Coordination of Expectations in Asset Pricing Experiments

Cars Hommes Joep Sonnemans Jan Tuinstra
Henk van de Velden

CeNDEF, Department of Economics, University of Amsterdam
Roetersstraat 11, 1018 WB Amsterdam, The Netherlands

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Corresponding Author: Joep Sonnemans, Department of Economics and CeNDEF, University of Amsterdam, Roetersstraat 11, 1018 WB, Amsterdam, The Netherlands. E-mail: j.h.sonnemans@uva.nl, Phone: ++31-20-525-4249.

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Abstract: We investigate expectation formation in a controlled experimental environment. Subjects are asked to predict the price in a standard asset pricing model. They do not have knowledge of the underlying market equilibrium equations, but they know all past realized prices and their own predictions. Aggregate demand of the risky asset depends upon the forecasts of the participants. The realized price is then obtained from market equilibrium with feedback from individual expectations. Each market is populated by six subjects and a small fraction of fundamentalist traders. Realized prices differ significantly from fundamental values. In some groups the asset price converges slowly to the fundamental price, in other groups there are regular oscillations around the fundamental price. In all groups participants coordinate on a common prediction strategy. The individual prediction strategies can be estimated and correspond, for a large majority of participants, to simple linear autoregressive forecasting rules.

Keywords: experimental economics, expectations, asset pricing, coordination

JEL classification code: C91, C92, D84, G12, G14

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1 Introduction

Expectations play an important role in economics. Decisions of economic agents are based upon their expectations and beliefs about the future state of the market. Through these decisions expectations feed back into the actual realization of the economic variables. This *expectations feedback mechanism* seems to be particularly important for financial markets. For example, if many traders expect the price of a certain asset to rise in the future, their demand for this asset increases which, by the law of supply and demand, will lead to an increase of the market price. This *self-confirming* nature of expectations is typical for speculative asset markets and it illustrates that the "psychology of the market" may be very important. A theory of expectation formation is therefore a crucial part of modeling economic and in particular financial markets.

It is hard to observe or obtain detailed information about individual expectations in real markets. One approach is to obtain data on expectations by survey data analysis, as done for example by Turnovsky (1970) on expectations about the Consumers' Price Index and the unemployment rate during the post-Korean war period. Frankel and Froot (1987) use a survey on exchange rate expectations and Shiller (1990) analyzes surveys on expectations about stock market prices and real estate prices. However, since in survey data research one can not control the underlying economic fundamentals, or the information that the forecaster possesses, it is hard to measure expectation rules in different circumstances.

An alternative approach is to study expectation formation in an experimental setting. In this paper we report the findings of a laboratory experiment about expectation formation in a simple asset pricing model. In this experiment we ask the participants to give their expectation of next period's price of an unspecified risky asset. Submitting predictions is the only task for the participants. They do not have knowledge of the underlying market equilibrium equation, but they know all past realized prices and, of course, their own predictions. Their earnings are inversely related to the prediction error they make. Given the price forecast of a participant, a computer program computes the associated aggregate demand for the risky asset and subsequently the market equilibrium price. The realized price thus becomes a function of the individual forecasts. Our experiment is designed in order to obtain explicit information about expectations of participants in such a controlled expectations feedback environment.

As mentioned above, the experimental approach has certain advantages over survey data research. A first advantage is that the experimenters have control over the underlying fundamentals. Uncertainty about economic fun-

damentals affects expectations of agents in real markets. In the experiment we can control the economic environment and the information subjects have about this environment. In our experiment economic fundamentals are constant over time. Participants have perfect information about the mean dividend and the interest rate, and could use this information to compute the, constant, fundamental price. A second advantage is that we get explicit information about individual expectations. Since in our setup there is no trade, our data is not disturbed by speculative trading behavior, or by changes in the underlying demand and/or supply functions of the participants. Prior to the experiment the only unknown to the experimenters is the way subjects form expectations. Hence, our experimental approach provides us with 'clean' data on expectations.

Finance is currently witnessing an important shift in research emphasis, according to some even a paradigmatic shift, from a modeling approach with perfect, rational agents to a behavioral finance approach with "boundedly rational" agents using simple "rule of thumb" trading strategies. The psychology of investors plays a key role in behavioral finance, and different types of psychology based trading and behavioral modes have been identified in the literature, such as positive feedback trading, trend extrapolation, noise trading, mistaken beliefs, overconfidence, overreaction, optimistic or pessimistic traders, upward or downward biased traders, correlated imperfect rational trades, overshooting, contrarian strategies, etc.. Some key references dealing with various aspects of investor psychology include e.g. Cutler et al. (1990), DeBondt and Thaler (1985), DeLong et al. (1990a, 1990b), Brock and Hommes (1997, 1998), Gervais and Odean (2001) and Hong and Stein (1999, 2003), among others; see e.g. Shleifer (2000) and Hirshleifer (2001) for extensive surveys and many more references on behavioral finance. Individual expectations about future asset prices play a key role and are intimately related to these different behavioral modes. Experiments are well suited to study which behavioral modes prevail in a controlled laboratory environment. Our experiments may be viewed as an attempt to classify individual forecasting rules and how they fit with the behavioral modes of financial traders. We will present experimental evidence for various of these behavioral modes, in particular for correlated imperfect rational forecasting due to trend extrapolation and overreaction. We will also investigate how far these behavioral modes deviate from perfect rationality and to which extent individual forecasting strategies are "irrational".

We study an experimental asset pricing model with 6 participants and a small fraction of computerized 'robot' traders. These robot traders (henceforth called *fundamentalist traders*) always predict the fundamental price and

trade on the basis of that prediction.[1] The robot traders may be viewed as a (small) fraction of traders that are well informed about market fundamentals, and trade on this information ignoring any patterns, such as trends, in prices. One would expect the presence of fundamentalists to make deviations from the fundamental price, such as (temporary) speculative bubbles, less likely and therefore coordination on the fundamental price more likely.

Our main experimental findings are the following. Realized experimental asset prices differ significantly from the (constant) fundamental price. We observe different types of behavior. In some groups the price of the asset converges (slowly) to the fundamental price and in other groups there are large oscillations around the fundamental price. For some groups these oscillations have a decreasing amplitude and prices seem to converge to the fundamental price slowly; in other groups the amplitude of the oscillations is more or less constant over the duration of the experiment or even increasing and there is no apparent convergence.

We are particularly interested in the *individual prediction strategies* used by the participants. Analysis of the predictions reveals that the dispersion between prediction strategies is much smaller than the forecast errors participants make on average. This indicates that participants within a group *coordinate on a common prediction strategy.* Although participants make forecasting errors, they are similar in the way that they make these errors. Estimation of the individual prediction strategies shows that participants tend to use simple linear prediction strategies, such as naive expectations, adaptive expectations or 'autoregressive' expectations. Again, participants within a group coordinate on using the same type of simple prediction strategy. We also find evidence for trend extrapolation and overreaction.

Although economic experiments are well suited for a detailed investigation of expectation formation in a controlled dynamic environment only little experimental work on expectation formation has been done. Williams (1987) considers expectation formation in an experimental double auction market which varies from period to period by small shifts in the market clearing price. Participants predict the mean contract price for 4 or 5 consecutive periods. The participant with the lowest forecast error earns $1.00. In Smith, Suchanek and Williams (1987) expectations and the occurrence of speculative bubbles are studied in an experimental asset market. In a series of related papers, Marimon, Spear and Sunder (1993) and Marimon and Sunder (1993, 1994, 1995) have studied expectation formation in inflationary overlapping

[1]The use of computerized traders in addition to "active" trading by participants is not uncommon in experimental asset markets. See e.g. Bloomfield (1996) and Bloomfield and O'Hara (1999) for experimental markets with computerized informed traders and computerized noise traders.

generations economies. Marimon, Spear and Sunder (1993) find experimental evidence for expectationally driven cycles and coordination of beliefs on a sunspot 2-cycle equilibrium, but only after agents have been exposed to exogenous shocks of a similar kind. Marimon and Sunder (1995) present experimental evidence that a "simple" rule, such as a constant growth of the money supply, can help coordinate agents' beliefs and help stabilize the economy. Although all these papers are clearly related to our work, they can not be viewed as pure experimental testing of the expectations hypothesis, *everything else being constant*, because in all these cases dynamic market equilibrium is affected not only by expectations feedback but also by other types of human behaviour, such as trading behaviour. A number of other laboratory experiments focus on expectation formation exclusively. Schmalensee (1976) presents subjects with historical data on wheat prices and asks them to predict the mean wheat price for the next 5 periods. Two other noteworthy experiments on expectation formation are Dwyer, Williams, Battalio and Mason (1993) and Hey (1994). In these papers a time series is generated by a stochastic process such as a random walk or a simple linear first order autoregressive process, and participants have to predict the next realization, sequentially. The drawback of the last two papers is that no economic context is given. Kelley and Friedman (2002) consider learning in an Orange Juice Futures price forecasting experiment, where prices are driven by a linear stochastic process with two exogenous variables (weather and competing supply). The main difference with our approach is that in the last three papers expectations feedback is ignored.

In our experiment we have explicitly accounted for this expectations feedback, which we believe to be very important for many economic environments, and especially for financial markets. Finally, Gerber, Hens and Vogt (2002) recently studied a repeated experimental beauty contest in which participants each period place either a buy or a sell order. Prices are determined by total market orders and noise. Although this is a positive feedback system like in our experiment, they do not measure expectations explicitly and their experimental environment is more stylized. Similar to our results, a high level of coordination is found.

The paper is organized as follows. Section 2 describes the design of the experiment and Section 3 discusses the underlying asset pricing model. Section 4 presents an analysis of the realized asset prices, whereas Section 5 focuses on the individual prediction strategies. Concluding remarks are given in Section 6.

2 Experimental design

In financial markets traders are involved in two related activities: *prediction* and *trade*. Traders make a prediction concerning the future price of an asset, and given this prediction, they make a trading decision. We designed an experiment that is exclusively aimed at investigating the way subjects form predictions. We solicit predictions from the subjects about the price of a certain asset for the next period. Given these predictions the computer derives the associated individual demand for the asset and subsequently the market clearing price (i.e. the price at which aggregate demand equals aggregate supply). Each subject therefore acts as an advisor or a professional forecaster and is paired with one trader, which may be thought of as a large pension fund. The subject has to make the most accurate prediction for this trader and then the trader (i.e. the computer) decides how much to trade. The earnings of the subjects in the experiment are inversely related to their prediction error.

The experiment is presented to the participants as follows. The participants are told that they are an advisor to a pension fund and that this pension fund can invest its money in a risk free asset (a bank account) with a risk free gross rate of return $R = 1 + r$, where r is the real interest rate, or it can decide to invest its money in shares of an infinitely lived risky asset. The risky asset pays uncertain dividends y_t in period t. Dividends y_t are IID distributed with mean $\overline{y}$. The mean dividend $\overline{y}$ and interest rate r are common knowledge. The task of the advisor (i.e. the participant) is to predict the price of the risky asset. Participants know that the price of the asset is determined by market equilibrium between demand and supply of the asset. Although they do *not* know the exact underlying market equilibrium equation they are informed that the higher their forecast is, the larger will be the fraction of money invested in the risky asset and the larger will be the demand for stocks. They do not know the investment strategy of the pension fund they are advising and the investment strategies of the other pension funds. The participants are not explicitly informed about the fact that the price of the asset depends on their prediction or on the prediction of the other participants. They also do not know the number of pension funds or the identity of the other members of the group.

The information for the participants is given in computerized instructions. Comprehension of the instructions is checked by two control questions. At the beginning of the experiment the participants are given two sheets of paper with a summary of all necessary information, general information, information about the stock market, information about the investment strategies of the pension funds, forecasting task of the financial advisor and information

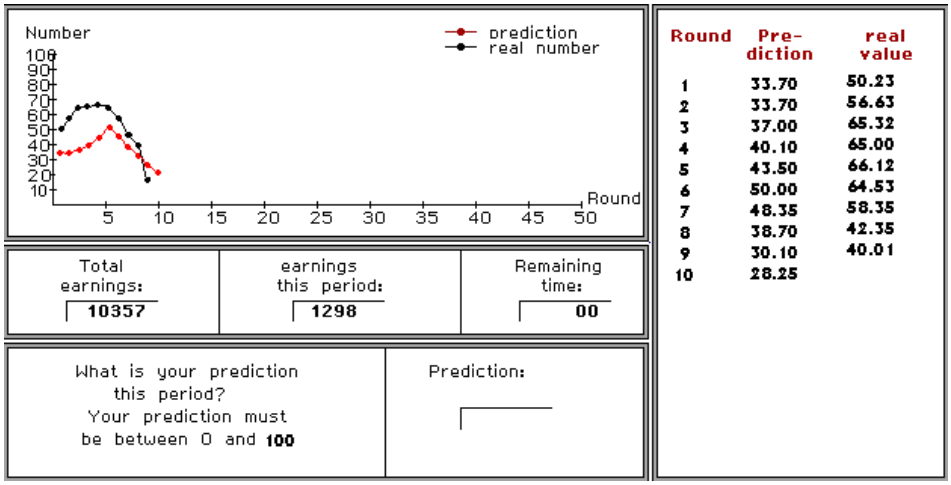


Figure 1: English translation of the computer screen as seen by the participants during the experiment. Predictions and prices have different colors.

about the earnings. The handout also contains information about the financial parameters (mean dividend and risk free rate of return) with which an accurate prediction of the fundamental price can be made. Finally they are given a table from which they can read, for a given forecast error, their earnings (see Appendix C). Appendix B contains an English translation of the information given to the participants.

In every period t in the experiment the task of the participants is to predict the price p_{t+1} of the risky asset in period $t+1$, given the available information. This information consist of past prices of the risky asset $p_{t-1}, p_{t-2}, \ldots, p_1$ and the participant his own past individual predictions $p^e_{ht}, p^e_{h,t-1}, \ldots, p^e_{h1}$, where $p^e_{h\tau}$ is the price participant h expects for period τ. Subjects are told that their price forecast has to be between 0 and 100 for every period. In periods 1 and 2 no information about past prices is available. At the end of period t, when all predictions for period $t+1$ have been submitted, the participants are informed about the price in period t and earnings for that period are revealed. Figure 1 shows an English translation of the computer screen the participants are facing during the experiment. On the screen the subjects are informed about their earnings in the previous period, total earnings, a table of the last twenty prices and the corresponding predictions and a time series of the prices and the predictions.

The earnings of the participants consist of a "show-up" fee of 5 Euro and of the earnings from the experiment which depended upon their forecasting errors. The number of points earned in period t by participant h is given by

the (truncated) quadratic scoring rule

$$e_{ht} = \max\left\{1300 - \frac{1300}{49}\left(p_t - p_{ht}^e\right)^2, 0\right\},$$

where 1300 points is equivalent to 0.5 Euro. Notice that earnings are zero in period t when $|p_t - p_{ht}^e| \geq 7$.[2]

An experimental asset market consists of 6 participants and a certain fraction of fundamentalist traders and it lasts for 51 periods. A total of 60 subjects (10 groups) participated in this experiment. Subjects (mostly undergraduates in economics, chemistry and psychology) were recruited by means of announcements on information boards in university buildings, and via e-mail. The computerized experiment was conducted in the CREED laboratory. It lasted for approximately 1.5 hours and average earnings were 21.46 Euro.

3 The price generating mechanism

3.1 The asset pricing model

The realized prices are generated by a standard asset pricing model with heterogeneous beliefs. For textbook treatments of this model see e.g. Cuthbertson (1996) or Campbell, Lo and MacKinlay (1997). Each trader can choose between investing his money in a risk free asset with a risk free gross rate of return $R = 1 + r$ or investing his money in shares of an infinitely lived risky asset. The price of this risky asset in period t is p_t. For each share dividends y_t are paid out in period t. These dividends are assumed to be independently and identically distributed with mean $\overline{y}$ and variance σ_y^2. The *fundamental value* (i.e. the discounted value of future dividends) of the risky asset is therefore equal to

$$p^f = \frac{\overline{y}}{r}.$$

The asset market is populated by 6 pension funds and a small fraction of fundamentalist traders, as discussed below. Each pension fund h is matched with a participant to the experiment and makes an investment decision at time t based upon this participant's prediction $p_{h,t+1}^e$ of the asset price. The fundamentalist traders always predict the fundamental price p^f and make a trading decision based upon this prediction. Moreover, the fraction n_t of

[2]Paying participants according to quadratic forecast error is equivalent (up to a constant) with paying them according to risk-adjusted profit of the traders (for details see Hommes (2001)).

these fundamental traders in the market is endogenous and depends positively upon the absolute distance between the asset price and the fundamental value.[3] The greater this distance the more these fundamental traders will invest, and the other way around. These fundamentalist traders therefore act as a 'stabilizing force' pushing prices in the direction of the fundamental price. Their presence therefore excludes the possibility of speculative bubbles in asset prices. DeGrauwe, DeWachter and Embrechts (1993) discuss a similar stabilizing force in an exchange rate model with fundamentalists and chartists. In the same spirit Kyle and Xiong (2001) introduce a long-term investor that holds a risky asset in an amount proportional to the spread between the asset price and its fundamental value.

The market clearing price is determined as follows. The amount of shares pension fund h wants to hold in period t depends positively upon the expected excess return $p^e_{h,t+1} + \overline{y} - Rp_t$. This means that an increase in the expected price of the asset for period $t+1$ leads to an increase in demand for the asset in period t. The market clearing price in period t is then given as (cf. Campbell, Lo and MacKinlay (1997), eq. 7.1.4 and Brock and Hommes (1998), eq. 2.7)

$$p_t = \frac{1}{1+r}\left[(1-n_t)\,\overline{p}^e_{t+1} + n_t p^f + \overline{y} + \varepsilon_t\right], \tag{1}$$

where $\overline{p}^e_{t+1} = \frac{1}{6}\sum_{h=1}^{6} p^e_{h,t+1}$ is the average predicted price for period $t+1$. The current period's asset price is therefore determined by (average) beliefs about next period's asset price and an extra noise term ε_t, where the latter corresponds to (small) stochastic demand and supply shocks. Note that the realized price at time t is determined by the price predictions for time $t+1$. Therefore, when traders have to make a prediction for the price in period $t+1$ they do not know the price in period t yet, and they can only use information on prices up till time $t-1$.

In the experiment the risk free rate of return, $r = 0.05$, and the mean dividend are fixed such that $p^f = 60$ (with $\overline{y} = 3$) in 7 of the groups and $p^f = 40$ (with $\overline{y} = 2$) in 3 of the groups. Small demand and supply shocks ε_t are independently drawn from $N\left(0, \frac{1}{4}\right)$. In order to be able to compare the different groups in the experiment, we used the same realizations of the demand and supply shocks for each group. Finally, the weight n_t of the

[3] This is similar to the model discussed in Brock and Hommes (1998) where the fraction n_{ht} of trader using prediction strategy h is also endogenous. In their paper this fraction depends positively upon past performance of the prediction strategy.

fundamentalist traders is given by

$$n_t = 1 - \exp\left(-\frac{1}{200}\left|p_{t-1} - p^f\right|\right), \tag{2}$$

which indeed increases as the price moves away from the fundamental price. Notice that $n_t = 0$ for $p_{t-1} = p^f$. Moreover, given that the fundamental value equals $p^f = 60$ or $p^f = 40$, the weight of the fundamentalist traders is bounded above by $\overline{n} = 1 - \exp\left(-\frac{3}{10}\right) \approx 0.26$. The weight of the other traders is the same for each trader and equal to $(1 - n_t)/6$.

An important feature of the asset pricing model is its *self-confirming* nature: if all traders have a high (low) prediction the realized price will also be high (low). This important feature is characteristic for a speculative asset market: if traders expect a high price, the demand for the risky asset will be high, and as a consequence the realized market price will be high, assuming that the supply is fixed.

3.2 Benchmark expectations rules

This subsection discusses some important benchmark expectations rules in the asset pricing model. In Sections 4 and 5 we will discuss which of these benchmarks gives a good description of the results from our asset pricing experiments. The development of the asset price depends upon the (subjective) expectations of the different trader types. Under rational expectations the subjective expectation E_{ht} of trader type h is equal to the objective mathematical conditional expectation E_t, for all h. Given that bubbles cannot occur in our framework this gives $E_t p_{t+1} = p^f$. Equation (1) then gives

$$p_t = p^f + \frac{1}{1+r}\varepsilon_t.$$

Therefore, under rational expectations p_t corresponds to independent drawings from the normal distribution with mean p^f and variance $(\sigma_\varepsilon/R)^2 = 100/441$. Figure 2 shows the asset price under rational expectations for the realization of the demand and supply shocks that was used in the experiment, when the fundamental value is given by $p^f = 60$.

The rational expectations hypothesis is quite demanding. It requires that participants know the underlying asset pricing model and use this to compute the conditional expectation for the future price and that they do not make structural forecast errors. In particular, rational expectations requires knowledge about the beliefs of all other participants. It will only prevail when participants are able to coordinate on the rational expectations equilibrium.

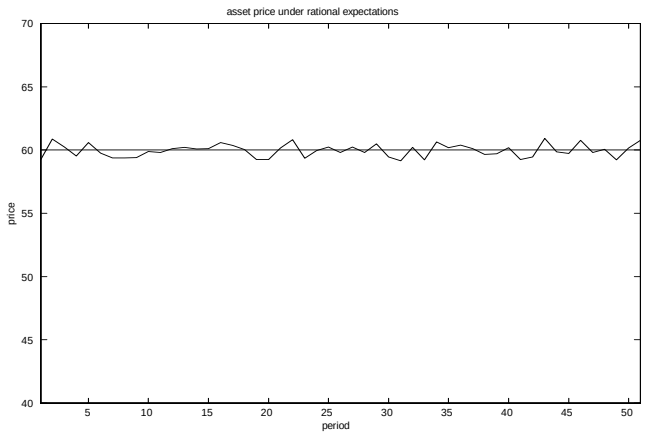


Figure 2: Asset price fluctuations when all participants have rational expectations and forecast $p^e_{h,t+1} = p^f$. The horizontal line at $p^f = \overline{y}/r = 60$ denotes the fundamental value.

Let us now consider asset price behavior when participants use simple forecasting rules instead of rational expectations. They do not have (exact) knowledge of the underlying model, but have their own beliefs about the development of asset prices and use this belief and the available time series observations to predict the price. The belief of a participant is sometimes called a *perceived law of motion*. Given those perceived laws of motion the price generating model is then referred to as the *implied actual law of motion*. The main objective of this paper is to get some insights into the nature of the perceived laws of motion people actually use. When participants have to predict a price for time $t+1$, they know the interest rate r (which is constant over time), the mean dividend $\overline{y}$, the realized prices up to time $t-1$ and their own price predictions up to time t. A general form of a participant's forecasting rule or prediction strategy therefore is

$$E_{ht}\left(p_{t+1}\right) = p^e_{h,t+1} = f_h\left(p_{t-1}, p_{t-2}, \ldots, p_1, p^e_{ht}, p^e_{h,t-1}, \ldots, p^e_{h1}, \overline{y}, r\right), \qquad (3)$$

where f_h can be any (possibly time-varying) function. There are no restrictions on the specification f_h and the possibilities are therefore unbounded. Given participants forecasting rules (3), the implied actual law of motion becomes

$$p_t = \frac{1}{R}\left[\sum_{h=1}^{6}\left(1-n_t\right) f_h\left(p_{t-1}, \ldots, p_1, p^e_{ht}, , \ldots, p^e_{h1}, \overline{y}, r\right) + n_t p^f + \overline{y} + \varepsilon_t\right].$$

The actual dynamics of prices is to a great extent characterized by the prediction strategies used by the traders. Depending on the prediction strategies

used by the agents (which may, for example, be nonlinear or discontinuous) almost any type of price behavior can occur.

We will now briefly discuss the dynamics of our asset pricing model under a number of simple and well known expectation rules. Notice that, since participants know the values of $\overline{y}$ and r, they have enough information to infer the fundamental value and predict it for any period, i.e. they can give $p^e_{h,t+1} = p^f$ as a forecast, for all t.

The perhaps simplest expectations scheme corresponds to *static* or *naive expectations*, where

$$p^e_{h,t+1} = p_{t-1},$$

that is, the participant's prediction for the next price corresponds to the last observed asset price. Under the assumption that all traders have naive expectations the price dynamics reduces to

$$p_t - p^f = \frac{1 - n_t}{1 + r}\left(p_{t-1} - p^f\right) + \frac{1}{1 + r}\varepsilon_t.$$

It can be easily seen that in this case prices will converge to the neighborhood of the fundamental price (see the left panel of Figure 3). Moreover, in the absence of any stochastic demand and supply shocks, prices converge monotonically to the fundamental price. This also holds true for another well known prediction strategy, *adaptive expectations*, which corresponds to

$$p^e_{h,t+1} = wp_{t-1} + (1 - w)\, p^e_{ht} = p^e_{ht} + w\left(p_{t-1} - p^e_{ht}\right),$$

where $0 < w \leq 1$. Hence, under adaptive expectations the prediction is adapted in the direction of the last observed price. The weight parameter w determines how fast predictions are updated. Notice that naive expectations corresponds to a special case of adaptive expectations, where $w = 1$.

We conclude this discussion on prediction strategies by looking at the class of linear autoregressive prediction strategies with 2 lags, that is

$$p^e_{h,t+1} = \alpha_h + \beta_{h1}p_{t-1} + \beta_{h2}p_{t-2}. \tag{4}$$

We will refer to (4) as the $AR\,(2)$ prediction rule. Notice that the endogeneity of the fraction of fundamentalist traders n_t introduces a nonlinearity in the price generating mechanism (1), even if all prediction strategies are linear. Now assume all participants use rule (4) and let $\beta_l = \frac{1}{6}\sum_{h=1}^{6}\beta_{hl}$, for $l = 1, 2$. Depending on the values of β_1 and β_2 one can have different types of dynamics. In particular, if $\beta_1^2 + 4R\beta_2 < 0$ the price will oscillate around the steady state price. In the absence of stochastic demand and supply shocks, these oscillations will converge to the steady state if $\beta_2 > -R$, but they will

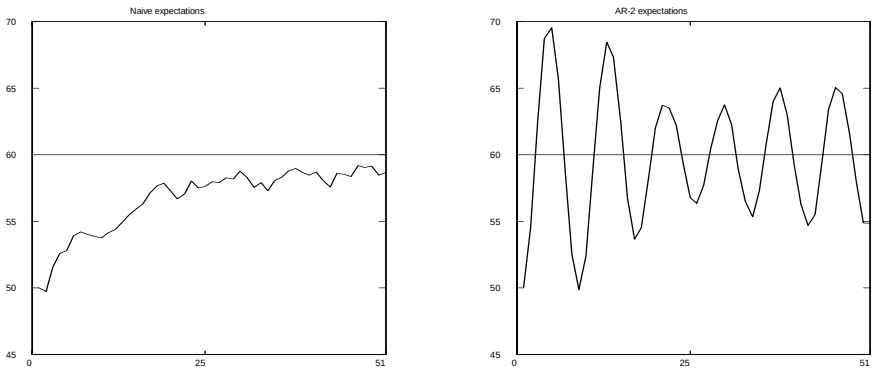


Figure 3: The realized price if all participants use naive expectations (left panel) or $AR\,(2)$ expectations $p^e_{h,t+1} = 30 + \frac{3}{2}p_{t-1} - p_{t-2}$ (right panel).

converge to a limit cycle when $\beta_2 < -R$. On the other hand, if $\beta_1^2 + 4R\beta_2 > 0$, the prices move monotonically or jump up and down, one period below the steady state and the next period above the steady state. If $|\beta_1| + |\beta_2| < R$, these price movements converge to the steady state.

The $AR\,(2)$ prediction strategy (4) can be rewritten as

$$p^e_{h,t+1} = \alpha + \beta p_{t-1} + \delta\,(p_{t-1} - p_{t-2})\,,$$

where $\beta \equiv \beta_1 + \beta_2$ and $\delta \equiv -\beta_2$. Expressed in this way it provides a nice intuition. Participants believe that the price will be determined by the last observation (the first two terms on the right-hand side) but they also try to follow the *trend* in the prices (expressed in the third term): if $\delta > 0$ they believe that an upward movement in prices will continue the next period, whereas if $\delta < 0$ they believe an upward movement in the prices will be (partially) offset by a downward movement in prices in the next period. The former correspond to *trend extrapolators* or *positive feedback traders*, whereas the latter correspond to so-called *contrarians*.

The left panel of Figure 3 shows the evolution of the realized price if everybody in the experiments uses naive expectations, the right panel of Figure 3 shows what happens if everybody uses $AR\,(2)$ expectations $p^e_{h,t+1} = 30 + \frac{3}{2}p_{t-1} - p_{t-2}$ (where we have taken the fundamental price to be equal to 60 in both cases). For both cases we assumed that $p_{h1} = p_{h2} = 50$, for all h. Furthermore, we used the same realization of demand and supply shocks ε_t as in the experiment.

4 Aggregate behavior of asset prices

Figure 4 shows the realized asset prices in the experiment for the ten groups. In the first seven groups the fundamental value equals 60, whereas in the last three groups the fundamental value equals 40. The horizontal line in the graphs corresponds to the fundamental price for that group.

We can classify the different groups in three different categories:

i) *monotonic convergence*: the price in groups 2 and 5 seems to converge monotonically to the fundamental price from below;

ii) *converging oscillations*: the price in groups 4, 7 and 10 oscillates around the fundamental price but the amplitude of the oscillations decreases over time indicating convergence to the fundamental price; and

iii) *persistent oscillations*: the price in groups 1, 6, 8 and 9 oscillates but the amplitude of this oscillations seems to be constant or even increasing. In these groups there does not seem to be convergence to the fundamental price.

Group 3 is more difficult to classify, it starts out with oscillations, but from a certain period on there seems to be monotonic convergence to the fundamental price.[4]

Comparing Figure 4 with Figures 2 and 3 one observes that realized prices under the naive expectations benchmark resemble realized prices in groups 2 and 5 of the experiment remarkably well. On the other hand, the oscillatory behavior of the realized price in groups 1, 4, $6-10$ in the experiment is qualitatively similar to the asset price behavior when participants use $AR\,(2)$ prediction strategies. Clearly, naive and $AR\,(2)$ prediction strategies give a qualitatively much better description of aggregate asset price fluctuations in the experiment than does the benchmark case of rational expectations. Recall from Section 3 that an $AR\,(2)$ rule can be interpreted as a trend following forecasting strategy.

Figure 5 shows the sample average and sample variance of realized prices for the 10 groups. The figure also represents sample averages and sample variances of three important benchmarks discussed in Section 3. They are denoted RE (where $p^e_{h,t+1} = p^f$, for all h), Naive (where $p^e_{h,t+1} = p_{t-1}$, for all

[4]The sudden fall of the asset price in group 3 from 55.10 in period 40 to 46.93 in period 41 is due to the fact that one of the participants predicts 5.25 for period 42. It is likely that this corresponds to a typing error (maybe his/her intention was to type 55.25), since this participants' 5 previous predictions all were between 55.00 and 55.40, giving him/her the very high average earnings of 1292 out of 1300 points in these periods.

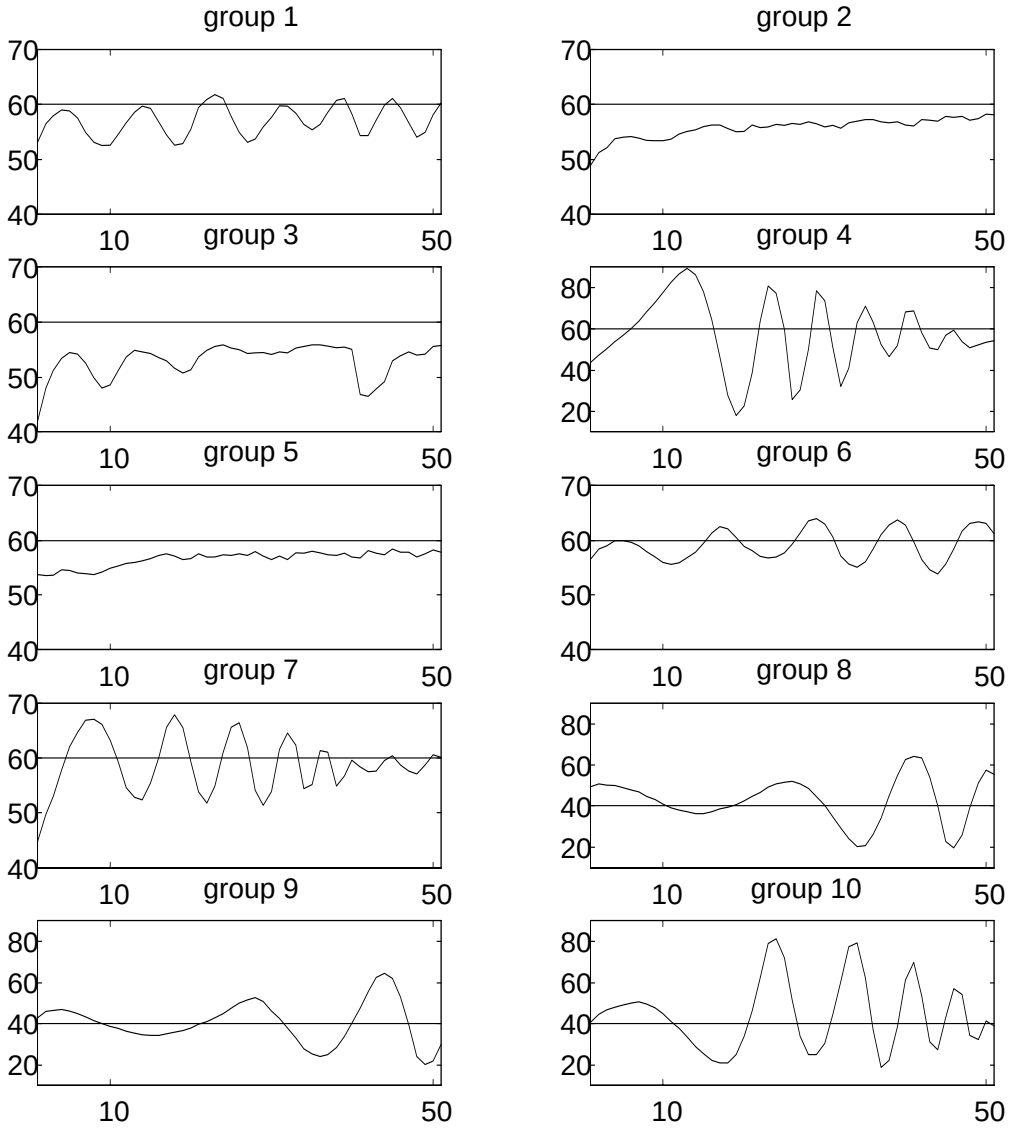


Figure 4: Realized prices for the different groups. The horizontal lines at $p = 60$ (groups 1 to 7) and $p = 40$ (groups $8, 9$ and 10) correspond to the fundamental price.

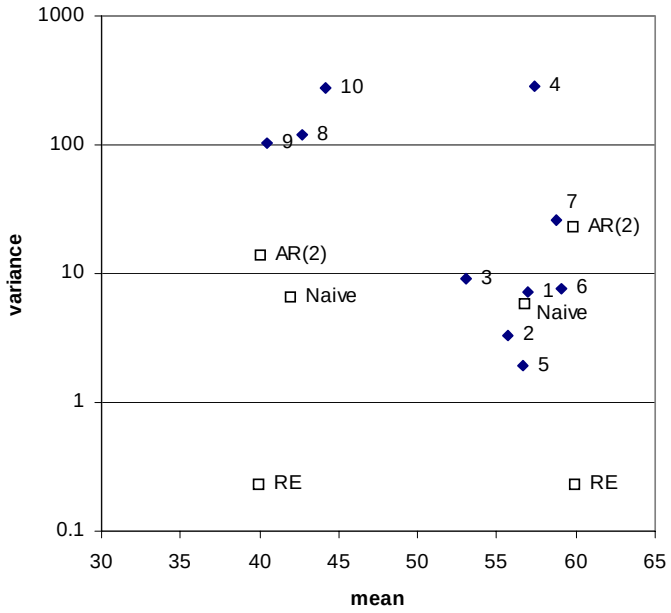


Figure 5: Mean and variance (on a logarithmic scale) of realized asset prices.

h and all $t > 1$) and $AR(2)$ (where $p^e_{h,t+1} = \frac{1}{2}p^f + \frac{3}{2}p_{t-1} - p_{t-2}$, for all h and all $t > 2$), respectively. All of these benchmarks are computed once for the case with fundamental value $p^f = 60$ and once for the case with $p^f = 40$. Inspection of Figure 5 confirms our earlier conclusion: naive expectations or $AR(2)$ expectations gives a much better description of aggregate price behavior than does rational expectations. Comparing the rational expectations benchmark with the 7 experimental groups with fundamental value $p^f = 60$, we see that the sample average is lower and the sample variance is higher in the experiment than under rational expectations. From this we conclude that in this experimental asset pricing model we have *i) undervaluation* of the asset; *ii) excess volatility* of the asset prices. Moreover, sample average and variance of the realized prices are more in line with those of naive and $AR(2)$ expectations. In terms of sample mean and sample variance naive and $AR(2)$ expectations yield much better results than rational expectations. The same holds, by and large, for the three groups with fundamental price $p^f = 40$, although the sample variance in these groups is rather large.

The undervaluation of the asset in the first seven groups can be explained as follows. We have restricted prices to lie between 0 and 100. Since agents have no prior information about the price generating process, many initial

guesses lie around 50. Most of the initial guesses will therefore be smaller than the fundamental price of 60. In fact, the first realized price p_1 is 48.96 on average (averaged over the seven groups), whereas the final realized price p_{51} is 58.18 on average. So, the undervaluation actually (slowly) disappears as time goes by. Also the volatility of prices decreases over time. In particular for the groups where there is slow but steady convergence to the fundamental price, the variance in the second subinterval approaches the variance under rational expectations. By the same argument we have overvaluation in the three groups with a fundamental value $p^f = 40$, which lies beneath the midpoint of the interval of admissible prices. Here we have $p_1 = 44.31$ on average (averaged over groups 8, 9 and 10) and $p_{51} = 41.46$ on average.

As a final remark on the realized asset prices we note that the influence of the fundamentalist traders on the asset pricing dynamics seems to be limited. For groups 4 and 10 the maximum weight of the fundamentalist trader becomes 0.191 and 0.186, respectively, reducing the weight of an individual participant to 0.135 and 0.136 for that period, respectively. For all other groups the maximum weight of the fundamentalist traders is smaller than 0.115, which is, even in that period, significantly smaller than the weight of an individual participant.

5 Individual prediction strategies

We now turn to the individual prediction strategies of the participants in our asset pricing experiment. In Subsection 5.1 we show that participants tend to coordinate on a common prediction strategy. Subsection 5.2 discusses earnings per group. Subsection 5.3 investigates whether participants use the available information efficiently. In Subsection 5.4 we present results on characterizing and estimating the individual prediction strategies. Subsection 5.5 presents four additional groups without fundamentalist traders.

5.1 Coordination

Figure 6 shows, for each group, the predictions of all participants. A striking feature of Figure 6 is that different participants within one group seem to *coordinate* on some common prediction strategy. This coordination of expectations is obtained in all ten groups.

In order to quantify this coordination on a common prediction strategy

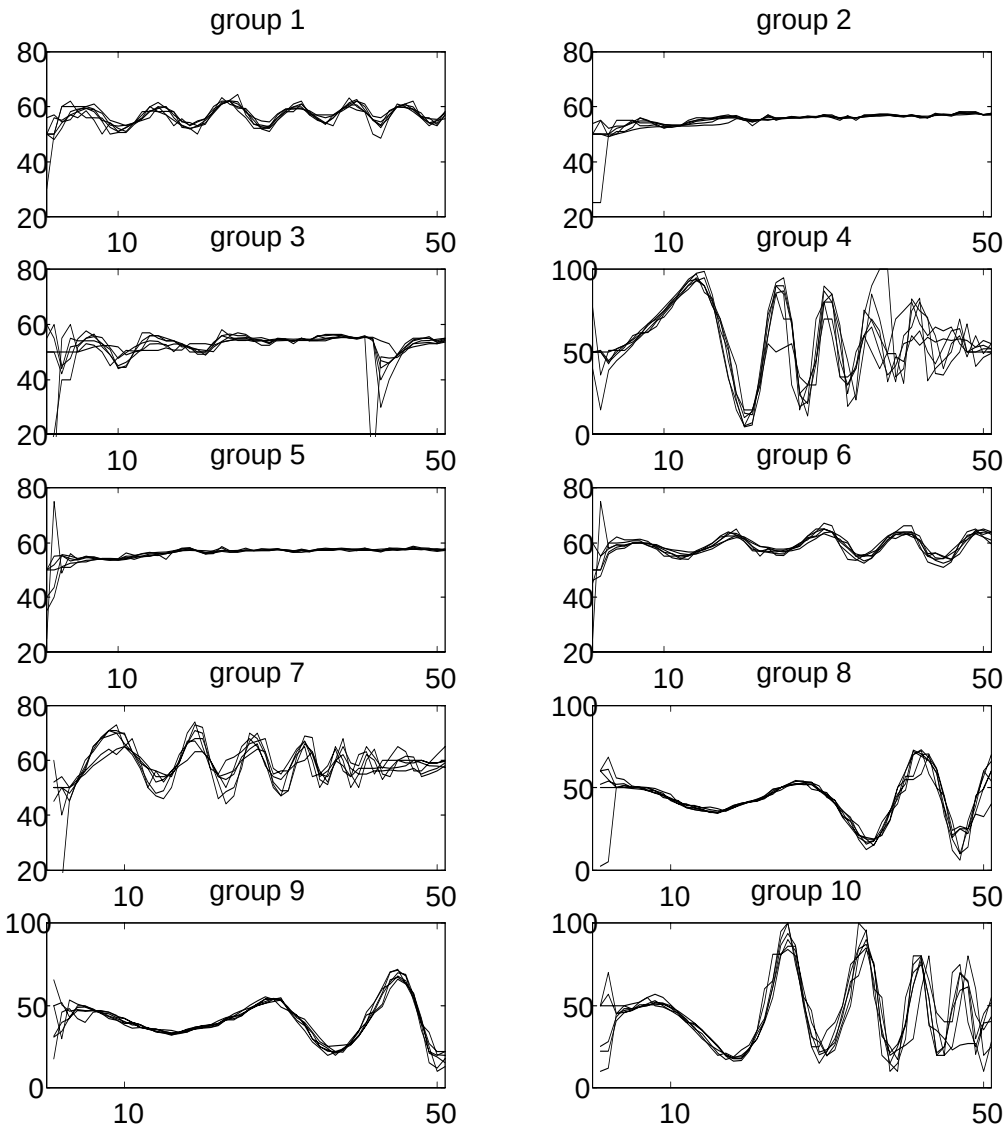


Figure 6: Individual predictions for each group.

we consider, for each group, the *average individual quadratic forecast error*

$$\frac{1}{6 \times 41} \sum_{h=1}^{6} \sum_{t=11}^{41} \left(p_{ht}^{e} - p_t\right)^2,$$

which corresponds to the individual quadratic forecast error averaged over time and over participants within a group. Note that the first 10 observations are neglected in order to allow participants to learn how to predict prices accurately. Defining $\overline{p}_t^e = \frac{1}{6} \sum_{h=1}^{6} p_{ht}^e$ as the average prediction for period t in a group (averaged over individuals in that group) we find that the average individual quadratic forecast error can be broken up into two separate terms, as follows

$$\frac{1}{6 \times 41} \sum_{h=1}^{6} \sum_{t=11}^{51} \left(p_{ht}^{e} - p_t\right)^2 = \frac{1}{6 \times 41} \sum_{h=1}^{6} \sum_{t=11}^{51} \left(p_{ht}^{e} - \overline{p}_t^e\right)^2 + \frac{1}{41} \sum_{t=11}^{51} \left(\overline{p}_t^e - p_t\right)^2. \tag{5}$$

The first term on the right-hand side of (5) measures the *dispersion between individual predictions*. It gives the distance between the individual prediction and the average prediction $\overline{p}_t^e$ within the group, averaged over time and participants. Note that it equals 0 if and only if all participants, in one group, use exactly the same prediction strategy. Hence, this term measures deviation from coordination on a common prediction strategy. The second term on the right-hand side of (5) measures the average distance between the mean prediction $\overline{p}_t^e$ and the realized price p_t. If individual expectations can be described as "rational expectations with error", where the error has mean zero and is serially uncorrelated and uncorrelated with the errors of the other participants, then we should expect that individual forecast errors cancel each other out in the aggregate. This is consistent with Muth (1961) who gives the following formulation of the rational expectations hypothesis (p.316):

"The hypothesis can be rephrased a little more precisely as follows: that expectations of firms (or, more generally, the subjective probability distribution of outcomes) tend to be distributed, for the same information set, about the prediction of the theory (or the "objective" probability distributions of outcomes)."

In other words, individual expectations may be wrong, but in the aggregate expectations should be approximately correct. If this is the case then this second term should be relatively small.

Table 1 shows, for each of the ten groups, how the average quadratic forecast error can be broken up in these two terms.[5]

	avg. individual error	avg. dispersion error	avg. common error
group	$\frac{1}{246}\sum_{h,t}\left(p^e_{ht}-p_t\right)^2$	$\frac{1}{246}\sum_{h,t}\left(p^e_{ht}-\overline{p}^e_t\right)^2$	$\frac{1}{41}\sum_{t}\left(\overline{p}^e_t-p_t\right)^2$
1	6.38	1.28 (20%)	5.10 (80%)
2	0.77	0.19 (25%)	0.58 (75%)
3	7.58	2.86 (38%)	4.72 (62%)
4	325.77	93.21 (29%)	232.56 (71%)
5	0.55	0.11 (20%)	0.44 (80%)
6	5.15	1.24 (24%)	3.91 (76%)
7	24.76	8.52 (34%)	16.24 (66%)
8	59.78	13.31 (22%)	46.48 (78%)
9	36.11	4.31 (12%)	31.80 (88%)
10	277.65	70.85 (26%)	206.80 (74%)

Table 1: Different measures for the individual prediction strategies

From inspection of Table 1 it is clear that only a relatively small part (ranging from 12% in group 9 to 38% in group 3) of the average quadratic forecasting error (first column) can be explained by the dispersion in expectations (second column). In fact, on average 75% of the average quadratic forecast error can be attributed to the average common error. This confirms our conjecture that there is coordination on a common prediction strategy. The observation that a relatively large part of the average quadratic forecast error is due to the difference between the average expectation and the realized price (third column) implies that "rational expectations with error" is not a good description of participants' expectation formation. In fact, it suggests that participants' mistakes are correlated. We therefore conclude that participants make significant forecasting errors, but they are alike in the way that they make these forecasting errors.

5.2 Earnings

Comparing Figure 6 with Figure 4 suggests that the participants are performing quite well. Indeed, earnings from predicting can be substantial. The total number of points they receive when always making the correct prediction is 66300 and under rational expectations earnings would be 65975. In

[5]For group 3, we have excluded the observation at time $t = 42$, where one of the participants appeared to make a typing error (see footnote 4), which has a big impact on these measures. If we include this observation we get 15.70, 11.10 and 4.60, respectively.

the experiment participants earn on average 46939 points. Participants in groups 4 and 10 earn a relatively small amount (20683 and 24470 respectively, on average), whereas participants in groups 2 and 5 on average make substantial earnings, close to the maximum (64168 and 63739 respectively). The other groups are somewhere in between. The prices in groups 2 and 5 are not equal to the fundamental price (the only rational expectations price) but the earnings in these groups are almost as high as earnings of rational forecasters. In this sense the behavior of these subjects can be considered as 'close' to rational. To some extent the same can be said about the other groups with the exception of groups 4 and 10. These last groups show a relatively high price volatility.

5.3 Informational efficiency

The analysis of Table 1 suggests that participants make structural forecast errors. However, if participants are rational their forecast error should be *unbiased* and *uncorrelated* with available information. To test whether participants are rational in this sense we considered the time series of the forecast errors $p_t - p^e_{ht}$, where we only used the last 41 observations. The sample average of these individual forecast errors is significantly different from 0 at the 5% level, for only 8 of the 60 participants. This means that for more than 85% of the individuals forecast errors are unbiased. Furthermore, we computed, for each participant, the first 10 lags of the autocorrelation function of the time series of forecast errors. The significant lags are presented in Table 2.

	part. 1	part. 2	part. 3	part. 4	part. 5	part. 6
1	1-3-4-5-7-8	1-3-4-7-8	1-3-4-5	1-3-4-5-7-8	1-3-4-7-8	1-3-4-5-7-8
2	1-2	–	1	1	1	2
3	–	–	1	1-2	1	1
4	1-3-4	1-3-4	1-3-6	1-3-4-6	1-3-4	1-3-8
5	1	2	–	–	2	1-8
6	1-4-5-6-9-10	1-4-5-6-9-10	1-4-5-6-10	1-4-5-10	1-4-5-6-9-10	1-4-5-6-9-10
7	1	1-2-3	1-2-3-8	1-3-4-8	1-2-3	1-3-4-8
8	1-4-5	1-2-4-5-6-7-8	1-4-5-6-7	1-4-5-6-7	1-4-5-6-7	1-4-5-6-7
9	1-2-5-6-7-8-9	1-2-5-6-7-8-9	1-2-5-6-7	1-5-6	1-5-6-7	1-2-5-6-7-8-9
10	1-4	1-3-4	1-3-4-5	1-3	1-3-4	1-3-4

Table 2: Autocorrelation structure in individual forecast errors. This table presents all significant lags at the 5% level.

Notice that the autocorrelation function of the forecast errors is signif-

icant at the first lag for many participants. However, participants do *not* have p_t in their information set, when predicting p_{t+1}. Hence, they are not able to exploit the first order autocorrelation structure in the forecast errors to improve their predictions. Therefore one should ignore the significant first order lags and focus on higher order lags of the autocorrelation function. We thus find that for about one fourth of the participants there is no exploitable (linear) structure in the forecast errors at all. Ignoring the first lag, we note that the second lag is only significant for 13 out of the 60 participants. Stated differently, the most easily detected linear structure has been exploited efficiently by 47 participants. In this sense individual forecasts of about 80% of all participants may be viewed as *boundedly rational*. Notice that most structure in the forecast errors can be found in the groups where the realized price oscillates around the fundamental price. Furthermore, there is much similarity between the autocorrelation structure of participants within a group, again indicating that participants in the same group seem to coordinate on a common prediction strategy.

5.4 Characterizing individual prediction strategies

We will now try to characterize and estimate the individual prediction strategies. Some participants try to extrapolate certain trends and by doing so overreact and predict too high or too low. Other participants are more cautious when submitting predictions. When prices are rising (declining) they usually predict a price lower (higher) than the actual price. Examples of the latter are participant 1 in group 2, participant 6 in group 6 and participant 4 in group 7 in Figure 7.(a)-(c). Figure 7.(d)-(f) shows three examples of trend extrapolators, participant 3 in group 4, participant 2 in group 6 and participant 3 in group 7. These prediction strategies exhibit an *overreaction* of predictions with respect to trends or changes in prices.

The individual degree of overreaction can be quantified as follows. Figure 8 shows, for each group, the average absolute (one-period) change in predictions of participant h,

$$\triangle_h^e = \frac{1}{41}\sum_{t=11}^{51}\left|p_{ht}^e - p_{h,t-1}^e\right|.$$

The average absolute change in the price, $\triangle = \frac{1}{41}\sum_{t=11}^{51}|p_t - p_{t-1}|$ is represented by the straight line. We will say that individual h *overreacts* if $\triangle_h^e > \triangle$ and we will say that individual h is *cautious* if $\triangle_h^e \leq \triangle$.

Figure 8 measures the *degree of overreaction*. For a vast majority of participants in groups 1, 3, 4 and $6-10$ the individual degrees of overreaction

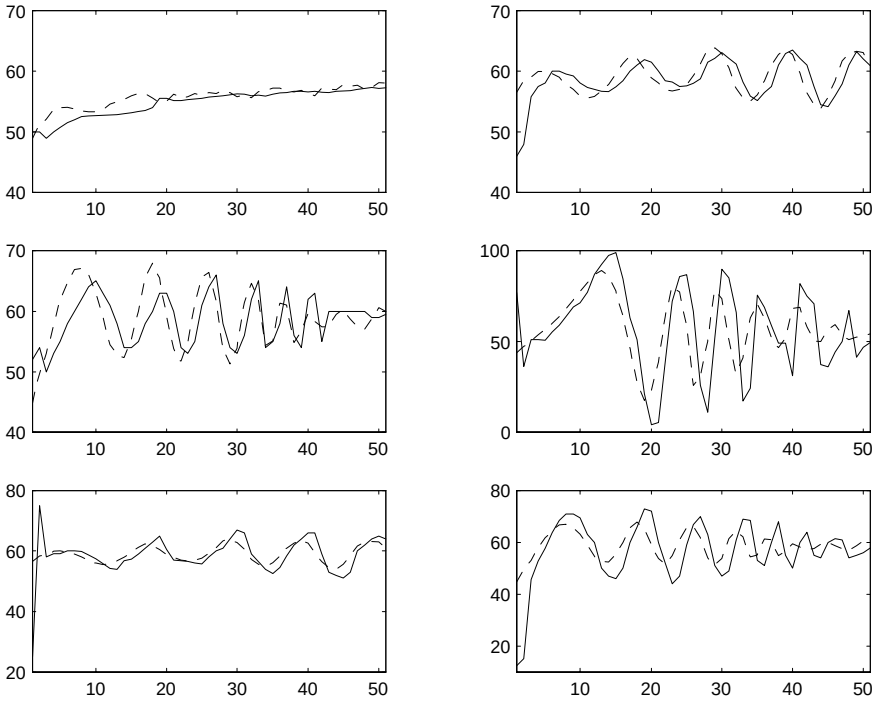


Figure 7: Predictions and realized prices. Solid lines correspond to individual predictions, dashed lines correspond to realized price. Upper left panel: participant 1 in group 2, upper right panel: participant 6 in group 6, middle left panel: participant 4 in group 7, middle right panel: participant 3 in group 4, lower left panel: participant 2 in group 6 and lower right panel: participant 3 in group 7.

are higher than the changes in the realized prices. Oscillatory behavior is thus caused by overreaction of a majority of agents. In groups 2 and 5 the changes in predictions are similar to the changes in prices. Convergence to the fundamental price occurs when a majority of traders is 'cautious'.

The final step in our analysis of the individual prediction strategies is to try to estimate simple forecasting rules. The prediction strategies of all 60 participants can be described by the following general simple linear model

$$p^e_{h,t+1} = \alpha_h + \sum_{i=1}^{4} \beta_{hi} p_{t-i} + \sum_{j=0}^{3} \gamma_{hj} p^e_{ht-j} + \nu_t, \tag{6}$$

where ν_t is an independently and identically distributed noise term. Notice that this general structure includes several interesting special cases: *i)*

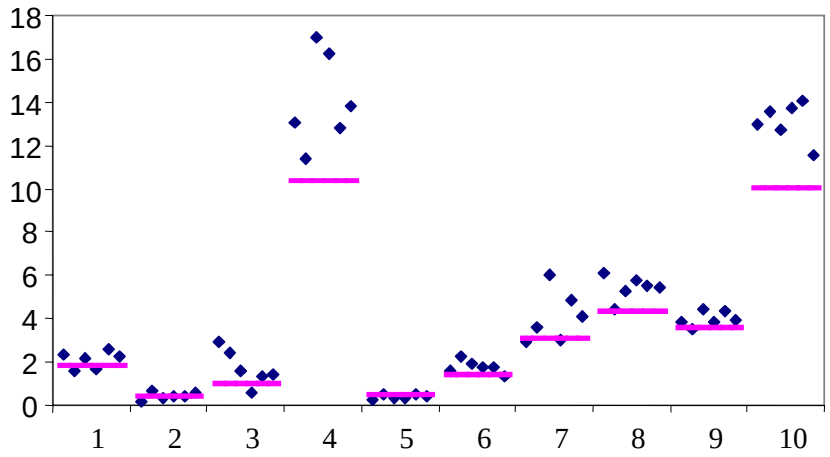


Figure 8: Average absolute changes in predictions and prices. The horizontal line for each group corresponds to average absolute price change $\triangle$, and the dots correspond to the average absolute price forecast $\triangle_h^e$ for the different participants.

naive expectations ($\beta_{h1} = 1$, all other coefficients equal to 0); *ii)* adaptive expectations ($\beta_{h1} + \gamma_{h0} = 1$, all other coefficients equal to 0) and *iii)* $AR(L)$ processes (all coefficients equal to 0, except α_h, $\beta_{h1}, \ldots, \beta_{hL}$). We estimated (6) for all 60 participants, using observations from $t = 11$ to $t = 51$. The estimation results can be found in Tables 4, 5 and 6 in Appendix A. These results are qualitatively summarized in Table 3.

Here $B(k, l)$ refers to a prediction strategy where k is the highest significant lag of the price and l is the highest significant lag of the prediction (which does not necessarily mean that all smaller lags are also significant) in the regression. We find 9 participants with $AR(1)$ beliefs (of which 3 participants use naive expectations), 29 participants with $AR(2)$ beliefs, 3 participants with $AR(3)$ beliefs and 3 participants with adaptive beliefs.[6] The remaining 16 participants use more complicated prediction rules. Notice that the $AR(1)$ and adaptive rules are all found in groups 2, 3 and 5, and the $AR(2)$ and $AR(3)$ rules are all found in the other groups. This is consistent with the finding that in groups 2 and 5 the price seems to converge

[6] We arrive at the naive and adaptive expectations strategies in the following way. For the $AR(1)$ processes we tested the joint hypothesis $\alpha_h = 0$ and $\beta_{h1} = 1$ (naive expectations). For processes where only the coefficients on p_{t-1} and p_{ht}^e are significant we tested the joint hypothesis $\alpha_h = 0$ and $\beta_{h1} + \gamma_{h0} = 1$ (adaptive expectations).

	$AR(1)$ (Naive)	$AR(2)$	$AR(3)$	Adaptive	Other
group 1	0	5	0	0	$B(4,2)$
group 2	4(3)	0	0	1	$B(1,2)$
group 3	2	3	0	1	–
group 4	0	3	1	0	$B(3,1)$, $B(4,3)$
group 5	3	1	0	1	$B(2,1)$
group 6	0	5	0	0	$B(2,2)$
group 7	0	4	1	0	$B(1,2)$
group 8	0	4	0	0	$B(1,1)$, $B(4,3)$
group 9	0	2	0	0	$B(1,1)$, $B(2,2)$, $B(2,3)$, $B(4,1)$
group 10	0	2	1	0	$2 \times B(1,1)$, $B(3,0)$
total	9	29	3	3	16

Table 3: Estimation results for individual prediction strategies

monotonically and that in groups 1, 4, and $6-10$ the price oscillates around some steady state. Group 3 takes a somewhat special position, starting out with oscillations and ending with monotonic convergence to the fundamental price. Prediction strategies within groups are more similar than strategies between groups which is consistent with the finding that participants within one group seem to coordinate on a common prediction strategy.

The $AR(2)$ prediction strategy can be rewritten as a trend following rule

$$p^e_{h,t+1} = \alpha_h + \beta_h p_{t-1} + \delta_h (p_{t-1} - p_{t-2}),$$

where $\beta_h \equiv \beta_{h1} + \beta_{h2}$ and $\delta_h \equiv -\beta_{h2}$. For all of the 26 $AR(2)$ prediction strategies in the "oscillating" groups $(1,\ 4,\ 6-10)$ we have $\widehat{\beta}_{h1} > 0$ and $\widehat{\beta}_{h2} < 0$. The latter inequality is equivalent with $\delta_h > 0$, which implies that all these participants try to follow the trend: they expect that a recent upward (or downward) movement in prices will continue in the near future. These participants therefore correspond to so-called *positive feedback traders*. Another interesting feature is that for the estimated $AR(2)$ strategies in the (oscillating) groups the variation in $\widehat{\beta}_{h1} + \widehat{\beta}_{h2}$ seems to be lower than the variation in $\widehat{\beta}_{h2}$. This suggest that participants within a group have the same value of $\beta_h = \beta_{h1} + \beta_{h2}$ but have different values of the trend coefficient δ. We tested this hypothesis for the 5 relevant groups. Only for groups 1, 4 and 9 we cannot reject the hypothesis that $\beta_h = \beta_{h1} + \beta_{h2}$ is the same for all relevant h.

In order to characterize the different estimated prediction strategies, we can determine, for each of them, what happens if all participants in a group

use that estimated prediction strategy. Recall that in this experiment, even if all participants use linear prediction rules, the asset price dynamics will be a nonlinear dynamical system because the weight n_t of the fundamentalist traders changes over time. We find that 10 of the estimated $AR(2)$ prediction strategies (3 in group 1, 1 in group 4, 3 in group 6, 1 in group 7 and 2 in group 8) are locally unstable and lead to persistent oscillations in the asset prices, if used by all participants in a group.[7] Two of the $AR(1)$ rules (1 in group 2 and 1 in group 3) are stable but lead to a very different steady state price when used by all participants in a group.[8] Moreover, if these $AR(1)$ rules are used by all participants in a group without fundamentalist traders and without an upper limit on predictions and asset prices, exploding bubbles emerge. For the more complicated non-$AR(2)$ rules used by the participants in groups 8, 9 and 10, we can say the following. The estimated prediction strategies of participants 2 and 4 in group 8 are unstable (in the sense that they would lead to bounded oscillations if all six participants used this rule). For group 9 each of the estimated prediction strategies of participants 3, 5 and 6 give rise to unstable dynamics, when used by the whole population. Finally, for group 10 the estimated prediction strategies of participants 3, 4 and 5 are unstable.

The estimated rules can also be used to get some insight in the following questions: what happens *i)* in the long run; *ii)* in the absence of fundamentalist traders. In order to investigate these issues we did the following numerical simulations. For each group the estimated individual prediction strategies were programmed and the experiment was ran with these programmed prediction strategies. First this numerical simulation was ran for more than 50 periods to investigate long run behavior. For the first seven groups we find that realized asset prices stabilize close to the fundamental value. For groups 8, 9 and 10, however, we find perpetual but bounded fluctuations of the realized asset prices. Secondly, we investigated what would happen in the absence of fundamentalist traders. Also here we found, for the first seven groups, convergence to a steady state close to the fundamental price. On the other hand, for groups 8 and 9 realized prices will explode in the absence of fundamentalist traders (and in the absence of an upper bound on asset prices). Finally, for group 10 the simulations show bounded oscillations in the absence of fundamentalist traders. Of course, analyses like these have to be considered with care, since we use the estimated prediction strategies in a context which is different from the context where they were

[7] Recall from Section 3 that an $AR(2)$ rule is locally unstable and leads to oscillating behavior when $\beta_1^2 + 4R\beta_2 < 0$ and $\beta_2 < -R$.

[8] Recall from Section 3 that an $AR(1)$ rule is locally unstable when $\beta_1 > R$.

used by the participants.

One final remark is in order. From the estimation results we should not draw the conclusion that these prediction strategies are typical for the different individuals, in the sense that these individuals will use the same rule in another context as well. Actually, participants coordinate on some kind of behavior and this behavior becomes self-fulfilling: the estimated relationships are consistent with that behavior.

5.5 The impact of the fundamentalist traders

In this subsection we discuss the influence of the fundamentalist traders. We ran four additional groups, where the only difference with the other sessions is that there are no fundamentalist traders ($n_t = 0$, for all t, in equation (1)). Figure 9 shows the realized asset prices and individual predictions per group. This figure shows that also in the case without fundamentalist traders coordination of individual forecasting strategies on a common prediction strategy occurs.

A total of 24 subjects participated in this session and their average earnings were 32664 points (17.56 Euro), which is below the average earnings of the ten other groups. For the four additional groups, the sample average of realized prices was 56.48, so that also in these groups the market is undervalued. The sample variance of realized prices is quite large, especially in groups 11–13 (647 on average). Hence, in accordance with what one would expect, without computerized fundamentalist traders market volatility is somewhat higher than in the presence of fundamentalist traders.

Figure 9 also shows that in three of the four groups temporary bubbles and crashes occur. The fourth group shows a steady oscillation around the fundamental value of $p^f = 60$. These results are similar to those from a related asset pricing experiment without fundamentalist traders, recently obtained in Hommes, Sonnemans, Tuinstra and van de Velden (2002), which we discuss here briefly. The main difference in these experiments is that participants have no a priori information about an upperbound on their prediction. The most striking feature of these experiments is that bubbles increasing up to a value of 1000 (i.e. more than 15 times the fundamental price) occur.[9] Therefore, in these experiments without fundamentalist traders and without an

[9] A priori no information about an upperbound for the asset price or the price forecast was given to the participants. However, when asset prices increase and an individual forecast larger than 1000 was given, the participant was informed that forecasts larger than 1000 were not allowed, and was asked for a forecast not exceeding 1000. This new information lead to subsequent crashes in asset prices, followed by subsequent bubbles and crashes in the experiments. See Hommes et al. (2002) for details.

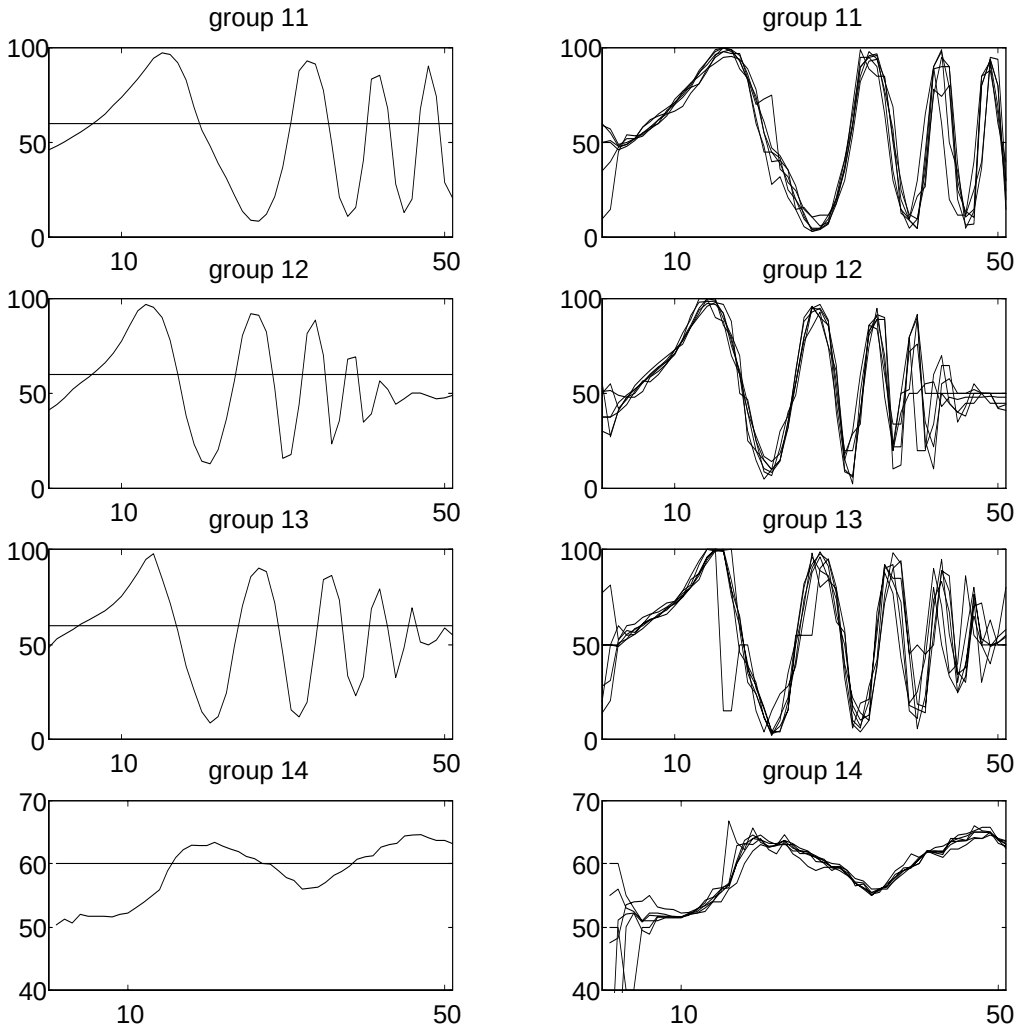


Figure 9: Realized prices (left side graphs) and predictions (right side graphs) for session without fundamentalist traders. Straight line at $p = 60$ corresponds to fundamental price.

a priori given upperbound, participants coordinate on a common prediction strategy, predicting (exponentially) growing asset prices.

From Figure 9 it is clear that also in the additional groups without fundamentalist traders, participants coordinate on a common prediction strategy. Computations similar to those in subsection 5.1 show that 75% of the average individual quadratic forecast error can be attributed to the common error. Estimating individual forecasting strategies, as in subsection 5.4, shows that the majority of the individual prediction strategies can be classified as $AR\,(2)$, $AR\,(3)$ or $AR\,(4)$ strategies. These results are similar to the results obtained for the oscillatory groups 1, 4, $6-10$ with fundamentalist trader.

In summary, also in the absence of the fundamentalist traders our key finding remains that there is coordination on a common prediction strategy. This coordination of expectations therefore seems to be a robust result in these asset pricing experiments.

6 Concluding Remarks

In this paper we investigated expectation formation in a simple experimental asset pricing model. Ten markets are populated by six participants and a certain fraction of computerized fundamentalist traders; four additional markets without computerized fundamentalist traders have also been investigated. We observe slow and monotonic convergence to the fundamental price, as well as regular oscillations around the fundamental price. In most groups the asset is undervalued and exhibits excess volatility. Simple expectation schemes (or *popular models* (Shiller (1990)) such as naive expectations or autoregressive expectations give a much better description of aggregate market behavior than do rational expectations. From the analysis of the individual prediction strategies we find that *participants within a group coordinate on a common prediction strategy.* Moreover, these popular models can be estimated rather accurately, and this reveals that participants indeed tend to use simple (linear) forecasting models. In the stable markets, a majority of participants is cautious and uses naive, adaptive or AR(1) forecasting strategies. In the oscillatory groups, a majority of participants exhibits overreaction and uses trend following strategies. Although the participants are not completely rational like standard economic theories assume, they perform very well. For a large majority of individuals, forecasting errors are unbiased and without autocorrelation in the smallest exploitable lag (lag 2) and their earnings are high. Our experimental outcomes thus support the common hypothesis in behavioral finance that individuals use simple, but reasonably successful, rules of thumb.

Let us finally try to develop some intuition for the emergence of expectational coordination. Participants in these experiments have an incentive to coordinate their prediction strategies, since the market clearing price is close to the average prediction. Participants who succeed in predicting the average prediction well, perform well in the experiment. This feature of the asset pricing experiment may be similar to real asset markets, and is consistent with the ideas of Keynes (1936, p.156) who, in a much quoted passage, compared behavior of traders in financial markets to so-called *beauty contests*:

"[P]rofessional investment may be likened to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole; so that each competitor has to pick, not those faces which he himself finds prettiest, but those which he thinks likeliest to catch the fancy of the other competitors, all of whom are looking at the problem from the same point of view. .... [W]e devote our intelligences to anticipating what average opinion expects the average opinion to be."

From our experiments we find that participants are rather successful in "anticipating what average opinion expects the average opinion to be".

If there are forces towards coordination of individual expectations, the question then is what kind of 'average' equilibrium outcome will individuals coordinate on? One possibility would be coordination on the fundamental price, but in our experiments (slow) convergence to the fundamental price only happens in a minority of cases. From a theoretical perspective there are other possibilities for coordination however, in particular the (rational) self-fulfilling bubble solutions growing at the risk free interest rate. In the absence of a robot trader and/or in the absence of upper- and lower bounds, these bubble solutions are rational expectations (perfect foresight) equilibria. The presence of a robot trader, who acts as a stabilizer in the direction of the fundamental price, makes coordination on these bubble solutions less likely. In the experiment however, coordination on temporary bubbles, triggered by simple trend following strategies, does occur even in the presence of computerized fundamentalist traders. These trends cannot continue forever and are reversed, due to the lower and upper bounds 0 and 100 and/or the presence of robot traders. The upward trend reverses and once reversed, trend extrapolating forecasting rules reinforce the downward trend. The result is then coordination of individual expectations on damped or permanent oscillatory price fluctuations with upward and downward trends, as observed in most of our groups. Our experiments thus provide evidence for a number

of behavioral modes popular in behavioral finance, in particular correlated imperfect rational forecasting due to trend extrapolation and overreaction.

It is illuminating to compare our results with those of van de Velden (2001), who uses the experimental approach to investigate expectation formation in a 'cobweb' commodity market with a supply response lag. He shows that, for an unstable cobweb model, the heterogeneity of expectations leads to excess volatility of realized prices but there does not seem to be any coordination on a common prediction strategy. An important difference between the asset pricing model discussed here and the cobweb model is the way in which expectations play a role in the model. The asset pricing model has an *expectations confirming* structure, which means that if people expect the price of the asset to increase it will indeed increase. We have already argued that this gives participants an incentive to coordinate on similar prediction strategies. On the other hand, the cobweb model has an *expectations reversing* structure: if a high price is expected, firms will produce a lot and by the equality of demand and supply the realized market equilibrium price will then be low. This structure is detrimental to coordination because, when most participants submit a prediction which lies above (below) the fundamental price, then it pays off to give a prediction below (above) the fundamental price. Market institutions therefore seem to play an important role in the emergence of coordination of expectations. Our results suggest that in speculative asset markets coordination on a trend may lead to (temporary) deviations from fundamentals.

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A Individual prediction strategies

group 1	α	β_1	β_2	β_3	β_4	γ_0	γ_1	γ_2	γ_3	R^2
part. 1	26.83	1.58	-1.05							0.83
part. 2	18.42	1.22	-0.55							0.79
part. 3	28.43	1.55	-1.05							0.81
part. 4	29.24	1.22	-0.72							0.84
part. 5	34.35	1.61	-1.23							0.77
part. 6	20.53	1.94	-2.24	1.88	-0.71	0.48	-0.60			0.95
group 2	α	β_1	β_2	β_3	β_4	γ_0	γ_1	γ_2	γ_3	R^2
part. 1	-2.59*	0.27				0.78				0.98
part. 2	-1.08*	1.02								0.88
part. 3	-6.38	1.11								0.92
part. 4	3.76	0.91				0.32	0.21			0.97
part. 5[a)]		1								1
part. 6	7.22*	0.87								0.77
group 3	α	β_1	β_2	β_3	β_4	γ_0	γ_1	γ_2	γ_3	R^2
part. 1[b)]	13.74	0.74								0.85
part. 2	-32.49	1.60								0.68
part. 3	0.25*	1.82	-0.83							0.94
part. 4	2.25*	0.24				0.71				0.81
part. 5	10.60	1.20	-0.41							0.88
part. 6	10.97	1.30	-0.51							0.85
group 4	α	β_1	β_2	β_3	β_4	γ_0	γ_1	γ_2	γ_3	R^2
part. 1	10.26*	1.28	-1.96	0.84		0.63				0.79
part. 2	4.36*	2.14	-3.27	3.08	-1.31		0.90	-1.45	0.84	0.80
part. 3	13.87	1.85	-1.10							0.82
part. 4	15.76	1.65	-0.89							0.85
part. 5	1.87*	1.86	-1.49	0.54						0.70
part. 6	16.82	1.38	-0.70							0.57

Table 4: Estimated individual prediction strategies for groups 1 to 4

This appendix contains the estimated individual prediction strategies for the 60 participants of this experiment. The estimated relationship has the following general structure

$$p^e_{h,t+1} = \alpha_h + \sum_{i=1}^{4} \beta_{hi} p_{t-i} + \sum_{j=0}^{3} \gamma_{hj} p^e_{ht-j}.$$

This was estimated on data from the experiment from $t = 11$ to $t = 51$. The first 10 periods are neglected in order to allow for some coordination or

learning. Tables 4, 5 and 6 show the estimation results. The constant term is always part of the regression although sometimes it is not significantly different from 0. These cases are indicated with a *. We tried to fit the simplest model, provided that there is no serial correlation in the residuals at the 5% significance level.

group 5	α	β_1	β_2	β_3	γ_0	γ_1	R^2
part. 1	1.36*	0.49	0.48				0.80
part. 2	11.32	0.80					0.79
part. 3	7.18	0.87					0.88
part. 4	2.00*	0.63			0.33		0.93
part. 5	12.08	0.79					0.75
part. 6	2.97*	0.79	-0.41		0.57		0.83
group 6	α	β_1	β_2	β_3	γ_0	γ_1	R^2
part. 1	3.17*	1.36	-0.41				0.96
part. 2	-9.60	2.48	-0.80			-0.52	0.96
part. 3	12.83	1.85	-1.06				0.90
part. 4	32.53	2.05	-1.60				0.93
part. 5	6.70	1.94	-1.06				0.97
part. 6	21.43	1.32	-0.69				0.95
group 7	α	β_1	β_2	β_3	γ_0	γ_1	
part. 1	41.77	0.85	-0.55				0.45
part. 2	41.54	0.99	-0.68				0.66
part. 3	11.08*	2.11	-1.31				0.84
part. 4	61.71	0.67				-0.72	0.47
part. 5	28.52	1.77	-1.82	0.56			0.67
part. 6	30.08	1.47	-1.00				0.71

Table 5: Estimated individual prediction strategies for groups 5 to 7

Some remarks:

1. The estimates indicated by a * are not significantly different from 0 at the 5% level.

2. Group 2: for participant 1 the null hypothesis of adaptive expectations, $H_0 : (\alpha = 0 \text{ and } \beta_1 + \gamma_0 = 1)$, cannot be rejected at the 5% significance level; for participants 2 and 6 the null hypothesis of naive expectations, $H_0 : (\alpha = 0,\ \beta_1 = 1)$ cannot be rejected at the 5% significance level, for participant 3 this hypothesis is rejected. [a)]For the sample considered participant 5 exactly uses naive expectations.

3. Group 3: for participant 4 the null hypothesis of adaptive expectations, $H_0 : (\alpha = 0,\ \beta_1 + \gamma_0 = 1)$ cannot be rejected. [b)]Participant 1

group 8	α	β_1	β_2	β_3	β_4	γ_0	γ_1	γ_2	γ_3	R^2
part. 1	11.35	2.32	-1.59							0.95
part. 2	0.42*	1.24			1.78			-1.03	-0.98	0.90
part. 3	8.46	1.97	-1.15							0.92
part. 4	-4.20*	2.32					-1.21			0.94
part. 5	1.44*	1.91	-0.93							0.95
part. 6	4.10*	1.71	-0.81							0.89
group 9	α	β_1	β_2	β_3	β_4	γ_0	γ_1	γ_2	γ_3	R^2
part. 1	4.08	0.81				0.87	-0.78			0.97
part. 2	4.89	1.52	-0.63							0.98
part. 3	7.88	2.63	-3.05	-1.59	1.32		1.49			0.98
part. 4	2.97*	1.92	-0.99							0.97
part. 5	3.29*	2.01	-2.06	2.23	-1.00	0.61	-0.87			0.99
part. 6	-3.06*	2.77	-1.12				-1.21	0.64		0.97
group 10	α	β_1	β_2	β_3	β_4	γ_0	γ_1	γ_2	γ_3	R^2
part. 1	9.29*	1.73	-0.97							0.70
part. 2	0.29*	1.58	-0.58							0.92
part. 3	-16.10	3.92	-2.52	1.20		-1.15				0.92
part. 4	7.74*	2.28					-1.41			0.83
part. 5	3.59*	2.61	-2.64	1.02						0.87
part. 6	13.45	1.56	-0.82							0.87

Table 6: Estimated individual prediction strategies for groups 8 to 10

submitted an expectation of 5.25 in period 42, where we have a strong belief that he planned to submit 55.25. We therefore replaced the 42'th observation on this participant by 55.25. We also estimated the individual prediction strategies in this group using only data from period $t = 11$ to period $t = 40$. We then find very similar results, namely one adaptive prediction strategy, one $AR(1)$ strategy and three $AR(2)$ strategies. The individual prediction strategy of participant 5 is a little more complicated in that case, since he uses the first two prices and the fourth lag of his own previous expectations (his prediction strategy can therefore be described by $B(2,3)$).

4. Group 5: for participant 1 the null hypothesis that this participant averages over the last two prices, $H_0 : \left(\alpha = 0,\ \beta_1 = \beta_2 = \frac{1}{2}\right)$ cannot be rejected; for participant 4 the null hypothesis of adaptive expectations cannot be rejected.

5. Group 10: for participant 2 the null hypothesis $H_0 : (\alpha = 0, \beta_1 + \beta_2 = 1)$ cannot be rejected.

6. For all groups with $AR(2)$ strategies we find that for the estimated $AR(2)$ strategies the variation in $\widehat{\beta}_{h1} + \widehat{\beta}_{h2}$ is much smaller than the variation in $\widehat{\beta}_{h2}$ alone. We know that this prediction strategy can be represented as

$$p^e_{h,t+1} = \alpha_h + \beta_h p_{t-1} + \delta_h \left(p_{t-1} - p_{t-2}\right),$$

where $\beta_h \equiv \beta_{h1} + \beta_{h2}$ and $\delta_h \equiv -\beta_{h2}$. Our hypothesis now is that β_h (and possibly α_h) is the same for all participants in a group and δ_h differs across participants in a group. We tested this hypothesis in all groups where the $AR(2)$ prediction strategy emerges. We cannot reject the hypothesis at a 5% level for the $AR(2)$ prediction strategies in groups 1, 4 and 9. The results are given in Table 7.

	α	β	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6
group 1	–	0.52	1.06	0.62	1.04	0.71	1.17	–
group 4	15.48	0.74	–	–	1.11	0.90	–	0.36
group 9	3.93	0.91	–	0.61	–	1.01	–	–

Table 7: Test for homogeneous positive feedback expectations

For group 1 we have no significant differences in $\beta_{h1} + \beta_{h2}$, for group 4 we have no significant differences in α_h and in $\beta_{h1} + \beta_{h2}$ and for group 9 we have no significant differences in α_h and in $\beta_{h1} + \beta_{h2}$. In all other groups the hypothesis is rejected.

B Information for Participants

General information.
You are a **financial advisor** to a pension fund that wants to optimally invest a large amount of money. The pension fund has two investment options: a risk free investment and a risky investment. The risk free investment is putting all money on a bank account paying a fixed interest rate. The alternative risky investment is an investment in the stock market. In each time period the pension fund has to decide which fraction of their money to put on the bank account and which fraction of the money to spend on buying stocks. In order to make an optimal investment decision the pension fund needs an accurate prediction of the price of stocks. As their financial advisor, you have to predict the stock market price (in guilder) during 52 subsequent time periods. Your earnings during the experiment depend upon your forecasting accuracy. The smaller your forecasting errors in each period, the higher your total earnings.

Information about the stock market.
The stock market price is determined by equilibrium between demand and supply of stocks. The supply of stocks is fixed during the experiment. The demand for stocks is mainly determined by the aggregate demand of a number of large pension funds active in the stock market. Some of these pension funds are advised by a participant to the experiment, others use a fixed strategy. There is also some uncertain, small demand for stocks by private investors but the effect of private investors upon the stock market equilibrium price is small. The price of the stocks is determined by market equilibrium, that is, the stock market price in period t will be the price for which aggregate demand equals supply.

Information about the investment strategies of the pension funds.
The precise investment strategy of the pension fund that you are advising and the investment strategies of the other pension funds are unknown. The bank account of the risk free investment pays a fixed interest rate of 5% per time period. The holder of the stocks receives an uncertain dividend payment in each time period. These dividend payments are uncertain however and vary over time. Economic experts of the pension funds have computed that the average dividend payments are 3 guilder per time period. The return of the stock market per time period is uncertain and depends upon (unknown) dividend payments as well as upon price changes of the stock. As the financial advisor of a pension fund you are **not** asked to forecast dividends, but you are only asked to forecast the price of the stock in each time period. Based upon

your stock market price forecast, your pension fund will make an optimal investment decision. The higher your price forecast the larger will be the fraction of money invested by your pension fund in the stock market, so the larger will be their demand for stocks.

Forecasting task of the financial advisor.
The only task of the financial advisors in this experiment is to forecast the stock market price in each time period as accurately as possible. The price of the stock will always be between 0 and 100 guilder. The stock price has to be predicted **two** time periods ahead. At the beginning of the experiment, you have to predict the stock price in the first **two** periods, that is, you have to give predictions for time periods 1 and 2. After all participants have given their predictions for the first two periods, the stock market price in the first period will be revealed and based upon your forecasting error your earnings for period 1 will be given. After that you have to give your prediction for the stock market index in the third period. After all participants have given their predictions for period 3, the stock market price in the second period will be revealed and, based upon your forecasting error your earnings for period 2 will be given. This process continues for 52 time periods.

To forecast the stock price p_t in period t, the available information thus consists of

- past prices up to period $t-2$,
- past predictions up to period $t-1$,
- past earnings up to period $t-2$

Earnings.
Earnings will depend upon forecasting accuracy only. The better you predict the stock market price in each period, the higher your aggregate earnings.

Earnings will be according to the following earnings table.

C Payoff Table

Payoff table **1300 points equal 1 guilder**									
error	points	error	points	error	points	error	points	error	points
0.1	1300	1.5	1240	3	1061	4.4	786	5.8	408
0.15	1299	1.55	1236	3.05	1053	4.45	775	5.85	392
0.2	1299	1.6	1232	3.1	1045	4.5	763	5.9	376
0.25	1298	1.65	1228	3.15	1037	4.55	751	5.95	361
0.3	1298	1.7	1223	3.2	1028	4.6	739	6	345
0.35	1297	1.75	1219	3.25	1020	4.65	726	6.05	329
0.4	1296	1.8	1214	3.3	1011	4.7	714	6.1	313
0.45	1295	1.85	1209	3.35	1002	4.75	701	6.15	297
0.5	1293	1.9	1204	3.4	993	4.8	689	6.2	280
0.55	1292	1.95	1199	3.45	984	4.85	676	6.25	264
0.6	1290	2	1194	3.5	975	4.9	663	6.3	247
0.65	1289	2.05	1189	3.55	966	4.95	650	6.35	230
0.7	1287	2.1	1183	3.6	956	5	637	6.4	213
0.75	1285	2.15	1177	3.65	947	5.05	623	6.45	196
0.8	1283	2.2	1172	3.7	937	5.1	610	6.5	179
0.85	1281	2.25	1166	3.75	927	5.15	596	6.55	162
0.9	1279	2.3	1160	3.8	917	5.2	583	6.6	144
0.95	1276	2.35	1153	3.85	907	5.25	569	6.65	127
1	1273	2.4	1147	3.9	896	5.3	555	6.7	109
1.05	1271	2.45	1141	3.95	886	5.35	541	6.75	91
1.1	1268	2.6	1121	4	876	5.4	526	6.8	73
1.15	1265	2.65	1114	4.05	865	5.45	512	6.85	55
1.2	1262	2.7	1107	4.1	854	5.5	497	6.9	37
1.25	1259	2.75	1099	4.15	843	5.55	483	6.95	19
1.3	1255	2.8	1092	4.2	832	5.6	468	error ≥ 7	0
1.35	1252	2.85	1085	4.25	821	5.65	453		
1.4	1248	2.9	1077	4.3	809	5.7	438		
1.45	1244	2.95	1069	4.35	798	5.75	423		