past performances:

$$\begin{aligned} n_t^A &\propto \exp\left[\alpha^A + \beta_A^A \pi_{A,t-1} + \beta_B^A \pi_{B,t-1} + \beta_C^A \pi_{C,t-1}\right], \\ n_t^B &\propto \exp\left[\alpha^B + \beta_A^B \pi_{A,t-1} + \beta_B^B \pi_{B,t-1} + \beta_C^B \pi_{C,t-1}\right], \\ n_t^C &\propto \exp\left[\alpha^C + \beta_A^C \pi_{A,t-1} + \beta_B^C \pi_{B,t-1} + \beta_C^C \pi_{C,t-1}\right]. \end{aligned} \tag{7}$$

Note that the fraction choosing an alternative may be directly affected by the past payoffs of any of the other alternatives, whereas this is not the case for the binary choice model.[33] Another important characteristic is that, although model (7) has twelve parameters, only eight of them can be identified. To see this, note that, for example, subtracting $\beta_A^C \pi_{A,t-1}$ from each of the profit sums in (7) gives rise to exactly the same normalized fractions. Selecting alternative C as the 'reference option', we can then write the three normalized fractions as

$$\begin{aligned} n_t^A &= \exp\left[(\alpha^A - \alpha^C) + (\beta_A^A - \beta_A^C)\pi_{A,t-1} + (\beta_B^A - \beta_B^C)\pi_{B,t-1} + (\beta_C^A - \beta_C^C)\pi_{C,t-1}\right]\Big/ Z, \\ n_t^B &= \exp\left[(\alpha^B - \alpha^C) + (\beta_A^B - \beta_A^C)\pi_{A,t-1} + (\beta_B^B - \beta_B^C)\pi_{B,t-1} + (\beta_C^B - \beta_C^C)\pi_{C,t-1}\right]\Big/ Z, \\ n_t^C &= 1/Z, \end{aligned} \tag{8}$$

where Z is a normalization factor which is chosen in such a way that $n_t^A + n_t^B + n_t^C = 1$.

We refer to (8) as the **General1** model. This model has eight parameters (two relative predisposition effects, $\alpha^h - \alpha^C$ for $h = A, B$, and six relative intensities of choice of the form $\beta_k^h - \beta_k^C$ for $h = A, B$ and $k = A, B, C$). This **General1** model nests the **PreAsym1**, **PreSym1** and **Sym1** models that were discussed (for two alternatives) in Section 5.1. In particular, imposing the restriction that $\beta_k^h = \beta_k^g$ for all k, h and g that are different from each other, gives the **PreAsym1** model

$$n_t^h = \frac{\exp\left[\alpha_h + \beta_h \pi_{h,t-1}\right]}{\exp\left[\alpha_A + \beta_A \pi_{A,t-1}\right] + \exp\left[\alpha_B + \beta_B \pi_{B,t-1}\right] + \exp\left[\beta_C \pi_{C,t-1}\right]} \quad \text{for } h = A, B, C$$

which has five parameters (note that $\alpha_C = 0$).[34] Imposing in addition that the three intensities

[33] The general form implied by (7) reduces to the **PreAsym1** model from Section 5.1 if there are only two alternatives, see Appendix C for details. Obviously, since the fractions need to be normalized to make them add up to one, the fractions in the binary choice **PreAsym1** model will *indirectly* be affected by payoffs of the other alternatives.

[34] These five parameters are related to the parameters of the **General1** model (8) as follows: the two (relative)

of choice are the same ($\beta_A = \beta_B = \beta_C = \beta$) we obtain the **PreSym1** model, with fractions given by

$$n_t^h = \frac{\exp\left[\alpha_h + \beta\pi_{h,t-1}\right]}{\exp\left[\alpha_A + \beta\pi_{A,t-1}\right] + \exp\left[\alpha_B + \beta\pi_{B,t-1}\right] + \exp\left[\beta\pi_{C,t-1}\right]} \quad \text{for } h = A, B, C.$$

Finally, the one-parameter canonical **Sym1** model is obtained when requiring that there are no predisposition effects ($\alpha_A = \alpha_B = 0$), giving

$$n_t^h = \frac{\exp[\beta\pi_{h,t-1}]}{\exp[\beta\pi_{A,t-1}] + \exp[\beta\pi_{B,t-1}] + \exp[\beta\pi_{C,t-1}]} \quad \text{for } h = A, B, C. \tag{9}$$

For the case of four alternatives the different models and restrictions are similar (see Appendix C for details).[35]

Given this general framework we will now try to find a multinomial logit model that fits the experimental data from the BH3, BH4, SI3 and SI4 blocks. We apply the same methodology as in Section 5.1. That is, we start with estimating the **General1** model and then investigate, using the information criteria and LR tests, whether a more parsimonious model exists that still gives a good description of the experimental data.

We start by estimating the different models for the experimental data from the SI3 and SI4 blocks. Tables 9 and 10 show the results. The estimated coefficients are presented in matrix form, with a description of the relevant parameters given in the first row of these tables (this description is based upon the specification in Eq. (8)). The last row of the tables shows the estimated **General1** models. For the restricted models (except for the **Sym1** model, which has only one parameter) we also show the values of all parameters (with the value equal to 0 if the parameter is restricted to be 0), but we only report standard errors once across parameters that are restricted to be the same. As before, the last column of the tables should be read from bottom to top. It shows the LR statistics and p-value for the hypothesis consisting of the restrictions leading to the model in the row immediately above.

Applying the results of the LR test at the 5% significance level to the estimated models for the experimenal data from the SI3 block, presented in Table 9, we conclude that for the **General1** model we cannot reject the restrictions leading to the **PreAsym1** model and for the **PreAsym1** model we cannot reject the restrictions leading to the **PreSym1** model. For the **PreSym1** model, however, we do reject the restrictions that lead to the **Sym1** model. This suggests that the estimated **PreSym1** model is the best parsimonious model describing the data. This is consistent with the BIC, which is indeed lowest for the **PreSym1** model, but not

predisposition effects are $\alpha_A = \alpha^A - \alpha^C$ and $\alpha_B = \alpha^B - \alpha^C$ and the three (relative) intensities of choice are given by $\beta_A = \beta_A^A - \beta_A^C$, $\beta_B = \beta_B^B - \beta_B^C$ and $-\beta_C = \beta_C^A - \beta_C^C$.

[35] If there are four alternatives, the **General1** model has fifteen parameters; the **PreAsym1** model has seven parameters (three relative predisposition effects and four intensities of choice); the **PreSym1** model has four parameters; and the **Sym1** model has one parameter, the intensity of choice β. The latter model was used in the case of four strategies discussed in Brock and Hommes (1998). Appendix C.2 provides formal equations, parameters to be estimated and restrictions.

Model	$\alpha^A - \alpha^C$ $\alpha^B - \alpha^C$	$\beta_A^A - \beta_A^C$ $\beta_A^B - \beta_A^C$	$\beta_B^A - \beta_B^C$ $\beta_B^B - \beta_B^C$	$\beta_C^A - \beta_C^C$ $\beta_C^B - \beta_C^C$	LL	AIC	BIC	McF R-sq	LR test
Sym1	−0.000	2.008 (0.158)	0.000	−2.008	−155.882	313.764	317.399	0.198	
PreSym1	0.688 (0.280) −1.014 (0.417)	1.974 (0.258) 0.000	0.000 1.974	−1.974 −1.974	−139.343	284.687	**295.591**	0.283	33.078 (0.000)
PreAsym1	−1.664 (1.127) −2.148 (1.027)	1.612 (0.297) 0.000	0.000 1.261 (0.431)	−1.123 (0.454) −1.123	−136.932	**283.864**	302.038	0.296	4.823 (0.090)
General1	−0.890 (1.280) −2.604 (2.096)	1.501 (0.377) 0.257 (0.693)	0.670 (0.468) 0.818 (0.863)	−1.720 (0.610) −0.837 (1.035)	−135.258	286.517	315.595	0.304	3.348 (0.341)

Table 9: Estimation of multivariate logit models for the experimental data from the SI3 block. The last 20 periods are used. The last column shows the results of LR test of the corresponding model against the previous (restricted) model.

Model	$\alpha^A - \alpha^D$ $\alpha^B - \alpha^D$ $\alpha^C - \alpha^D$	$\beta_A^A - \beta_A^D$ $\beta_A^B - \beta_A^D$ $\beta_A^C - \beta_A^D$	$\beta_B^A - \beta_B^D$ $\beta_B^B - \beta_B^D$ $\beta_B^C - \beta_B^D$	$\beta_C^A - \beta_C^D$ $\beta_C^B - \beta_C^D$ $\beta_C^C - \beta_C^D$	$\beta_D^A - \beta_D^D$ $\beta_D^B - \beta_D^D$ $\beta_D^C - \beta_D^D$	LL	AIC	BIC	McF R-sq	LR test
Sym1	0.000	2.694 (0.162)	−0.000	−0.000	−2.694	−418.554	839.109	843.437	0.259	
PreSym1	1.303 (0.234) −0.602 (0.582) −0.078 (0.113)	3.072 (0.248) 0.000 0.000	0.000 3.072 0.000	0.000 0.000 3.072	−3.072 −3.072 −3.072	−391.487	790.974	808.286	0.307	54.135 (0.000)
PreAsym1	5.630 (0.958) 0.660 (1.510) −0.453 (0.503)	3.186 (0.234) −0.000 0.000	0.000 3.915 (0.390) −0.000	0.000 −0.000 4.204 (0.341)	−4.059 (0.366) −4.059 −4.059	−374.731	763.463	793.758	0.337	33.511 (0.000)
General1	5.305 (3.551) 24.027 (13.552) 7.385 (2.880)	1.407 (0.492) 1.573 (2.415) 0.500 (0.436)	2.982 (0.817) 10.230 (4.974) 1.875 (0.621)	1.004 (0.664) −4.162 (3.931) 5.497 (0.720)	−5.677 (1.243) −11.678 (5.213) −8.944 (1.080)	−337.499	**704.999**	**769.918**	0.403	74.464 (0.000)

Table 10: Estimation of the multivariate logit models for the data from the SI4 blocks. The last 20 periods are used (pooled over experienced and inexperienced participants). The last column shows the results of LR test of the corresponding model against the previous (restricted) model.

with the AIC (which selects the **PreAsym1** model, although the AIC values are quite close).

We observe that the same specification (the **PreSym1** model) was chosen for the SI2 blocks (recall that the time series for alternatives A and B are exactly the same in the SI2 and SI3 blocks). Moreover, the predisposition effect towards alternative A that we found for the SI2 blocks (see Table 6) is preserved when adding a third time series (from Table 9 we see that the estimated predisposition for alternative A, relative to the new alternative C, is 0.688, whereas the predisposition for alternative B, again relative to alternative C, is −1.014). The estimated intensity of choice, however, although still relatively high at $\beta = 1.974$, is about twice as small as the intensity of choice for the **PreSym1** model estimated on the data from the SI2 blocks. Inspection of the time series in the upper left and middle left panels of Fig. 4 may help explain this. Although in periods 1 until about 10 payoffs for alternatives A and B are roughly the same, in the subsequent fifteen periods alternative A consistently outperforms alternative B.

Participants that only need to choose between these two alternatives may therefore learn that the strategy of selecting the alternative that had the highest payoff in the previous period is very successful, which is reflected in a high value of the intensity of choice. The success of such a strategy is slightly inhibited, however, when adding alternative C. This alternative sometimes, but not always, outperforms alternative A in the first 25 periods. This means that choosing the best alternative from the previous period, although still a sensible strategy, will give the optimal choice less often, which is consistent with the decrease in the estimated intensity of choice for the SI3 block.

Remarkably, where the data from both the SI2 and SI3 blocks give rise to the **PreSym1** model, the chosen model for the data from the SI4 blocks, the **General1** model, is considerably more complicated. Although not all estimated coefficients in the **General1** model presented in the last row of Table 10 are significantly different from zero, the restrictions that would reduce that model to the **PreAsym1** model are rejected (let alone simplifying it further to the **PreSym1** model), a finding that is also consistent with the two information criteria. Note that the first three time series in the SI4 blocks are the same as in the SI3 block, and that only the time series corresponding to alternative D is added for the SI4 blocks. This new time series has a rather straightforward relation to the time series of the other three alternatives. In particular, as can be seen from the lower left panel of Fig. 4, it is the worst of the four alternatives in each of the periods 1 until 15, but the best alternative for almost all of the last fifteen periods. One would therefore not expect the addition of this alternative to complicate the decision problem of the participants substantially. A more reasonable explanation may be that participants are in general better able to make decisions between two or three alternatives than between four alternatives, which is consistent with the finding that a much more complex model structure is required to describe their behavior satisfactorily for this latter case.[36]

We now consider the experimental data from the BH3 and BH4 blocks. These two sets of time series are special in the sense that the different time series in each block are not linearly independent. In particular, for the time series from the BH3 blocks we have that $\pi_{A,t} = \frac{1}{2}(\pi_{B,t} + \pi_{C,t})$ for every t, and for the time series from the BH4 block we have $\pi_{A,t} = c_B\pi_{B,t} + c_C\pi_{C,t} + c_D\pi_{D,t}$ with $c_B = c_C \approx 2.8810$ and $c_D \approx -4.7619$ for every t (note that, contrary to the SI blocks, there exists no relationship between the time series from the BH3 (BH4) blocks and those of the BH2 and BH4 (BH3) blocks). This linear dependence has important consequences for estimating the multinomial logit model. More specifically, it means that the parameters of the multinomial logit model are not identified.

For the models to be estimated for the BH3 blocks we address this problem by using the

[36]Note that indeed the (relative) intensities of choice associated with the payoff for alternative D (the three coefficients in the last column of the **General1** model in Table 10) are relatively high, suggesting that participants conditioned their behavior to a substantial extent on payoffs of that alternative. Also note that, although payoffs of one particular alternative may directly effect the fractions of all alternatives, the strongest effect is typically on the fraction of participants using that particular alternative (that is, typically $\beta_h^h > \beta_k^k > 0$ for $k \neq h$ and $k, h \in \{A, B, C\}$).

linear dependence to modify the **General1** model (8) as follows

$$
\begin{aligned}
n_t^A &= \exp\Big[(\alpha^A - \alpha^C) + (\tilde\beta_B^A - \tilde\beta_B^C)\pi_{B,t-1} + (\tilde\beta_C^A - \tilde\beta_C^C)\pi_{C,t-1}\Big]\Big/ Z\,,\\
n_t^B &= \exp\Big[(\alpha^B - \alpha^C) + (\tilde\beta_B^B - \tilde\beta_B^C)\pi_{B,t-1} + (\tilde\beta_C^B - \tilde\beta_C^C)\pi_{C,t-1}]\Big]\Big/ Z\,,\\
n_t^C &= 1/Z\,,
\end{aligned}
\tag{10}
$$

where, for every alternative $h = A, B, C$, we define the modified (and identifiable) coefficients $\tilde\beta_B^h = \beta_B^h + \frac{1}{2}\beta_A^h$ and $\tilde\beta_C^h = \beta_C^h + \frac{1}{2}\beta_A^h$ and where Z is again required for normalization. Note that the interpretation of the parameters $\tilde\beta_k^h$ differs from that of β_k^h (for $k = B, C$). A high value of the former can derive from the fact that participants are sensitive to payoffs generated by alternative k or that they are sensitive to payoffs generated by alternative A (or by a combination of both). This makes interpretation of the estimated model more difficult.

By imposing the restrictions $\tilde\beta_k^h = \tilde\beta_k^l$ for all h, k and l that are different from each other, the **General1** model (10) reduces to the **PreAsym1** model

$$
\begin{aligned}
n_t^A &= \frac{\exp[\alpha_A]}{\exp[\alpha_A] + \exp[\alpha_B + \tilde\beta_B\pi_{B,t-1}] + \exp[\tilde\beta_C\pi_{C,t-1}]}\,,\\
n_t^B &= \frac{\exp[\alpha_B + \tilde\beta_B\pi_{B,t-1}]}{\exp[\alpha_A] + \exp[\alpha_B + \tilde\beta_B\pi_{B,t-1}] + \exp[\tilde\beta_C\pi_{C,t-1}]}\,,
\end{aligned}
$$

where $\alpha_A = \alpha^A - \alpha^C$, $\alpha_B = \alpha^B - \alpha^C$, $\tilde\beta_B = \tilde\beta_B^B - \tilde\beta_B^C$ and $\tilde\beta_C = -(\tilde\beta_C^B - \tilde\beta_C^C)$. Note that this **PreAsym1** model does not include the intensity of choice for alternative A. Indeed, we cannot identify all of the intensities of choice simultaneously; and the way how we reduced parameters in (10) to obtain identification, dictates that the intensity of choice for A is absent.[37] With the additional assumption $\tilde\beta = \tilde\beta_B = \tilde\beta_C$ we obtain the **PreSym1** model given by

$$
\begin{aligned}
n_t^A &= \frac{\exp[\alpha_A]}{\exp[\alpha_A] + \exp[\alpha_B + \tilde\beta\pi_{B,t-1}] + \exp[\tilde\beta\pi_{C,t-1}]},\\
n_t^B &= \frac{\exp[\alpha_B + \tilde\beta\pi_{B,t-1}]}{\exp[\alpha_A] + \exp[\alpha_B + \tilde\beta\pi_{B,t-1}] + \exp[\tilde\beta\pi_{C,t-1}]}.
\end{aligned}
$$

Finally, the **Sym1** model – where predisposition effects are absent – reads

$$
\begin{aligned}
n_t^A &= \frac{1}{1 + \exp[\tilde\beta\pi_{B,t-1}] + \exp[\tilde\beta\pi_{C,t-1}]},\\
n_t^B &= \frac{\exp[\tilde\beta\pi_{B,t-1}]}{1 + \exp[\tilde\beta\pi_{B,t-1}] + \exp[\tilde\beta\pi_{C,t-1}]}.
\end{aligned}
$$

For the models describing the data from the BH4 block we can reduce the number of parameters in the **General1** model from fifteen to twelve by defining for every fund $h =$

[37] If we would express performance of B (resp. C) as a linear combination of the performances of A and C (resp. A and B), then the **PreAsym1** model would not have the intensity of choice for B (resp. C).

Model	$\alpha^A - \alpha^C$ / $\alpha^B - \alpha^C$	$\tilde{\beta}^A_B - \tilde{\beta}^C_B$ / $\tilde{\beta}^B_B - \tilde{\beta}^C_B$	$\tilde{\beta}^A_C - \tilde{\beta}^C_C$ / $\tilde{\beta}^B_C - \tilde{\beta}^C_C$	LL	AIC	BIC	McF R-sq	LR test
Sym1	0.000	0.000	−0.039	−654.266	**1310.531**	**1314.928**	0.005	
			(0.012)					
PreSym1	0.110	0.000	−0.047	−653.891	1313.781	1326.972	0.006	0.750
	(0.135)		(0.017)					(0.687)
	0.063	0.047	−0.047					
	(0.099)							
PreAsym1	0.175	0.000	−0.059	−653.612	1315.224	1332.811	0.006	0.558
	(0.162)		(0.024)					(0.455)
	0.192	0.033	−0.059					
	(0.199)	(0.026)						
General1	−0.230	0.061	−0.029	−650.573	1313.147	1339.528	0.011	6.077
	(0.235)	(0.032)	(0.028)					(0.048)
	0.181	0.060	−0.080					
	(0.227)	(0.031)	(0.029)					

Table 11: Estimation of multivariate logit model for BH3. Last 20 periods of data are used (pooled over experiences). The last column shows the results of LR test of the corresponding model against the previous (restricted) model.

A, B, C, D the parameters $\tilde{\beta}^h_B = c_B\beta^h_A + \beta^h_B$, $\tilde{\beta}^h_C = c_C\beta^h_A + \beta^h_C$, and $\tilde{\beta}^h_D = c_D\beta^h_A + \beta^h_D$. The **PreAsym1**, the **PreSym1** and the **Sym1** models are then found by imposing restrictions that are similar to those discussed above for the models for the BH3 blocks.

Table 11 provides the estimation and test results for the data from the BH3 blocks. The parameter restrictions leading to the **PreAsym1** model are rejected, but only weakly (the p-value is 0.048). Furthermore, for the **PreAsym1** model the restrictions that lead to the **PreSym1** model are not rejected and for the **PreSym1** model the restrictions that lead to the **Sym1** model are also not rejected. In addition, the two information criteria select the **Sym1** model as the best model for the BH3 blocks. Overall, therefore, it seems that the **Sym1** model gives a good description for participants' decisions in the BH3 blocks. Note that this is the only set of time series for which decisions are consistent with the canonical discrete choice model (1).[38] Relevant in this respect might be that the time series from the BH3 blocks are relatively unpredictable (see the middle left and right panels of Fig. 3).

The results are, again, notably different when there are four alternatives, as can be seen from Table 12. Here the two information criteria as well as the LR tests select the **General1** model as the optimal model, which is consistent with the findings for the SI4 blocks.[39] These results suggest that more complex multinomial logit models are required to explain choice behavior when there are four or more alternatives.

[38]The underlying data generating process used a **Sym1** model as well.

[39]Note that the time series in the BH4 block seem to be (slightly) more predictable than those in the BH3 blocks (see Fig. 3), which is also consistent with the better performance of participants in the BH4 block as compared to the BH3 blocks (see Table 5). Again, therefore, it is hard to argue that the decision problem for the BH4 block is substantially more complicated than the decision problem for the BH3 blocks.

Model	$\alpha^A - \alpha^D$ $\alpha^B - \alpha^D$ $\alpha^C - \alpha^D$	$\tilde{\beta}_B^A - \tilde{\beta}_B^D$ $\tilde{\beta}_B^B - \tilde{\beta}_B^D$ $\tilde{\beta}_B^C - \tilde{\beta}_B^D$	$\tilde{\beta}_C^A - \tilde{\beta}_C^D$ $\tilde{\beta}_C^B - \tilde{\beta}_C^D$ $\tilde{\beta}_C^C - \tilde{\beta}_C^D$	$\tilde{\beta}_D^A - \tilde{\beta}_D^D$ $\tilde{\beta}_D^B - \tilde{\beta}_D^D$ $\tilde{\beta}_D^C - \tilde{\beta}_D^D$	LL	AIC	BIC	McF R-sq	LR test
Sym1	0.000	0.000	−0.000	0.031	−387.197	776.395	780.029	−0.241	
				(0.022)					
PreSym1	5.590	0.000	0.000	−0.302	−292.944	593.888	608.427	0.061	188.507
	(0.762)			(0.051)					(0.000)
	3.444	0.302	−0.000	−0.302					
	(0.718)								
	3.806	−0.000	0.302	−0.302					
	(0.715)								
PreAsym1	9.132	0.000	0.000	−0.863	−291.893	595.785	617.594	0.065	2.103
	(3.538)			(0.500)					(0.349)
	7.247	0.258	0.000	−0.863					
	(3.552)	(0.072)							
	7.120	0.000	0.341	−0.863					
	(3.531)		(0.071)						
General1	1.074	11.338	11.327	−22.185	−256.207	**536.415**	**580.032**	0.179	71.371
	(8.101)	(12.651)	(13.358)	(24.731)					(0.000)
	2.445	8.423	7.771	−15.924					
	(8.115)	(12.656)	(13.366)	(24.743)					
	2.656	8.539	9.194	−17.498					
	(8.103)	(12.653)	(13.359)	(24.732)					

Table 12: Estimation of multivariate logit model for BH4. Last 20 periods of data are used. The last column shows the results of LR test of the corresponding model against the previous (restricted) model.

6 Concluding Remarks

In this paper we presented the results of a laboratory experiment where participants observe past performance of two, three or four different investment alternatives (framed as financial funds) and subsequently choose one of these alternatives. Participants do this for a number of consecutive periods and are paid, for each period, according to the payoffs generated by the alternative they chose. The experimental data show that participants switch regularly between alternatives. This switching is driven, to a large extent, by the past payoffs of the different alternatives – in particular payoffs from the most recent period. Moreover, participants are quite successful in exploiting salient patterns in the time series of payoffs. We also find that experience with a *different* set of time series has no discernible effect on choices that participants make, although participants do learn to make better decisions when gaining experience with the *same* set of time series.

Our main goal was to analyze whether the well known *discrete choice model* provides a reasonable description of the decisions participants make. As this discrete choice framework plays a central role in heterogeneous agent models, evidence for its relevance in laboratory experiments would provide a sound empirical microfoundation for these models. Such a microfoundation has been lacking until now. We found that behavior of participants can be described by discrete choice models reasonably well, although a number of important qualifications have to be made.

First, the canonical discrete choice model, used in Brock and Hommes (1997, 1998) and in the vast majority of heterogeneous agent models, tends to be too stylized to describe the decisions of participants in our experiment. In particular, in most cases a *predisposition* towards one of the alternatives, see, e.g., Branch (2004), has to be included. On the other hand, when there is either substantial or almost no structure in the time series of payoffs it is sufficient to consider a discrete choice model which uses only one lag in payoffs, and has symmetric responses to these payoffs (that is, the intensities of choice do not vary across alternatives). For time series with a more complicated autocorrelation structure or a cyclical pattern discrete choice models with asymmetric responses and/or multiple lags give a better description of participants' decisions.

Second, the intensities of choice are not the same for different types of time series: estimated values of the intensity of choice increase with the predictability in the time series of payoffs. This makes intuitive sense, but is worth noting because the typical heterogeneous agent model does not take this relation between predictability and the value of the intensity of choice into account. This suggests that an interesting and relevant extension to the standard theoretical heterogeneous agent model would be to make the intensity of choice in that model time-varying and dependent on the autocorrelation structure in the performance of the different alternatives.

Third, although discrete choice models (with predisposition effects, one lag in payoffs and symmetric responses) tend to give a good description of the choice behavior of participants when they are faced with two or three alternatives, they fail to do so when there are more than three alternatives.

The experimental work presented in this paper can be extended in many directions. For example, one could consider an experiment where participants are only informed about the payoff of the alternative they actually chose, in order to investigate the trade-off between exploration and exploitation they then face.[40] Alternatively, information about past performance of the alternatives might only be shown in tabular form, since a graphical presentation makes the structure in the time series more salient, and may therefore have a positive effect on the estimated intensities of choice. Another extension would be to subtract a small fee (similar to a so-called "back-end load" sometimes charged by mutual funds in order to discourage subscribers from moving out of the fund) from the participants' earnings every time they switch between alternatives to investigate how large and persistent payoff differentials have to be in order for participants to switch alternatives. Herding and imitation behavior can be investigated by allowing (some) participants to observe decisions of other participants.

By considering exogenously generated time series of payoffs in the experiment we abstract from any influence the decisions of participants may have on the actual outcomes. While this can be justified for some market environments (the performance of mutual funds is indeed not strongly influenced by decisions of individual investors) it might not hold for all financial assets. In fact, aggregate market dynamics may be influenced by trader beliefs: if many investors believe that the return of a particular stock will increase, demand for that stock goes up and so will its price and return. One could consider a laboratory experiment in which this feedback is explicitly incorporated. Payoffs of the participants will then also depend upon the decisions the other participants in the experiment make and it will be interesting to see whether, and how, this changes choice behavior.

[40] This would make the experiment closely related to the literature on so-called "multi-armed bandits".

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APPENDIX

A Experimental Instructions

In this experiment you will observe the time series of the financial returns of 2, 3 or 4 investment funds: fund A, B (or fund A, B, C, or fund A, B, C, D) for 100 consecutive periods. You will have to make an investment decision by choosing one of these funds. Your payoff in the experiment will depend on the return of the fund chosen as your investment decision.

The actual returns of the funds will always be positive. The information you have when making your forecasts and investment decision consists of the actual returns of the funds in the recent past. After each 50 periods of predictions, we will show you the returns of new funds, which might not have the same pattern as the former ones.

Earnings

You may earn points for every period of the experiment. The earned points will be transformed to the payoff in Euros at the end of the experiment. **The number of points you get is the return from the fund you have chosen this period.**

At the end of the experiment you are paid 1 euro for each 20 points you earned during the experiment. In addition you will get a show-up fee of 5 euros. As an example, if for one period you choose between 3 funds, and your choice and the actual returns are as in the table below: Then: you will get 3 points for this period, which is the actual return on fund A.

The Fund You choose	Profit of Fund A	Profit of Fund B	Profit of Fund C	Your Payoff
A	3	1.3	4.6	3

B Testing the effect of experience

In Section 4.1 we test the effect of experience with another set of time series on the choice behavior with a new set of time series. For each time period $t = 1, \ldots, 40$ and for every alternative h (with h being A, B, C or D) we observe the shares of choices $s_{t,h}^{\mathrm{InExp}}$ and $s_{t,h}^{\mathrm{Exp}}$ made by the inexperienced and experienced participants, respectively. Under the null hypothesis that the discrete choices in these two populations are drawn from an identical multinomial distribution we can compute the actual, n, and "expected", $\hat{n}$, number of choices of each of the alternatives in the inexperienced and experienced sessions:

$$
\begin{aligned}
n_{t,h}^{\mathrm{InExp}} &= s_{t,h}^{\mathrm{InExp}} \cdot N^{\mathrm{InExp}}, & \hat{n}_{t,h}^{\mathrm{InExp}} &= (n_{t,h}^{\mathrm{InExp}} + n_{t,h}^{\mathrm{Exp}}) \cdot \frac{N^{\mathrm{InExp}}}{N^{\mathrm{InExp}} + N^{\mathrm{Exp}}}, \\
n_{t,h}^{\mathrm{Exp}} &= s_{t,h}^{\mathrm{Exp}} \cdot N^{\mathrm{Exp}}, & \hat{n}_{t,h}^{\mathrm{Exp}} &= (n_{t,h}^{\mathrm{InExp}} + n_{t,h}^{\mathrm{Exp}}) \cdot \frac{N^{\mathrm{Exp}}}{N^{\mathrm{InExp}} + N^{\mathrm{Exp}}}.
\end{aligned}
$$

where N^{InExp} and N^{Exp} are the numbers of participants in the "inexperienced" and "experienced" blocks, respectively. Under the null hypothesis Pearson's statistic

$$
\sum_h \frac{(n_{t,h}^{\mathrm{InExp}} - \hat{n}_{t,h}^{\mathrm{InExp}})^2}{\hat{n}_{t,h}^{\mathrm{InExp}}} + \sum_h \frac{(n_{t,h}^{\mathrm{Exp}} - \hat{n}_{t,h}^{\mathrm{Exp}})^2}{\hat{n}_{t,h}^{\mathrm{Exp}}}
$$

is distributed according to the Chi-squared distribution with $H-1$, degrees of freedom, where H is the number of alternatives. Table 13 shows the $p-$value of this statistic for different time periods and sets of time series.

Period	**WN2**	**BH2**	**BH3**	**SI2**	**SI4**	**Period**	**WN2**	**BH2**	**BH3**	**SI2**	**SI4**
1	0.88	0.12	0.00	0.87	0.93	21	0.33	0.53	0.34	0.96	0.25
2	0.10	0.68	0.38	0.22	0.79	22	0.78	0.71	0.69	0.23	0.35
3	0.90	0.32	0.97	0.98	0.00	23	0.91	0.67	0.00	0.93	0.14
4	0.18	0.88	0.81	0.74	0.95	24	0.81	0.40	0.00	0.43	0.68
5	0.64	0.92	0.75	0.40	0.59	25	0.18	0.71	0.69	0.52	0.54
6	0.78	0.84	0.55	0.02	0.11	26	0.81	0.67	0.69	0.65	0.41
7	0.91	0.66	0.38	0.39	0.43	27	0.90	0.04	0.55	0.29	0.25
8	0.48	0.22	0.72	0.81	0.91	28	0.55	0.72	0.00	0.57	0.38
9	0.81	0.00	0.78	0.74	0.68	29	0.68	0.72	0.38	0.37	0.72
10	0.81	0.32	0.00	0.94	0.53	30	0.89	0.14	0.64	0.48	0.82
11	0.93	0.04	0.81	0.86	0.24	31	0.32	0.00	0.81	0.42	0.25
12	0.19	0.71	0.00	0.93	0.01	32	0.32	0.04	0.93	0.68	0.25
13	0.89	0.67	0.00	0.19	0.08	33	0.55	0.72	0.87	0.55	0.53
14	0.23	0.04	0.00	0.74	0.07	34	0.38	0.00	0.86	0.54	0.25
15	0.32	0.71	0.00	0.97	0.52	35	0.66	0.00	0.69	0.48	0.59
16	0.55	0.00	0.00	0.86	0.09	36	0.53	0.00	0.69	0.59	0.66
17	0.81	0.50	0.69	0.38	0.25	37	0.23	0.00	0.69	0.66	0.38
18	0.90	0.00	0.00	0.93	0.39	38	0.90	0.87	0.00	0.04	0.72
19	0.99	0.00	0.57	0.46	0.43	39	0.76	0.04	0.46	0.46	0.50
20	0.81	0.00	0.00	0.53	0.18	40	0.10	0.04	0.00	0.64	0.53

Table 13: $p-$values for Pearson chi-squared statistics.

C Estimation of Discrete Choice Models

In Section 5 we have presented results on estimating the standard logit model on choices between two, three or four alternatives, with the past performance of these alternatives as explanatory variables. In this appendix we elaborate on the underlying models and estimation procedures.

The most general version of the multivariate logit model predicts that the probability of choosing alternative h at time t by participant i, denoted by $\Pr(c_{i,t} = h)$, is proportional to the exponent of a linear combination of past payoffs of *all* alternatives, i.e., all information available to the participant. We again use capital letters A, B, C and D for the different alternatives but abuse notation slightly by denoting the last alternative by H (so that H is B, C or D when there are two, three or four alternatives, respectively) and the penultimate alternative by H^- (so that H^- denotes A, B or C, respectively). The most general model then has

$$\Pr(c_{i,t} = h) \propto \exp\left[\alpha^h + \beta^h_{A,1}\pi_{A,t-1} + \cdots + \beta^h_{H,1}\pi_{H,t-1} + \beta^h_{A,2}\pi_{A,t-2} + \cdots + \beta^h_{H,2}\pi_{H,t-2} + \ldots\right], \tag{11}$$

for any alternative h. Here α^h and the $\beta^h_{k,\ell}$'s are the coefficients to be estimated, where ℓ indicates the lag (which can go up to a fixed maximum lag L). Coefficient $\beta^h_{k,\ell}$ measures the sensitivity of choice of alternative h with respect to alternative k's payoff ℓ periods ago. Note that the model is indeterminate since adding, for a given other alternative k and lag l, the same constant to the sensitivities of choices for each of the alternatives (i.e., for given k and ℓ change $\beta^h_{k,\ell}$ to $\widetilde{\beta}^h_{k,\ell} = \beta^h_{k,\ell} + \xi$ for $h = A, B, \ldots, H$) will not change the probabilities (the same reasoning holds for the α^h).

We can therefore choose one alternative as the "reference option" (in our case alternative H) and rewrite expression (11) for all $h = A, B, \ldots, H$ as follows

$$\begin{aligned}\Pr(c_{i,t} = h) \propto \exp\big[&(\alpha^h - \alpha^H) + (\beta^h_{A,1} - \beta^H_{A,1})\pi_{A,t-1} + \cdots + (\beta^h_{H,1} - \beta^H_{H,1})\pi_{H,t-1} \\ &+ (\beta^h_{A,2} - \beta^H_{A,2})\pi_{A,t-2} + \cdots + (\beta^h_{H,2} - \beta^H_{H,2})\pi_{H,t-2} + \ldots\big].\end{aligned} \tag{12}$$

The coefficients to be estimated for this model are the predispositions of alternatives $h = A, B, \ldots, H^-$ *relative* to that of alternative H, $\alpha^h - \alpha^H$, and the *relative* intensities of choice of alternative $h = A, B, \ldots, H^-$ with respect to alternative H, $\beta^h_{k,l} - \beta^H_{k,l}$, given the payoffs of the different choices at different lags.

In this paper we estimate different versions of model (12), including those that are described by equations (1) and (2) in the main text (see below how these models are derived from (12)) and test whether behavioral simplifications often implicitly made in theoretical work are consistent with the experimental data.

C.1 Models with One Lag

Consider the one-lag version of model (12) which, after normalizing the probabilities, gives (the subscripts for the single lag have been suppressed)

$$\Pr(c_{i,t} = h) = \frac{\exp\left[(\alpha^h - \alpha^H) + (\beta_A^h - \beta_A^H)\pi_{A,t-1} + \cdots + (\beta_H^h - \beta_H^H)\pi_{H,t-1}\right]}{\sum_{k=A}^{H^-} \exp\left[(\alpha^k - \alpha^H) + (\beta_A^k - \beta_A^H)\pi_{A,t-1} + \cdots + (\beta_H^k - \beta_H^H)\pi_{H,t-1}\right] + 1}, \quad (13)$$

for $h = A, B, \ldots, H^-$, with the remaining probability given by $\Pr(c_{i,t} = H) = 1 - \sum_{h=A}^{H^-} \Pr(c_{i,t} = h)$. We refer to model (13) as the **General1** model.

C.1.1 Binary Choice

When there are only two alternatives, A and B, the (binomial) logit model gives

$$\Pr(c_{i,t} = A) = \frac{\exp\left[(\alpha^A - \alpha^B) + (\beta_A^A - \beta_A^B)\pi_{A,t-1} + (\beta_B^A - \beta_B^B)\pi_{B,t-1}\right]}{\exp\left[(\alpha^A - \alpha^B) + (\beta_A^A - \beta_A^B)\pi_{A,t-1} + (\beta_B^A - \beta_B^B)\pi_{B,t-1}\right] + 1}. \quad (14)$$

There are three (composite) coefficients to be estimated, which we will denote by

$$\alpha = \alpha^A - \alpha^B, \ \beta_A = \beta_A^A - \beta_A^B \text{ and } -\beta_B = \beta_B^A - \beta_B^B. \quad (15)$$

In the case of binary choice, therefore, the most general one-lag model (**General1**) coincides with the **PreAsym1** model (5), which we discussed in the main text.

The **PreAsym1** model is characterised by a predisposition effect, α, and – potentially different – intensities of choices, β_A and β_B. If we impose the restriction that the two intensities of choice are the same, $\beta_A = \beta_B = \beta$, we obtain the **PreSym1** model, corresponding to

$$\Pr(c_{i,t} = A) = \frac{\exp\left[\alpha + \beta(\pi_{A,t-1} - \pi_{B,t-1})\right]}{\exp\left[\alpha + \beta(\pi_{A,t-1} - \pi_{B,t-1})\right] + 1} = \frac{\exp\left[\alpha + \beta\pi_{A,t-1}\right]}{\exp\left[\alpha + \beta\pi_{A,t-1}\right] + \exp\left[\beta\pi_{B,t-1}\right]},$$

which only has the parameters α and β.

Finally, imposing the additional restriction $\alpha = 0$ results in model (1):

$$\Pr(c_{i,t} = A) = \frac{\exp\left[\beta\pi_{A,t-1}\right]}{\exp\left[\beta\pi_{A,t-1}\right] + \exp\left[\beta\pi_{B,t-1}\right]},$$

which we refer to as the **Sym1** model and has only one parameter (the intensity of choice β).

C.1.2 Choice between Three Alternatives

When there are three alternatives, A, B and C, the **General1** model becomes

$$\Pr(c_{i,t} = A) = \frac{\exp[(\alpha^A - \alpha^C) + (\beta_A^A - \beta_A^C)\pi_{A,t-1} + (\beta_B^A - \beta_B^C)\pi_{B,t-1} + (\beta_C^A - \beta_C^C)\pi_{C,t-1}]}{\exp[(\alpha^A - \alpha^C) + \sum_{h=A}^{C}(\beta_h^A - \beta_h^C)\pi_{h,t-1}] + \exp[(\alpha^B - \alpha^C) + \sum_{h=A}^{C}(\beta_h^B - \beta_h^C)\pi_{h,t-1}] + 1} \tag{16}$$

$$\Pr(c_{i,t} = B) = \frac{\exp[(\alpha^B - \alpha^C) + (\beta_A^B - \beta_A^C)\pi_{A,t-1} + (\beta_B^B - \beta_B^C)\pi_{B,t-1} + (\beta_C^B - \beta_C^C)\pi_{C,t-1}]}{\exp[(\alpha^A - \alpha^C) + \sum_{h=A}^{C}(\beta_h^A - \beta_h^C)\pi_{h,t-1}] + \exp[(\alpha^B - \alpha^C) + \sum_{h=A}^{C}(\beta_h^B - \beta_h^C)\pi_{h,t-1}] + 1},$$

with $\Pr(c_{i,t} = C) = 1 - \Pr(c_{i,t} = A) - \Pr(c_{i,t} = B)$. Eight parameters need to be estimated: two relative predispositions, $\alpha^A - \alpha^C$ and $\alpha^B - \alpha^C$, and six relative intensities of choice, $\beta_k^A - \beta_k^C$ and $\beta_k^B - \beta_k^C$ for $k = A, B, C$.

In order to obtain the **PreAsym1** model we impose the following three restrictions

$$\beta_A^B = \beta_A^C, \; \beta_B^A = \beta_B^C \text{ and } \beta_C^A = \beta_C^B . \tag{17}$$

These restrictions imply that the probabilities of choosing any two different alternatives respond in the same way to a change in the payoff associated to the third alternative. If these restrictions are not rejected and we denote

$$\beta_A = \beta_A^A - \beta_A^C, \; \beta_B = \beta_B^B - \beta_B^C, \text{ and } -\beta_C = \beta_C^A - \beta_C^C , \tag{18}$$

then model (16) simplifies to

$$\Pr(c_{i,t} = h) = \frac{\exp[\alpha^h + \beta_h \pi_{h,t-1}]}{\exp[\alpha^A + \beta_A \pi_{A,t-1}] + \exp[\alpha^B + \beta_B \pi_{B,t-1}] + \exp[\alpha^C + \beta_C \pi_{C,t-1}]}, \quad h = A, B, C .$$

This model has five parameters: two relative predisposition effects, $\alpha^A - \alpha^C$ and $\alpha^B - \alpha^C$, and three intensities of choice, β_A, β_B and β_C.

The **PreSym1** model can be obtained from the **PreAsym1** model with two additional restrictions, $\beta_B = \beta_A$ and $\beta_C = \beta_A$ (using the notation from (18)). The resulting model is (denoting β_A by β):

$$\Pr(c_{i,t} = h) = \frac{\exp[\alpha^h + \beta \pi_{h,t-1}]}{\exp[\alpha^A + \beta \pi_{A,t-1}] + \exp[\alpha^B + \beta \pi_{B,t-1}] + \exp[\alpha^C + \beta \pi_{C,t-1}]}, \quad h = A, B, C .$$

This model has three parameters: the two relative predisposition effects, $\alpha^A - \alpha^C$ and $\alpha^B - \alpha^C$, and the intensity of choice β.

Finally, imposing the restrictions $\alpha^A = \alpha^C$ and $\alpha^B = \alpha^C$ we end up with the **Sym1** model which is

$$\Pr(c_{i,t} = h) = \frac{\exp[\beta \pi_{h,t-1}]}{\exp[\beta \pi_{A,t-1}] + \exp[\beta \pi_{B,t-1}] + \exp[\beta \pi_{C,t-1}]}.$$

This model has only one parameter, the intensity of choice, $\beta = \beta_A^A - \beta_A^C$.

C.2 Choice between Four Alternatives

Finally, when the choice is between four alternatives (A, B, C and D) the **General1** model becomes

$$\Pr(c_{i,t}=A) = \exp\left[(\alpha^A-\alpha^D)+(\beta_A^A-\beta_A^D)\pi_{A,t-1}+(\beta_B^A-\beta_B^D)\pi_{B,t-1}+(\beta_C^A-\beta_C^D)\pi_{C,t-1}+(\beta_D^A-\beta_D^D)\pi_{D,t-1}\right]\Big/Z$$
$$\Pr(c_{i,t}=B) = \exp\left[(\alpha^B-\alpha^D)+(\beta_A^B-\beta_A^D)\pi_{A,t-1}+(\beta_B^B-\beta_B^D)\pi_{B,t-1}+(\beta_C^B-\beta_C^D)\pi_{C,t-1}+(\beta_D^B-\beta_D^D)\pi_{D,t-1}\right]\Big/Z$$
$$\Pr(c_{i,t}=C) = \exp\left[(\alpha^C-\alpha^D)+(\beta_A^C-\beta_A^D)\pi_{A,t-1}+(\beta_B^C-\beta_B^D)\pi_{B,t-1}+(\beta_C^C-\beta_C^D)\pi_{C,t-1}+(\beta_D^C-\beta_D^D)\pi_{D,t-1}\right]\Big/Z\,,$$

where Z is the normalizing constant and $\Pr(c_{i,t}=D) = 1-\Pr(c_{i,t}=A)-\Pr(c_{i,t}=B)-\Pr(c_{i,t}=C)$. In this case, fifteen parameters need to be estimated: three relative predispositions, $\alpha^A-\alpha^D$, $\alpha^B-\alpha^D$ and $\alpha^C-\alpha^D$, and twelve relative intensities of choice: $\beta_k^A-\beta_k^D$, $\beta_k^B-\beta_k^D$ and $\beta_k^C-\beta_k^D$ for $k=A,B,C,D$.

Similar to the case with three choices we obtain the **PreAsym1** model if we impose that the respective probabilities of choosing two different alternatives respond in a same way to a change in the payoffs of a third alternative. This gives the following eight restrictions

$$\beta_A^B=\beta_A^C=\beta_A^D,\ \beta_B^A=\beta_B^C=\beta_B^D,\ \beta_C^A=\beta_C^B=\beta_C^D \text{ and } \beta_D^A=\beta_D^B=\beta_D^C.$$

If these restrictions are not rejected and we introduce the following notation

$$\beta_A=\beta_A^A-\beta_A^D,\ \beta_B=\beta_B^B-\beta_B^D\,,\ \beta_C=\beta_C^C-\beta_C^D \text{ and } -\beta_D=\beta_D^A-\beta_D^D, \tag{19}$$

then we can write the **PreAsym1** model as

$$\Pr(c_{i,t}=h)=\frac{\exp[\alpha^h+\beta_h\pi_{h,t-1}]}{\exp[\alpha^A+\beta_A\pi_{A,t-1}]+\exp[\alpha^B+\beta_B\pi_{B,t-1}]+\exp[\alpha^C+\beta_C\pi_{C,t-1}]+\exp[\alpha^D+\beta_D\pi_{D,t-1}]},$$

for $h=A,B,C,D$. In this model seven parameters can be identified: three relative predisposition effects, $\alpha^A-\alpha^D$, $\alpha^B-\alpha^D$ and $\alpha^C-\alpha^D$, and four intensities of choice, β_A,β_B,β_C and β_D.

The **PreSym1** model is obtained from the **PreAsym1** model with the three restrictions $\beta_A=\beta_B=\beta_C=\beta_D$ (using the notation from (19)). The resulting **PreSym1** model (denoting the common intensity of choice by β) is

$$\Pr(c_{i,t}=h)=\frac{\exp[\alpha^h+\beta\pi_{h,t-1}]}{\exp[\alpha^A+\beta\pi_{A,t-1}]+\exp[\alpha^B+\beta\pi_{B,t-1}]+\exp[\alpha^C+\beta\pi_{C,t-1}]+\exp[\alpha^D+\beta\pi_{D,t-1}]},$$

for $h=A,B,C,D$. This model has four parameters: three relative predisposition effects, $\alpha^A-\alpha^D$, $\alpha^B-\alpha^D$ and $\alpha^C-\alpha^D$, and the intensity of choice β.

Finally, the **Sym1** model is a restricted version of the **PreSym1** model with the three restrictions $\alpha^A=\alpha^B=\alpha^C=\alpha^D$. The resulting model is

$$\Pr(c_{i,t}=A)=\frac{\exp[\beta\pi_{h,t-1}]}{\exp[\beta\pi_{A,t-1}]+\exp[\beta\pi_{B,t-1}]+\exp[\beta\pi_{C,t-1}]+\exp[\beta\pi_{D,t-1}]},$$

for $h = A, B, C, D$ and only has the intensity of choice, β, as a parameter.

C.3 Models with Several Lags

For the blocks with two alternatives, A and B, we also estimate the binary choice models with multiple lags. In the most general model the choice is affected by L_A lags of performance for alternative A and L_B lags of performance for alternative B. This model gives (with alternative B as "reference option")

$$\begin{aligned}\Pr(c_{i,t} = A) \propto \exp\Big[&(\alpha^A - \alpha^B) + (\beta^A_{A,1} - \beta^B_{A,1})\pi_{A,t-1} + (\beta^A_{B,1} - \beta^B_{B,1})\pi_{B,t-1} + \\ &+ (\beta^A_{A,2} - \beta^B_{A,2})\pi_{A,t-2} + (\beta^A_{B,2} - \beta^B_{B,2})\pi_{B,t-2} + \\ &+ \cdots + (\beta^A_{A,L_A} - \beta^B_{A,L_A})\pi_{A,t-L_A} + \cdots + (\beta^A_{B,L_B} - \beta^B_{B,L_B})\pi_{B,t-L_B}\Big].\end{aligned}$$

By introducing the notation $\alpha = \alpha_A - \alpha_B$, $\beta_{A,\ell} = \beta^A_{A,\ell} - \beta^B_{A,\ell}$ for $1 \le \ell \le L_A$ and $-\beta_{B,\ell} = \beta^A_{B,\ell} - \beta^B_{B,\ell}$ for $1 \le \ell \le L_B$ the model can be rewritten as

$$\Pr(c_{i,t} = A) = \frac{\exp\left[\alpha + \sum_{\ell=1}^{L_A} \beta_{A,\ell}\pi_{A,t-\ell}\right]}{\exp\left[\alpha + \sum_{\ell=1}^{L_A} \beta_{A,\ell}\pi_{A,t-\ell}\right] + \exp\left[\sum_{\ell=1}^{L_B} \beta_{B,\ell}\pi_{B,t-\ell}\right]}. \tag{20}$$

In the main text we discuss the special case of $L_A = L_B = L$, when the model becomes (6), which we call **PreAsymL**. The model has $2L + 1$ parameters to be estimated. Note that the number of lags necessary to include can be decided using various tests. For instance, model **PreSymL-1** is nested within **PreSymL** when the two restrictions $\beta_{A,L} = 0$ and $\beta_{B,L} = 0$ are imposed.

Imposing the L restrictions $\beta_{A,\ell} = \beta_{B,\ell}$ for every $1 \le \ell \le L$, we obtain model **PreSymL** with $L + 1$ parameters, where there is no asymmetry in responses to the performances of the two alternatives. With one additional restriction, $\alpha = 0$, we obtain model **SymL** with L parameters.

Finally, we might impose the following $L - 2$ restrictions in model **PreSymL**

$$\frac{\beta_L}{\beta_{L-1}} = \cdots = \frac{\beta_\ell}{\beta_{\ell-1}} = \cdots = \frac{\beta_2}{\beta_1}, \text{ for all } 2 \le \ell \le L$$

and denoting the ratio between subsequent intensities of choice as η (and $\beta = \beta_1$) we get the following model

$$\Pr(c_{i,t} = A) = \frac{\exp\left[\alpha + \beta\left(\pi_{A,t-1} + \eta\pi_{A,t-2} + \cdots + \eta^{L-1}\pi_{A,t-L}\right)\right]}{\exp\left[\alpha + \beta\sum_{\ell=1}^{L} \eta^{\ell-1}\pi_{A,t-\ell}\right] + \exp\left[\alpha + \beta\sum_{\ell=1}^{L} \eta^{\ell-1}\pi_{B,t-\ell}\right]}. \tag{21}$$

This model (typically with $\alpha = 0$) has been occasionally used in the literature on heterogeneous agent modelling, see footnote 32. The parameter η is then referred to as the *memory* parameter.

C.4 Method of Maximisation of Likelihood

In the paper we report the estimations of the general model (12) and its various restricted versions on aggregate data from different blocks of time series. To explain the estimation procedure we introduce the following general notation. Let N be the number of observations for one block, K the number of parameters to be estimated, H the number of alternatives and L the number of lags in the model.

Now we denote by $\boldsymbol{X}$ a $N \times (1 + HL)$ matrix of exogenous regressors that is arranged in the following way. The first column of $\boldsymbol{X}$ is a vector of ones. The next H columns (that is, column 2 until column $H+1$) correspond to the performances of the H alternatives, lagged by one period. Columns $H+2$ until $2H+1$ correspond to the performances of the H alternatives, lagged by two periods, etc.

Subsequently, let $\boldsymbol{\beta}$ denote the vector of $K = (1 + HL)(H - 1)$ estimates, arranged as $\boldsymbol{\beta} = \left((\boldsymbol{\beta}^A)', (\boldsymbol{\beta}^B)', \ldots, \left(\boldsymbol{\beta}^{H^-}\right)' \right)'$, where the vector $\boldsymbol{\beta}^h$ (for $h = A, B, \ldots, H^-$) collects the $1 + HL$ coefficients that effect the proportionality coefficient of choosing alternative h given in equation (12), that is it consists of $\alpha^h - \alpha^H$, $\beta^h_{A,1} - \beta^H_{A,1}$ up until $\beta^h_{H,L} - \beta^H_{H,L}$. The vector of estimates $\boldsymbol{\beta}$ therefore can be written as

$$\boldsymbol{\beta} = \Big[\underbrace{\alpha^A - \alpha^H \big| \beta^A_{A,1} - \beta^H_{A,1} \big| \ldots \big| \beta^A_{H,1} - \beta^H_{H,1} \big| \ldots \big| \beta^A_{A,L} - \beta^H_{A,L} \big| \ldots \big| \beta^A_{H,L} - \beta^H_{H,L}}_{(\boldsymbol{\beta^A})'} \Big|$$
$$\underbrace{\alpha^B - \alpha^H \big| \beta^B_{A,1} - \beta^H_{A,1} \big| \ldots \big| \beta^B_{H,1} - \beta^H_{H,1} \big| \ldots \big| \beta^B_{A,L} - \beta^H_{A,L} \big| \ldots \big| \beta^B_{H,L} - \beta^H_{H,L}}_{(\boldsymbol{\beta^B})'} \Big| \ldots \Big]'.$$

For example, in the **General1** model with four alternatives, discussed in Section C.2, matrix $\boldsymbol{X}$ has five columns, and there are five scalars in each of $\boldsymbol{\beta^A}$, $\boldsymbol{\beta^B}$ and $\boldsymbol{\beta^C}$, leading to a total of $K = 15$ parameters to be estimated.

Using this notation the likelihood function can now be derived. Let $I^h_{i,t}$ be an index function of participant i choosing alternative h at time t, so that

$$I^h_{i,t} = \begin{cases} 1 & \text{if } c_{i,t} = h, \\ 0 & \text{otherwise}. \end{cases}$$

For binary choice, for instance, we have $I^B_{i,t} = 1 - I^A_{i,t}$, and $I^C_{i,t} = I^D_{i,t} = 0$. The likelihood function for an individual i is given by

$$\mathcal{L}_i(\boldsymbol{\beta}) = \prod_{t=t_0}^{T} \Pr(c_{i,t} = A)^{I^A_{i,t}} \Pr(c_{i,t} = B)^{I^B_{i,t}} \Pr(c_{i,t} = C)^{I^C_{i,t}} \Pr(c_{i,t} = D)^{I^D_{i,t}}, \tag{22}$$

given that the model is estimated on data from period t_0 to T (with $t_0 = 21$ and $T = 40$ for the estimations presented in the main text). The product of all individual likelihood functions within the blocks with the same time series gives a likelihood function for the discrete choice model estimated on the aggregate data, which is

$$\mathcal{L}(\boldsymbol{\beta}) = \prod_i \mathcal{L}_i(\boldsymbol{\beta}). \tag{23}$$

For aggregate estimation we maximize function $\mathcal{L}(\boldsymbol{\beta})$ given by (23). The sample size is $(T - t_0 + 1)n_p$, where the number of participants n_p for a given block (or for two blocks with the same time series) is as reported in the last column of Table 2. Using (22) and (23) as well as (12) the log-likelihood function becomes

$$\log \mathcal{L}(\boldsymbol{\beta}) = \sum_i \log \mathcal{L}_i(\boldsymbol{\beta}) = \sum_i \sum_{t=t_0}^{T} \left[\sum_{k=A}^{H^-} I_{i,t}^k \cdot \boldsymbol{X_t}\boldsymbol{\beta^k} - \log \left(1 + \sum_{k=A}^{H^-} \exp\left(\boldsymbol{X_t}\boldsymbol{\beta^k}\right) \right) \right], \tag{24}$$

where $\boldsymbol{X_t}$ denotes the $(t - t_0 + 1)$'th row of matrix $\boldsymbol{X}$ (i.e., containing the performances from the previous time period).

For function (24) we compute the gradient vector whose j'th element ($j = 1, \ldots, 1 + HL$) of the first part (corresponding to the derivative with respect to $\boldsymbol{\beta^A}$) is

$$\left[\frac{\partial \log \mathcal{L}(\boldsymbol{\beta})}{\partial \boldsymbol{\beta^A}}\right]_j = \sum_i \sum_{t=t_0}^{T} \left(I_{i,t}^A - \frac{\exp(\boldsymbol{X_t}\boldsymbol{\beta^A})}{1 + \exp(\boldsymbol{X_t}\boldsymbol{\beta^A}) + \exp(\boldsymbol{X_t}\boldsymbol{\beta^B}) + \exp(\boldsymbol{X_t}\boldsymbol{\beta^C})} \right) [\boldsymbol{X_t}]_j,$$

where $[\boldsymbol{X_t}]_j$ is the j'th element of $\boldsymbol{X_t}$. Similar expressions describe the derivatives with respect to the components of vectors $\boldsymbol{\beta^B}$ and $\boldsymbol{\beta^C}$. Setting the gradient vector to zero we get a system of first-order conditions, which we solve numerically to find the estimates.[41]

The Hessian matrix for the log-likelihood function is given by

$$\boldsymbol{H} = \begin{pmatrix} \frac{\partial^2 \log \mathcal{L}(\boldsymbol{\beta})}{\partial \boldsymbol{\beta^A} \partial (\boldsymbol{\beta^A})'} & \frac{\partial^2 \log \mathcal{L}(\boldsymbol{\beta})}{\partial \boldsymbol{\beta^A} \partial (\boldsymbol{\beta^B})'} & \frac{\partial^2 \log \mathcal{L}(\boldsymbol{\beta})}{\partial \boldsymbol{\beta^A} \partial (\boldsymbol{\beta^C})'} \\ \frac{\partial^2 \log \mathcal{L}(\boldsymbol{\beta})}{\partial \boldsymbol{\beta^B} \partial (\boldsymbol{\beta^A})'} & \frac{\partial^2 \log \mathcal{L}(\boldsymbol{\beta})}{\partial \boldsymbol{\beta^B} \partial (\boldsymbol{\beta^B})'} & \frac{\partial^2 \log \mathcal{L}(\boldsymbol{\beta})}{\partial \boldsymbol{\beta^B} \partial (\boldsymbol{\beta^C})'} \\ \frac{\partial^2 \log \mathcal{L}(\boldsymbol{\beta})}{\partial \boldsymbol{\beta^C} \partial (\boldsymbol{\beta^A})'} & \frac{\partial^2 \log \mathcal{L}(\boldsymbol{\beta})}{\partial \boldsymbol{\beta^B} \partial (\boldsymbol{\beta^C})'} & \frac{\partial^2 \log \mathcal{L}(\boldsymbol{\beta})}{\partial \boldsymbol{\beta^C} \partial (\boldsymbol{\beta^C})'} \end{pmatrix}$$

[41] We apply the optimization routine `fminunc` in MATLAB with the minus sign since we actually minimize $-\log \mathcal{L}_i(\boldsymbol{\beta})$. The code is available at request. The formula above is implemented in the program `loglikelihood` to compute the value of variable `llf`. Note that in the data file, the choices are 1 (for A), 2 (for B), 3 (for C) and 4 (for D).

The diagonal blocks of this matrix are

$$\frac{\partial^2 \log \mathcal{L}(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}^{\boldsymbol{A}} \partial (\boldsymbol{\beta}^{\boldsymbol{A}})'} = -\sum_i \sum_{t=t_0}^{T} \frac{\exp(\boldsymbol{X_t}\boldsymbol{\beta}^{\boldsymbol{A}})\big(1+\exp(\boldsymbol{X_t}\boldsymbol{\beta}^{\boldsymbol{B}})+\exp(\boldsymbol{X_t}\boldsymbol{\beta}^{\boldsymbol{C}})\big)}{\big(1+\exp(\boldsymbol{X_t}\boldsymbol{\beta}^{\boldsymbol{A}})+\exp(\boldsymbol{X_t}\boldsymbol{\beta}^{\boldsymbol{B}})+\exp(\boldsymbol{X_t}\boldsymbol{\beta}^{\boldsymbol{C}})\big)^2} \boldsymbol{X}_{\boldsymbol{t}}'\boldsymbol{X_t},$$

$$\frac{\partial^2 \log \mathcal{L}(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}^{\boldsymbol{B}} \partial (\boldsymbol{\beta}^{\boldsymbol{B}})'} = -\sum_i \sum_{t=t_0}^{T} \frac{\exp(\boldsymbol{X_t}\boldsymbol{\beta}^{\boldsymbol{B}})\big(1+\exp(\boldsymbol{X_t}\boldsymbol{\beta}^{\boldsymbol{A}})+\exp(\boldsymbol{X_t}\boldsymbol{\beta}^{\boldsymbol{C}})\big)}{\big(1+\exp(\boldsymbol{X_t}\boldsymbol{\beta}^{\boldsymbol{A}})+\exp(\boldsymbol{X_t}\boldsymbol{\beta}^{\boldsymbol{B}})+\exp(\boldsymbol{X_t}\boldsymbol{\beta}^{\boldsymbol{C}})\big)^2} \boldsymbol{X}_{\boldsymbol{t}}'\boldsymbol{X_t},$$

$$\frac{\partial^2 \log \mathcal{L}(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}^{\boldsymbol{C}} \partial (\boldsymbol{\beta}^{\boldsymbol{C}})'} = -\sum_i \sum_{t=t_0}^{T} \frac{\exp(\boldsymbol{X_t}\boldsymbol{\beta}^{\boldsymbol{C}})\big(1+\exp(\boldsymbol{X_t}\boldsymbol{\beta}^{\boldsymbol{A}})+\exp(\boldsymbol{X_t}\boldsymbol{\beta}^{\boldsymbol{B}})\big)}{\big(1+\exp(\boldsymbol{X_t}\boldsymbol{\beta}^{\boldsymbol{A}})+\exp(\boldsymbol{X_t}\boldsymbol{\beta}^{\boldsymbol{B}})+\exp(\boldsymbol{X_t}\boldsymbol{\beta}^{\boldsymbol{C}})\big)^2} \boldsymbol{X}_{\boldsymbol{t}}'\boldsymbol{X_t},$$

with the off-diagonal blocks given by

$$\frac{\partial^2 \log \mathcal{L}(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}^{\boldsymbol{A}} \partial (\boldsymbol{\beta}^{\boldsymbol{B}})'} = \sum_i \sum_{t=t_0}^{T} \frac{\exp(\boldsymbol{X_t}\boldsymbol{\beta}^{\boldsymbol{A}})\exp(\boldsymbol{X_t}\boldsymbol{\beta}^{\boldsymbol{B}})}{\big(1+\exp(\boldsymbol{X_t}\boldsymbol{\beta}^{\boldsymbol{A}})+\exp(\boldsymbol{X_t}\boldsymbol{\beta}^{\boldsymbol{B}})+\exp(\boldsymbol{X_t}\boldsymbol{\beta}^{\boldsymbol{C}})\big)^2} \boldsymbol{X}_{\boldsymbol{t}}'\boldsymbol{X_t},$$

$$\frac{\partial^2 \log \mathcal{L}(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}^{\boldsymbol{A}} \partial (\boldsymbol{\beta}^{\boldsymbol{C}})'} = \sum_i \sum_{t=t_0}^{T} \frac{\exp(\boldsymbol{X_t}\boldsymbol{\beta}^{\boldsymbol{A}})\exp(\boldsymbol{X_t}\boldsymbol{\beta}^{\boldsymbol{C}})}{\big(1+\exp(\boldsymbol{X_t}\boldsymbol{\beta}^{\boldsymbol{A}})+\exp(\boldsymbol{X_t}\boldsymbol{\beta}^{\boldsymbol{B}})+\exp(\boldsymbol{X_t}\boldsymbol{\beta}^{\boldsymbol{C}})\big)^2} \boldsymbol{X}_{\boldsymbol{t}}'\boldsymbol{X_t},$$

$$\frac{\partial^2 \log \mathcal{L}(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}^{\boldsymbol{B}} \partial (\boldsymbol{\beta}^{\boldsymbol{C}})'} = \sum_i \sum_{t=t_0}^{T} \frac{\exp(\boldsymbol{X_t}\boldsymbol{\beta}^{\boldsymbol{B}})\exp(\boldsymbol{X_t}\boldsymbol{\beta}^{\boldsymbol{C}})}{\big(1+\exp(\boldsymbol{X_t}\boldsymbol{\beta}^{\boldsymbol{A}})+\exp(\boldsymbol{X_t}\boldsymbol{\beta}^{\boldsymbol{B}})+\exp(\boldsymbol{X_t}\boldsymbol{\beta}^{\boldsymbol{C}})\big)^2} \boldsymbol{X}_{\boldsymbol{t}}'\boldsymbol{X_t}.$$

We numerically compute matrix $\boldsymbol{H}$ for the estimated $\hat{\boldsymbol{\beta}}$ and then compute the standard errors of the estimates as the square roots of the diagonal elements of the inverse of this matrix.

Goodness-of-Fit

To evaluate the goodness of fit of a particular model we compute the McFadden R^2 (also known as likelihood ratio index) as follows

$$1 - \frac{\log \mathcal{L}(\hat{\boldsymbol{\beta}})}{\log \mathcal{L}(0)},$$

where $\log \mathcal{L}(0)$ denotes the resulting log-likelihood when only the constant term is estimated.[42]

Model Comparison

To compare different models we use several criteria. First, we report and compare the log-likelihood, $\log \mathcal{L}(\hat{\boldsymbol{\beta}})$, as well as the values for two information criteria penalizing for additional parameters. The first of those, the Akaike Information Criterion, is given by

$$\mathrm{AIC} = 2k - 2\log \mathcal{L}(\hat{\boldsymbol{\beta}}),$$

[42] This measure cannot be larger than one, and it is never negative (except for model **Sym1**, where the constant term is absent). Its interpretation as the percentage explained by the model with respect to the most elementary model is not always useful, though the higher its value, the better.

where k corresponds to the number of parameters. The second criterion, the Bayesian Information Criterion, is computed as

$$\mathrm{BIC} = k \log(N) - 2 \log \mathcal{L}(\hat{\boldsymbol{\beta}}).$$

Minimizing each of these measures over different models can be used as a criterion for model selection.

Second, we will apply the log-likelihood ratio test to compare *nested* models. Let H_0 denote the hypothesis that r given restrictions hold for the model. If $\log \mathcal{L}_{\mathfrak{R}}$ denote the maximised value of log-likelihood for the restricted version of the model and $\log \mathcal{L}_{\mathfrak{U}}$ is the maximised value of the log-likelihood for the unrestricted version, then the log-likelihood ratio

$$\mathrm{LR} = -2(\log \mathcal{L}_{\mathfrak{R}} - \log \mathcal{L}_{\mathfrak{U}}) \,.$$

follows a χ^2_r distribution. If LR is high enough so that the p-value, $1 - F_{\chi^2_r}(\mathrm{LR})$, is small, namely

$$1 - F_{\chi^2_r}(\mathrm{LR}) < \alpha$$

the H_0 hypothesis is rejected at the α significance level (we will typically use $\alpha = 0.05$ and $\alpha = 0.01$).