Introduction to Special Issue on Complexity in Economics and Finance

Mikhail Anufriev
CeNDEF, University of Amsterdam

William A. Branch
University of California, Irvine

February 2, 2009

This special issue of the *Journal of Economic Dynamics and Control* is a collection of 10 papers presented at a workshop on *Complexity in Economics and Finance*, held at the Lorentz center in Leiden, the Netherlands, on October 22-27, 2007.

The benchmark approach to economic and financial economic theory is to assume the existence of a fully rational, representative agent that exists inside an economy with complete markets. In dynamic settings, this approach typically assumes that the representative agent maximizes her expected lifetime utility subject to a (perceived) budget constraint. One implication is that optimizing agents will be forward-looking, and hence, there will be a crucial role for expectations in individual decisions. It follows that depending on the nature of expectations, macroeconomic and financial outcomes will be impacted by the beliefs of agents.

Although expectations play a primary role in macroeconomic and financial outcomes, there is no consensus on how agents actually form expectations. The Rational Expectations (RE) paradigm, still the benchmark model in economics, assumes that agents' subjective expectations are consistent with the actual stochastic process generated by those beliefs. A large literature questions the RE hypothesis on theoretical and empirical grounds. Theoretically, it can be argued that RE are not realistic because of the strong cognitive and computational assumptions required for agents' beliefs to be rational. To actually compute rational beliefs, agents would need to know the precise structure and laws of motion for the economy, even though this structure depends on agents' beliefs (c.f. Evans and Honkapohja (2001)). Moreover, rational expectations models often may possess multiple equilibria, on which individual agents must coordinate. Empirical studies have shown that survey data and macroeconomic data are not consistent with rational expectations. Mankiw, Reis, and Wolfers (2003) and Branch (2004) show that survey data on inflation expectations exhibit a wide degree of time-varying heterogeneity that can not be generated by a representative rational expectation. Milani (2007) finds that a simple New Keynesian model where rational expectations are replaced by an adaptive learning rule is preferred by the data to the best fitting model solved under the assumption of rational expectations. The standard economic approach also fails to explain many important features of economic systems, e.g., the emergence of speculative bubbles and crashes in financial markets, fat tails, long memory and clustered volatility in the returns distribution of

financial assets and nonlinear mean-reverting dynamics of asset prices, see e.g., extensive surveys of LeBaron (2006), Hommes (2006) and Lux (2008).

These findings suggest that a promising avenue for research is to replace the representative agent, rational expectations framework with heterogeneous, boundedly rational agents. The primary question addressed by this literature is whether a model of heterogeneous bounded rationality will support the predictions of the rational expectations paradigm, and if not, what are the implications of a more reasonable model of expectation formation. Another alternative research program is a complexity approach that views markets as *complex evolving systems*. The complexity view emphasizes non-equilibrium price adjustments through the interaction of many heterogeneous agents. According to this view, aggregate market outcomes are thought of as emerging properties of complex systems.

Research on complexity in economics and finance is characterized by an interdisciplinary approach, attracting a number of distinguished scholars from physics, mathematics, computer science, economics and psychology, and using tools developed in statistical physics, chaos and bifurcation theory, theory of nonlinear stochastic processes, laboratory experiments with human subjects and agent-based simulation techniques. A broad spectrum of approaches and viewpoints was a defining feature of the workshop *Complexity in Economics and Finance*. The 10 papers of this special issue reflect this diversity and can be, roughly, grouped into five topical categories. The papers of Branch and McGough and Assenza and Berardi explore the implications of heterogeneous agents in standard macroeconomic settings. The papers of Anufriev and Panchenko, Franke, and Kozhan and Salmon examine the implications for interacting heterogeneous agents in finance. The papers of Heemeijer et al. and Sutan and Willinger turn to experimental settings for evidence in favor of heterogeneous agents models. The papers of Blume and Easley and Cherkashin et al. look at the role of wealth evolution in the market with endogenous prices. A paper of Marsili et al. adopts the statistical econophysics approach to financial market data.

Lawrence Blume and David Easley extend their earlier work in Blume and Easley (1992, 2006). The current paper analyses a dynamic model with rational traders who allocate part of their wealth to the financial market. Asset prices, as well as the agents' wealth, are endogenously determined. An important feature of the model is that even though the traders are utility-maximizing, they have heterogeneous, incorrect beliefs. An intriguing question about the role of markets in such heterogeneous populations then arises: will the market somehow "balance" the heterogeneous beliefs of all different traders and lead to market prices more accurate than individual beliefs? Or, will the market "select" some traders, perhaps those with more accurate beliefs, and market prices will come to reflect the (possibly) misspecified beliefs of these traders? The Blume and Easley model supports the second view. In particular, it implies that assets could be mispriced even in a complete markets setting populated by rational traders. Blume and Easley extend their previous paper to allow for the market survival of multiple agents and demonstrate that the necessary conditions for survival of a single agent are not sufficient in the multiple agents setting. They then provide the appropriate sufficient conditions.

In the contribution by William Branch and Bruce McGough heterogeneous expectations are introduced into a New Keynesian monetary model. The New Keynesian model is a micro founded

monetary model that leads to reduced form equations for output and inflation, the so-called "IS" and "New Keynesian Phillips Curve." This model assumes monopolistically competitive firms, who update their prices occasionally and in a staggered fashion (e.g. Calvo). The firms are owned by households who decide on their lifetime consumption and bond holdings. Branch and McGough replace the assumption of homogeneous expectations by assuming there are a continuum of two different types of agents, each with different methods with which they form their expectations. The agents in these models seek to satisfy their intertemporal Euler equations, which depend on their particular forecasting mechanism, and their perceived budget constraints. Branch and McGough provide a set of axioms for heterogeneous beliefs that yields an aggregation of individual consumption, bond holdings, and price-setting that has the same reduced-form as the New Keynesian model under rational expectations, only with the expectations operator replaced by a convex combination of heterogeneous expectations operators. As an example of the implications of these results, it is shown that a model with agents split between rational and adaptive agents can have important ramifications for monetary policy: in models where policy is set to ensure a unique rational expectations equilibrium may lead to multiple equilibria under heterogeneous expectations.

Peter Heemeijer, Cars Hommes, Joep Sonnemans, and Jan Tuinstra adopt an experimental framework to test whether agents are heterogeneous, and whether that heterogeneity is stable or tends to diminish over time. They place subjects in a laboratory through a variety of simple market experiments where in some treatments the feedback onto price is positive (e.g. asset pricing models) and others it is negative (e.g. cobweb). In Brock and Hommes (1997, 1998) it was shown that if agents are heterogeneous, and the degree of heterogeneity responds to past predictor performance, then the market dynamics may be complex. The experimental framework of Heemeijer, et al., is a natural setting to test these previous theoretical findings. The contribution in this special issue shows that under negative feedback experiments prices converge quickly to their rational expectations values. Under positive feedback, persistent deviations from the rational expectations price occur frequently. Heemeijer, et al., explain this finding by noting that positive feedback models reward speculative behavior and make coordination less likely than negative feedback models, where deviating from the equilibrium can cause wide swings in market prices away from traders' forecasts.

Mikhail Anufriev and Valentyn Panchenko focus on how the institutional features of a financial market may impact the interaction of heterogeneous agents who form their trading strategy based on simple forecasting rules. Most financial market models assume the existence of a Walrasian auctioneer who clears the demand and supply of financial assets. In the landmark paper by Brock and Hommes (1997, 1998), it was shown that if market participants form their demand for assets by selecting from a set of parsimonious forecasting rules then the price dynamics may be unstable and complex. Anufriev and Panchenko address the degree to which this unrealistic, but tractable, market assumption affects the instability results. In addition to a Walrasian auctioneer, this contribution assesses the implications of interacting heterogeneous agents under different micro-structures such as price- and order- driven markets. Surprisingly, even though trade in order-driven markets is typically thought to be less stable than Walrasian markets, Anufriev and Panchenko find that when agents switch between fundamentalist and chartist trading strategies based on relative profit performance, the order-driven markets tend to be more stable. With a sufficiently high sensitivity to relative profit differences, the order-driven markets may still be unstable and yield complex price

and order dynamics. The authors explain this result by the interplay between agents' heterogeneous beliefs and the market micro-structure.

Similar to the contribution by Blume and Easley, Dmitriy Cherkashin, Doyne Farmer and Seth Lloyd are interested in the implications of the competition between different investment strategies for market outcomes. Their work allows for the possibility that strategy payoffs depend on the aggregate behavior of agents, as is the case in many economic situations with feedback between actions and outcomes. Cherakshin, et al., formalize this situation as a betting game. They distinguish between "self-defeating" games where the objective probability of an event decreases with the amount of wealth bet on this event (i.e. negative feedback) and "self-reinforcing" games where the objective probability of an event increases with the betting amount (i.e. positive feedback) Their paper illustrates the long-run outcomes of such a game, i.e., the wealth distribution of players and the strategy payoffs, and they provide results on the speed of convergence. As an extension, the authors then investigate the effect of the presence of a hyper-rational player who knows the strategies of other players together with their wealth distribution. Finally, adopting as a measure of efficiency the absence of profitable opportunities, they find that over time the game becomes more efficient, but convergence is slow. It is especially slow in the self-reinforcing games.

Roman Kozhan and Mark Salmon take the interacting heterogeneous agent approach to the data on exchange rates. One long-standing puzzle in exchange rate models is why it is difficult for any model to forecast better than a pure random walk model of exchange rates. Kozhan and Salmon argue that any representative agent model will fail to outperform a random walk model unless the model imposes time-varying risk premia. Instead, this contribution assumes that agents are heterogeneous in their forecasting model and in their utility functions. Foreign exchange traders choose a forecasting model that is either based on fundamentals or by extrapolating trends (chartist). Kozhan and Salmon observe that one rationale for selecting a forecasting rule is because traders are uncertain about the true underlying process, and argue that agents should incorporate this uncertainty into their decision making along the lines of Gilboa and Schmeidler (1989). Thus, in the Kozhan-Salmon model traders also decide whether to be uncertainty neutral or uncertainty averse (e.g. max-min expected utility). The paper then turns to whether there is evidence in the foreign exchange market for heterogeneity in beliefs and whether uncertainty aversion is an important part of the puzzle. They find that there is heterogeneity, that fundamentalists tend to be uncertainty neutral, and a fraction of chartists are uncertainty averse. These empirical findings suggest that future work in foreign exchange rate prediction should focus on models with heterogeneous agents.

Angela Sutan and Marc Willinger use an experimental setting to study the effect of positive and negative feedback in beauty contest games. Participants in their experiments are asked to guess a number between 0 and 100, the winner will be the one whose guess is closest to the average guess. In a positive feedback setting, the payoff is increasing in the mean guess, while in a negative feedback experiment it is decreasing. Sutan and Willinger find that in negative feedback environments, the experimental findings are closer to the rational expectations outcome then in positive feedback environment. The intuition for this finding is similar to Heemeijer, et al.

Reiner Franke analyses an asset pricing model with three types of agents: fundamentalists, trend-followers and a market-maker. Although, there is a large literature with heterogeneous traders in asset markets, there has been much less focus on the evolution of the market-maker's inventories.

To design a simple, prototype model, where such effects could be studied, Franke incorporates a heterogeneous agent framework into the Beja and Goldman (1980). The resulting model in continuous time is analyzed both analytically and numerically, and the stability properties for various parameter configurations is presented.

The contribution by Tiziana Assenza and Michele Berardi focus on the effect heterogeneity can have in a model of credit cycles. Assenza and Berardi alter the seminal Kiyotaki and Moore (1997) credit model to incorporate out-of-equilibrium dynamics and heterogeneous expectations. The Kiyotaki and Moore model assumes two types of agents, farmers (who borrow) and gatherers (who lend). Because of limited enforcement in the event of default, the lenders impose collateral constraints on farmers. Kiyotaki and Moore assume agents are homogeneous in their beliefs and have perfect foresight. In a Kiyotaki-Moore perfect foresight equilibrium, there is no default and farmers use their loans to invest in their land. Assenza and Berardi instead assume that farmers and lenders have an adaptive learning rule, and the forecasts from these rules may be different. Assenza and Berardi allow default by letting farmers borrow and then use the proceeds from the loan for consumption whenever their own perception of the return to investing is less than the cost to repaying the loan. Because farmers and gatherers may have different beliefs, gatherers may be willing to lend to farmers without anticipating the ensuing default. Thus, heterogeneity in beliefs could account for default in settings where there is no default under rational expectations.

Matteo Marsili, Giacomo Raffaelli and Benedicte Ponsot provide a (potential) explanation of two financial market "stylized facts": an excessive correlation of stock prices and non-stationarity of financial asset correlations. The paper starts with presenting new evidence in favor of these empirical findings and then focuses on an ad hoc model for the price dynamics in a market with different types of traders: fundamentalists, noise traders, trend-followers and a market-maker. In contrast with a standard economic approach and in the spirit of (phenomenological) models of physics, the mathematical structure of the pricing equation is directly assumed in this model and kept as simple as possible to be just sufficient to explain the observed phenomena. Both through simulations and analytic derivations, Marsili, Raffaelli and Ponsot show that the model can generate realistically unstable dynamics of correlations for a large region in the parameter space, where the law of large numbers cannot be applied to the agents' adaptive estimation of the risk and return. The analysis of factors contributing to instability is then straight-forward and informative. The authors show that the region of instability expands with an increase of the volume traded, an increase of the across-assets correlation and, surprisingly, with an increase of the targeted return. On the other hand, the presence of the unstable region does not depend on the risk measure used.

To conclude, the 10 papers in this special issue demonstrate that heterogeneous agent models have distinct theoretical, experimental, and empirical findings from rational expectations.

References

[1] Beja, Avraham, and M. Barry Goldman, 1980, "On the dynamics behavior of prices in disequilibrium," *Journal of Finance*, 35, 235 - 248.

[2] Blume, Lawrence E., and David Easley, 1992, "Evolution and market behavior," *Journal of Economic Theory*, 58, 9-40.

[3] Blume, Lawrence E., and David Easley, 2006, "If you're so smart, why aren't you rich? Belief selection in complete and incomplete markets," *Econometrica*, 74(4), 929 - 966.

[4] Branch, William A., 2004, "The Theory of Rationally Heterogeneous Expectations: Evidence from Survey Data on Inflation Expectations," *Economic Journal*, 114, 592 - 621.

[5] Brock, William A., and Cars H. Hommes, 1997, "A Rational Route to Randomness," *Econometrica*, 65, 1059-1160.

[6] Brock, William A., and Cars H. Hommes, 1998, "Heterogeneous Beliefs and Routes to Chaos in a Simple Asset Pricing Model," *Journal of Economic Dynamics and Control*, 22, 1235-1274.

[7] Evans, George W., and Seppo Honkapohja, 2001, *Learning and Expectations in Macroeconomics*, Princeton University Press.

[8] Hommes, Cars, 2006, "Heterogeneous Agent Models in Economics and Finance," in: *Handbook of Computational Economics*, ed. Tesfatsion, Leigh, and Kenneth L. Judd, 1109-1186.

[9] Kiyotaki, Nobuhiro and John Moore, 1997, "Credit Cycles," *Journal of Political Economy*, 105, 2, 211-248.

[10] LeBaron, Blake, 2006, "Agent-Based Computational Finance," in: *Handbook of Computational Economics*, ed. Tesfatsion, Leigh, and Kenneth L. Judd, 1187-1232.

[11] Lux, Thomas, 2009, "Applications of Statistical Physics to Finance and Economics", in *Handbook on Complexity Research*, ed. Rosser, J. Barkley Jr., *forthcoming*.

[12] Mankiw, N. Gregory, Ricardo Reis, and Justin Wolfers, 2003, "Disagreement about Inflation Expectations," *NBER Macroeconomics Annual 2003*, 18, 209-248.

[13] Milani, Fabio, 2007, "Expectations, Learning and Macroeconomic Persistence," *Journal of Monetary Economics*, 54, 7, 2065-2082.