Incomplete Contract and Divisional Structures*

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Abstract

This paper analyzes the internal divisional structure within an organization in the framework of incomplete contract theory. We use the framework of Aghion and Tirole (1997) and define the managerial control structure as “sequence of search”. A key feature of this paper which differentiate it from other works in the literature is that we add an \textit{ex post} bargaining phase in which the managers can agree on the project which maximize their joint private benefit. Our model shows the share of cooperative surplus the managers can get from bargaining and their default pay off plays a key role in determining their search effort. When there is no spill over effect between the agents’ effort, internal separation is always dominated by internal integration with control right assigned to the agent (manager) with high interest congruence with the principal (head quarter). When there are dissonance effect during integration, the principal want to have separation to avoid it. But more importantly, the optimal divisional structure depends heavily on whether the bargaining is interest congruence enhancing (increase the interest congruence between agents and the principal) or destroying (decrease the interest congruence).

\textbf{JEL Classification:} D23, L22

\textbf{Key words:} organizational form, divisional structure, incomplete contract, bargaining

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1 Introduction

Researchers in economics and business have long recognized the importance of organizational forms for the performance of teams, corporations, and economies. Milgrom and Roberts (1992) shows the internal reorganization of General Motors in the 1920s by imposing more decentralized forms may be the key factor that leads to its surpassing over more centralized Ford in the next decades. Maskin et al (2000) also attribute the striking difference in the performance of China and former Soviet Union economies to the M-form and U-form decision structure in the government divisions which make economic plans.

This paper pays special interest on one kind of organizational form: the divisional structure when there are (potentially) many divisions performing the same kind of task. There is a rich literature on the organizational form that deals with divisional structure when the divisions have different functions or are self-sufficient regions. (e.g. Maskin et al 2000, Dessein and Santos, 2006.) Besides literature on divisional structure, there are also many seminal works done on leadership, the delegation structure between the leader and the divisional managers. (Rotemberg and Saloner, 1993, Bolton and Dewartripont, 1994). There are a bunch of real world examples which draw our attention to the pure endogenous choice on divisional structure where multiple division may coexist while they perform very similar kinds of tasks: a university may have many research centers in one research area on top of the department of this filed; a large hospital may have more than one department of internal medicine or surgery; a large corporation may have more than one brand of the same product even with very similar positioning, e.g. it may be very hard to tell the difference between daily used goods produced by Lux, Hazeline, Dove, Ponds, which are all owned by Uniliever. All these facts suggest there might be forces beyond pure functional concerns that drive the determination of optimal divisional structure making.

The methodology of former literature on divisional structure basically goes to two categories: the communication and information transmission models (Marschak and Radner, 1972, van Zandt, 1990, Argyres, 1994, Bolton and Dwartripont, 1994, Dessein and Santos, 2006), which mainly discuss the trade off between coordination (or communication cost) and specialization (or local adaptation), and incentive models (Rotemberg and Saloner, 1993, Poitevin, 1995, Maskin et al, 2000) which focuses on the CEO’s best decision on the incentive scheme for the division managers. For more detailed discussion of these literature, Mookherjee (2006) provides a comprehensive survey.

The communication models usually assume better use of local information, and therefore benefit from specialization when the CEO or headquarter makes more separate division,
or delegates more decision power to the local divisions, and poor coordination between divisions, or communication cost for doing so. When the benefit of specialization outweighs the loss of coordination, it is better to have less divisions, or less decision power delegated to the local divisions, and vice versa. The incentive models usually consider different contracts between CEO and division managers, or different working style of CEO (democratic or autocratic) and the induced effort by the managers. A special kind of organizational form will be chosen if it induces the highest amount of manager’s effort.

Our work will be similar to the incentive models, but instead of considering complete contracts where the principal (CEO of top manager in the headquarter) maps performance to pay-offs or working styles where the principal also needs to make their own production decisions, we consider a model using incomplete contract approach. In our model the principal just need to decide whether she wants to have two separate divisions, and make both agents the the manager of their own divisions, or integrate the divisions, and make only one of them the general manager. In other words, we are considering the internal boundary of a firm. We are looking at the cost and benefit of integration (separation) of homogenous divisions within a firm, as Grossman and Hart (1986) did for the vertical integration (separation) between two firms.

A straight result from this kind of analysis is that the control right over division is usually a strong incentive for the agent, the party assigned with the control is usually more motivated to make effort, while the party losing control is discouraged. But a key difference between the internal and external boundary is that the welfare analysis is no more with respect to the joint pay-off of the two agents (division managers), but the goal of the principal (CEO). So we must introduce interest congruence analysis from Aghion and Tirole (1997). As we have at least three parties (one principal and two agents) here, we are faced with a generally more complicated model: both the interest congruence between the principal and agents, and the interest congruence between different agents should be taken into account. Like Grossman and Hart (1986), the agents can make ex post bargaining when the joint payoff can be improved from the controlling agent’s decision. We show the bargaining in these settings are basically more important here than in the models on external boundary of the firm, because the agents have to be both informed before they bargain, so the bargaining may change the ex post interest congruence between the agents and the principal. We find separation is always dominated by integration and giving the control to the manager whose effort is more important when bargaining does not change interest congruence between the principal and agents, and large area of separation domination otherwise.
The structure of this paper is organized as following: Section 2 shows the model setup; Section 3 discuss the simplest case without contract incompleteness; Section 4 discuss the model with incomplete contract but without bargaining; Section 5 discusses the model with incomplete model and bargaining; Section 6 introduces spill-over effect of the agents’ effort. Finally, Section 7 concludes.

2 The Model Setup

We want to start with a model with one principal $P$ (we use “she” to address this party, usually the headquarter or the CEO), who is not productive and two agents, $A_1$ and $A_2$ (we use “he” to address each of them, usually the local offices or managers of divisions). Similar to Aghion and Tirole (1997) and Riyanto (2000). There are two divisions $D_1$ and $D_2$. There are $n \geq 3$ projects for each division. The project $i \in \{i_1, \ldots, i_n\}$ chosen in $D_1$ is associated with a profit $B_i$ to $P$, and private benefit $b^1_i$, $b^2_i$ to $A_1$, $A_2$. The project $j \in \{j_1, \ldots, j_n\}$ chosen in $D_2$ is associated with a profit $B_j$ to $P$, and private benefit $b^1_j$, $b^2_j$ to $A_1$, $A_2$. For simplicity we assume the distribution of projects in $D_1$ and $D_2$ are exactly the same. We assume some projects yields “sufficiently negative” payoff so uninformed party will confess ignorance. When no project is chosen, every party get 0. So each agent has income from both $D_1$ and $D_2$.

The agents search for information independently. At private cost $g_{A_1}(e)$ agent $A_1$ perfectly learns payoff of all projects with probability $e_1$. At private cost $g_{A_2}(e)$ agent $A_1$ perfectly learns payoff of all projects with probability $e_2$. For simplicity we assume the function form of cost are the same, meaning $g_{A_1}(e) = g_{A_2}(e) = g_{A}(e)$. All the cost functions are increasing and strictly convex.

For each division, the favorite project for $P$ gives her $B$. The favorite project for agent $A_1$ gives him $b_1$. $A_1$ and $A_2$ get $\beta_1 b$ and $\beta_2 b$ when the favorite of $P$ is chosen (This is just a benchmark, and will actually not happen because the principal does not search). The favorite project for $A_2$ gives him $b_2$. For simplicity we first assume $b_1 = b_2 = b$. $P$ does not search, so she has no “favorite project” for the current setting. $P$ receives $\alpha_1 B$ and $\alpha_2 B$ on expectation respectively when the favorite projects for $A_1$ and $A_2$ is chosen. $A_1$ and $A_2$ get $\gamma_1 b$ and $\gamma_2 b$ when the other’s favorite is chosen.

For the intuition behind this model, we can consider $P$ as the CEO of a big firm. $D_1$, $D_2$ are two sales offices, and $A_1$, $A_2$ are two sales managers. The firm produces several kinds of products, and sells them to several big customers. $P$ may come from other departments.
before she becomes CEO, so she knows nothing about sales. Therefore she must rely on the expertise of $A_1$ and $A_2$ to get access to the customers. Different products and customers may bring different private benefit to $A_1$ and $A_2$. For instance, $A_1$ may personally like one kind of product better and feel satisfaction when getting order for it, and $A_2$ may have very good relationship with one customer, and enjoy doing business with him. There are many potential sales opportunities from the customers in both sales offices, but in order to make revenue the agents have to invest time and energy to search. If the agents do not search, he is not informed the right demand information of the customer. An uninformed agent does not dare to choose a project, because when doing this he steps the risk of selling to the wrong customer or even dishonest ones, and bringing in loss for the company. A successful deal (project) is good for all parties, because the firm gets profit, the agent signing the contract gets private benefit, and the other agent may be also better off because the successful deal adds to the reputation of the firm.$^1$

Here the interest congruence between the principal and the agents can be considered as a kind of “productivity” (or loyalty) of the agent. So later we may say agent $i$ is more productive than agent $j$ if $\alpha_i > \alpha_j$. We assume $A_1$ and $A_2$ always receive some benefits from the project in both $D_1$ and $D_2$. It sounds like a kind of spill over when the principal makes separation decision (each agent is assigned the control over one division), the agent can still get some benefit from the division under the other agent’s control. This is a kind of spill over between agents which influences only the private benefit of the agents. There is another better known spill over effect, which is the spill over between agents which influences their productivity, and therefore the benefit of the principal. In this paper, we refer “spill over” to the one related to the principal’s benefit only, and treat the one related to the agents’ benefit just as a set of ordinary parameters ($\gamma_1$ and $\gamma_2$).

We assume $\alpha_1, \alpha_2, \beta_1, \beta_2, \gamma_1, \gamma_2 \in (0, 1]$. Here, because the interest congruence is always none negative, the uninformed party will always accept the project chosen by the informed party. It means all parties always would like to participate in the game (The IR condition is always satisfied).

The principal does not search, and only has to decide the allocation of control to $A_1$ and $A_2$. She has to decide whether to have the two division controlled separately, or together by a general manager. If they are controlled separately, for simplicity we only discuss the case when $A_1$ has the right to choose project for $D_1$, and $A_2$ has the right to choose for $D_2$, as the reverse case is analogous. If one of them is informed and the other is

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$^1$For example, $A_1$’s favorite customer is a small firm, and $A_2$’s favorite firm is a Fortune 500 firm. When the firm decides to choose $A_2$’s favorite customer, $A_1$ is also better off, because in the future, he can also boast to the potential customers “our firm does business with Fortune 500 firms”.

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Table 1: The columns are the situation where the party’s favorite project is chosen. The rows are the party’s payoff given the project is chosen in the column.

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<td>$A_1$</td>
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<td>$A_1$ &amp; $A_2$ Jointly</td>
<td>$(\beta_1 + \beta_2)b$</td>
<td>$b + \gamma_2b$</td>
<td>$\gamma_1b + b$</td>
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not, the uninformed party will accept what the informed party tells him to do because $\gamma_i b$ is surely better than 0. If it is controlled by a single manager, the manager choose projects for both divisions when he is informed, and the other agent can only choose when the manager is uninformed and he is informed. *Control allocation mean sequence to choose his favorite projects when informed.*

Like the story in Grossman and Hart (1986), although $A_1$ and $A_2$ can not make side agreement *ex ante*, they may want to bargain with each other *ex post* when they see a possible improvement of joint utility (the joint pay off is larger if $A_2$ can choose the favorite project of $A_1$ if $b + \gamma_1b < \gamma_2b + b$, or the other way around). We assume they split the cooperative surplus by Generalized Nash Bargaining. $A_1$ gets $w$ from the cooperative surplus, and $A_2$ gets $1 - w$.

Figure 1: The control structures in a graph. The left one is the organization under separation control. The middle is the case of integration with control right assigned to $A_1$, and the right one is the case of integration with control right assigned to $A_2$.

Allowing for *ex post* bargaining between the parties is also one feature that differentiate our work from the past searching models on divisional structure. This means although they can not sign contract on which project to choose and what effort to use *ex ante*, because
such contracts between agents with potential threat to the interest of their principal is generally not enforceable. They can however make agreement on which project to choose if any two of them are informed \textit{ex post}. So in this sense the information here is not hard information assumed by Aghion and Tirole (1997). Because one party can not convey it to other parties when the others are uninformed. Their default pay-offs in the bargaining will be the controlling party (party $i$) gets $b$ and the other (party $j$) gets $\gamma_j b$, so the allocation of control determines the default payoff, which we call bargaining position in this model. But the controlling party might also choose the other’s favorite if joint pay-off can be improved and he can cream off part of the surplus. Such improvement can be achieved via bargaining.

We denote $\Pi_i^P, \Pi_i^1, \Pi_i^2$ as the return to $P, A_1, A_2$ from division $i$. For simplicity and relevance to reality we assume $B \gg b$, so $B + \beta_1 b > \alpha_1 B + b$ and $B + \beta_2 b > \alpha_2 B + b$ hold for arbitrary parameters $\alpha$ and $\beta$. The joint pay-off can not be improved when the principal’s favorite project is chosen. So the principal never wants to bargain with the agents when she is informed. Moreover, as we already assumed the principal does not search (so she is actually never informed). \textbf{The possibility for bargaining between the principal and agents are ruled out by the assumption of unproductive (and therefore always uninformed) principal.}

Although the possibility of bargaining between the principal and agents is ruled out, it is surely possible for the agents to bargain. For example, if $A_1$ and $A_2$ are in one department, and $A_1$ is in control, when both $A_1$, $A_2$ learns pay-off of all projects. If $A_1$ just pick up his own favorite, the pay is $b$ to himself and $\gamma_2 b$ for $A_2$. When $A_1$ choose the favorite of $A_2$, the payoff is $\gamma_1 b$ for himself and $b$ for $A_2$. When $b + \gamma_2 b < b + \gamma_1 b$, $A_1$ might come to $A_2$ and say:“ look, I can choose your favorite project, but in return for that, I want $w(\gamma_1 - \gamma_2)b$ on top of what I can get for sure if my favorite project is chosen, so that we get $b + w(\gamma_1 - \gamma_2)b : (1 - w)\gamma_1 + w\gamma_2 b$, instead of either $b : \gamma_2 b$, or $\gamma_1 b : b$. The split of cooperative surplus is realized by side-payment, which is commonly used in models that study coordination of technological standard in multidivisional firms (Argyres, 1994).

By doing this we can introduce the \textit{ex post} efficiency (within the bargaining parties) into our model. And \textit{ex ante} control structure and bargaining power allocation still matter for incentive to make effort. Bargaining can happen when the joint pay-off (of agents) can be improved. The allocation of control only determines the bargaining position, the default pay-off. Here we use \textbf{Bargaining power} to describe the share of cooperative surplus one agent can get, and \textbf{Bargaining position} to say his relative default pay-off.

We assume the information on the parameters $\alpha, \beta, \gamma$ etc. is common knowledge. Although the principal does not search, she still has an idea of the productivity of the agents.
The pay-offs are realized. The agent in control chooses the project or bargains with the other when he is informed. Otherwise the other agent gets the right to choose.

The agents put in effort to search. P decides the divisional structure.

Figure 2: The time line of the game.

3 Social Optimum without Contract Incompleteness

If there is no contract incompleteness, the principal P can make contract to make sure her favorite project is chosen for both divisions when at least one agent is informed. \( \Pi_1^P = \Pi_2^P = B, \Pi_1^1 = \Pi_1^2 = \beta_1 b, \Pi_2^1 = \Pi_2^2 = \beta_2 b \), so the joint pay-off is \( S_P = 2(B + \beta_1 b + \beta_2 b) \).

The expected social welfare will be:

\[
2(e_1 + e_2 - e_1 e_2)(B + \beta_1 b + \beta_2 b) - g_A(e_1) - g_A(e_2)
\] (1)

In order to maximize this social welfare function, the efforts by the two agents should satisfy, the superscript * means social optimal:

\[
g_A'(e_1^*) = 2(1 - e_2^*)(B + \beta_1 b + \beta_2 b) \] (2)

\[
g_A'(e_2^*) = 2(1 - e_1^*)(B + \beta_1 b + \beta_2 b) \] (3)

These first order conditions imply the agents choose effort such that the marginal cost of searching is equal to the marginal return to all three parties. Here because the favorite project of P will be chosen either \( A_1 \) or \( A_2 \) is informed, so one agent’s effort is a pure waste if the other is already informed. In each division, the return to one unit of effort by agent \( i \) is one unit chance higher to get \( B \) to the principal, \( \beta_1 b \) to \( A_1 \) and \( \beta_2 b \) to \( A_2 \) given the other agent is not informed (multiplied by \( (1 - e_j^*), j \in \{1, 2\}, j \neq i \) ). There are two divisions,
so the “social” return to each unit of effort is two times the return to three parties in one division.

Because the cost function $g(e)$ of effort is increasing in $e$, we can infer that the derivative of the equilibrium effort with respect to the variables or parameters in the right hand side of the above equations share the sign of that of the cost function with respect to the same variables or parameters. It is straightforward that $\partial g_A'(e_i^*)/\partial e_j^* < 0$, $\partial g_A'(e_i^*)/\partial B > 0$, $\partial g_A'(e_i^*)/\partial b > 0$, $\partial g_A'(e_i^*)/\partial \beta_1 > 0$, $\partial g_A'(e_i^*)/\partial \beta_2 > 0$, so $\partial e_i^*/\partial e_j^* < 0$, $\partial e_i^*/\partial B > 0$, $\partial e_i^*/\partial b > 0$, $\partial e_i^*/\partial \beta_1 > 0$, $\partial e_i^*/\partial \beta_2 > 0$. The equilibrium effort by $A_i$ is decreasing in the other agent’s effort, and increasing in the benefits of all agents, and the parameters of interest congruence between the principal and agents.

4 Agents’ Effort under Incomplete Contract without Bargaining

Now we introduce contract incompleteness into the model. There are many rationales in the reality for this assumption, e.g. due to unforeseeable contingency of the project.

4.1 The Case of Separation

When $\gamma_1 = \gamma_2$, the joint benefit of $A_1$ and $A_2$ are the same, and therefore leaves no room for bargaining. In this case the benefit to the two agents from the two divisions are:

\[
\Pi_1^1 = e_1b + (1 - e_1)e_2\gamma_1b \\
\Pi_1^2 = e_2\gamma_1b + (1 - e_2)e_1b \\
\Pi_2^1 = e_1\gamma_2b + (1 - e_1)e_2b \\
\Pi_2^2 = e_2b + (1 - e_2)e_1\gamma_2b
\]

Add them up and subtract the cost of searching,

\[
\Pi^1 = (2e_1 + 2e_2\gamma_1 - e_1e_2 - e_1e_2\gamma_1)b - g_A(e_1) \\
\Pi^2 = (2e_2 + 2e_1\gamma_2 - e_1e_2 - e_1e_2\gamma_2)b - g_A(e_2)
\]
The agents’ effort should satisfy, let hat denotes separation without bargaining:

\[ g'_A(\hat{e}_1) = [2 - \hat{e}_2(1 + \gamma_1)]b \]  \hspace{1cm} (4)

\[ g'_A(\hat{e}_2) = [2 - \hat{e}_1(1 + \gamma_2)]b \]  \hspace{1cm} (5)

These first order conditions imply the agents choose effort such that the marginal cost of searching is equal to the marginal return to himself. The marginal benefit from the division where the agent \( i \) is in control is one unit chance higher for getting \( b \) given the other is not informed (multiplied by \( (1 - \hat{e}_j), j \in \{1, 2\}, j \neq i \)), but there is a marginal opportunity cost that if \( i \) does not search, it will increase the possibility that \( i \) is not informed but the other agent \( j \) is informed, and \( j \)'s favorite project will bring him \( \hat{e}_j \gamma_i b \). So the net marginal profit from \( D_i \) to \( i \) is \( (1 - \hat{e}_j \gamma_i) b \). The net marginal profit for \( i \) from \( D_j \) is \( (1 - \hat{e}_j) b \), because the effort generates no return if \( j \) is informed, and \( b \) when \( j \) is not informed. So the net marginal return to \( i \) from two divisions are \( [2 - \hat{e}_j(1 + \gamma_i)]b \).

As we already know \( B \) is much larger than \( b \), and the difference between \( 2(1 - e^*) \) and \( [2 - \hat{e}(1 + \gamma)] \) is at most 2, the effort in this case should be lower than first best in most of the cases. So in most cases we have \( \hat{e}_i < e^*_i, i \in \{1, 2\} \).

4.2 The Case of Integration

We just analyze the case where \( A_1 \) is assigned control, and the case where \( A_2 \) has control is analogous. In this case it is just like \( A_1 \) has two divisions of his own, and \( A_2 \) is last to search in both divisions.

\[ \Pi^1 = 2[e_1 + (1 - e_1)e_2\gamma_1]b - g_A(e_1) \]
\[ \Pi^2 = 2[e_1\gamma_2 + (1 - e_1)e_2]b - g_A(e_2) \]

The agents’ effort should satisfy. We use upper bar to denote \( A_1 \) control:

\[ g'_A(\overline{e}_1) = 2(1 - \gamma_1\overline{e}_2)b \]  \hspace{1cm} (6)

\[ g'_A(\overline{e}_2) = 2(1 - \overline{e}_1)b \]  \hspace{1cm} (7)

These first order conditions imply the agents choose effort such that the marginal cost of searching is equal to the marginal return to himself. And integration under \( A_1 \) control is just analogous to the case when both divisions are \( A_1 \)’s “own division” analyzed in the separation case. Each 1 unit of effort by \( A_1 \) gives him more chance to get \( b \) but less chance
to get $\gamma_1 b$ from freeriding $A_2$’s effort. Each unit of effort brings $A_2$ private benefit $b$ only when $A_1$ is uninformed.

Compare this to the first order condition in separation, it is clear that for the same $e_2$, $e_1$ is larger in this case and for the same $e_1$, $e_2$ is smaller in this case. Moreover, in both cases, $e_1$ ($e_2$) is decreasing in $e_2$ ($e_1$). So we can prove by simple contradiction that $\tau_1 > \hat{e}_1, \tau_2 < \hat{e}_2$. By the same argument in the above, $\tau_1, \tau_2$ is smaller than in the first best. To sum up the case of separation and integration, we can find:

**Proposition 1.** Due to conflict of interest, the searching effort chosen by the agents is always smaller under incomplete contract than under the case without contract incompleteness.

**Proposition 2.** Compared to the case of separation, the party who is assigned the control when the divisions are integrated has stronger incentive to search, and the party who loses the control during the integration has lower incentive to search.

### 5 Agents’ Effort under Incomplete Contract with Bargaining

#### 5.1 The Case of Separation

We can assume the joint pay-off is larger under favorite of $A_1$ ($b + \gamma_2 b > \gamma_1 b + b$). So in $D_1$, there is no room for bargaining, because by choosing favorite of $A_2$, the joint pay-off is worse. In $D_2$, $A_2$ can choose favorite of $A_1$, but claim part of the surplus in the joint benefit. The sharing rule is set so that a share of $w$ goes to $A_1$ and $(1 - w)$ goes to $A_2$. So on expectation:
Here for simplicity we assume there is no change in the interest congruence parameter between the agents and the principal during the bargaining. We know that the parameters of interest congruence are based on the parties’ ex ante expectation on the pay-off structure of the projects. This expectation is like a prior, and will be updated once the agent is informed. So we can surely use these parameters to analyze the cases when at least one party is uninformed, but take care when dealing with bargaining where both agents must be informed. We leave the discussion on what will happen if we relax this condition to the next section.

The efforts by the two agents should satisfy the conditions as below, the superscript SP means separation:

\[ g'_A(e^{SP}_1) = (2 - e^{SP}_2 (1 + \gamma_1 - w(\gamma_2 - \gamma_1))b \]  
\[ g'_A(e^{SP}_2) = (2 - e^{SP}_1 [1 + \gamma_2 - (1 - w)(\gamma_2 - \gamma_1))]b \]  

These first order conditions imply the agents choose effort such that the marginal cost of searching is equal to the marginal return to himself. In \( D_1 \) the marginal return of one
unit of effort by the agents is the same with the case when there is no bargaining. In $D_2$, one unit of effort will bring $A_1$ one unit chance higher to get benefit $b$ if $A_2$ is not informed, one unit chance higher to get $[(1 - w)\gamma_1 + w\gamma_2]b$, and opportunity cost of one unit chance to get $\gamma_1 b$ from freeriding $A_2$.

5.2 The Case of Integration

We can still assume the joint pay-off is larger under favorite of $A_1$ ($b + \gamma_2 b > \gamma_1 b + b$).

If $A_1$ controls, $A_1$ always searches first in both divisions, and then $A_2$. There is no need for bargaining.

$$\Pi^1 = 2[e_1 b + (1 - e_1)e_2\gamma_1 b] - g_A(e_1)$$

$$\Pi^2 = 2[e_1\gamma_2 b + (1 - e_1)e_2 b] - g_A(e_2)$$

$$\Pi^P = 2[e_1\alpha_1 B + (1 - e_1)e_2\alpha_2 B]$$

FOC (UP for $A_1$ control):

$$g'_A(e_1^{UP}) = 2(1 - e_2^{UP}\gamma_1)b$$

$$g'_A(e_2^{UP}) = 2(1 - e_1^{UP})b$$

If $A_2$ controls, there is need for bargaining. (DOWN for $A_2$ control) Say the generalized Nash bargaining determines the share of surplus is $w : (1 - w)$

$$\Pi^1 = 2[e_2(1 - e_1)\gamma_1 + e_1 e_2[(1 - w)\gamma_1 + w\gamma_2]$$

$$+ (1 - e_2)e_1 b - g_A(e_1)$$

$$\Pi^2 = 2[e_2(1 - e_1) + e_1 e_2[1 + (1 - w)(\gamma_2 - \gamma_1)]$$

$$+ (1 - e_2)e_1\gamma_2 b - g_A(e_2)$$

$$\Pi^P = 2[e_2(1 - e_1)\alpha_2 + e_1 e_2\alpha_1$$

$$+ e_1(1 - e_2)\alpha_1]B$$

$$g'_A(e_1^{DOWN}) = 2(1 - e_2^{DOWN}(1 + \gamma_1 - [(1 - w)\gamma_1 + w\gamma_2])b$$

$$g'_A(e_2^{DOWN}) = 2(1 - e_1^{DOWN}(1 + \gamma_2 - [1 + (1 - w)(\gamma_2 - \gamma_1)])b$$
5.3 The Principal’s Decision on Divisional Structure

It can be seen that as long as $A_1$ and $A_2$ are informed, they always reach the project that maximizes their joint pay-off \textit{ex post}. But this does not necessarily mean the choice of project is always efficient for the principal. Because the informed agents choose $A_1$’s favorite project if $b + \gamma_2 > \gamma_1 b + b \Leftrightarrow \gamma_2 > \gamma_1$ \footnote{The intuition here the agents want to choose the project that give the party not in control more utility, when the party in control’s utility is fixed.} when the principal is uninformed, but $A_1$’s favorite project is better for the principal only when $\alpha 1 > \alpha 2$.

Let parameter $\phi = 1 + \gamma_1 - [(1 - w)\gamma_1 + w\gamma_2] = 1 - w(\gamma_2 - \gamma_1) \in [0, 1], \delta = 1 + \gamma_2 - [1 + (1 - w)(\gamma_2 - \gamma_1)] = w\gamma_2 + (1 - w)\gamma_1 \in [0, 1], \rho = \frac{\phi + \gamma_1}{2} \in [0, 1], \sigma = \frac{\delta + 1}{2} \in [0, 1]$, and all functions $g(e) = e^2 b$, this will give us the equation systems:

\begin{align*}
\epsilon_1^{SP} &= 1 - \rho \epsilon_2^{SP}, \epsilon_2^{SP} = 1 - \sigma \epsilon_1^{SP} \\
\epsilon_1^{UP} &= 1 - \gamma_1 \epsilon_2^{UP}, \epsilon_2^{UP} = 1 - \epsilon_1^{UP} \\
\epsilon_1^{DOWN} &= 1 - \phi \epsilon_2^{DOWN}, \epsilon_2^{DOWN} = 1 - \delta \epsilon_1^{DOWN}
\end{align*}

Solve them, we can get:

\begin{align*}
\epsilon_1^{SP} &= \frac{1 - \rho}{1 - \rho \sigma} = -\frac{2(-1 + \gamma_1 + w\gamma_1 - w\gamma_2)}{3 + (-1 + w^2) \gamma_1^2 + w^2 \gamma_2^2 - 2 \gamma_1 (1 + w^2 \gamma_2)} \\
\epsilon_2^{SP} &= \frac{1 - \sigma}{1 - \rho \sigma} = \frac{2 + 2(-1 + w)\gamma_1 - 2w\gamma_2}{3 + (-1 + w^2) \gamma_1^2 + w^2 \gamma_2^2 - 2 \gamma_1 (1 + w^2 \gamma_2)} \\
\epsilon_1^{UP} &= 1 \\
\epsilon_2^{UP} &= 0 \\
\epsilon_1^{DOWN} &= \frac{1 - \phi}{1 - \delta \phi} = \frac{w(\gamma_2 - \gamma_1)}{1 - ((1 - w)\gamma_1 + w\gamma_2)(1 - w(\gamma_2 - \gamma_1))} \\
\epsilon_2^{DOWN} &= \frac{1 - \delta}{1 - \delta \phi} = \frac{1 - (1 - w)\gamma_1 - w\gamma_2}{1 - ((1 - w)\gamma_1 + w\gamma_2)(1 - w(\gamma_2 - \gamma_1))}
\end{align*}

Let $\epsilon_1^{DOWN} = p, \epsilon_2^{DOWN} = q$, so $\epsilon_1^{SP} = m, \epsilon_2^{SP} = n$, it is easy to prove $\rho < \phi$ and $\sigma > \delta$, so $m > p, n < q, \phi + \delta = \rho + \sigma = 1 + \gamma_1$. Because $\phi, \delta, \rho, \sigma \in [0, 1]$, $p, q, m, n$ are also in $[0, 1]$. Because these parameters are also the negative cross derivative of one agent’s effort $\epsilon_i$ on the effort of the other $\epsilon_j$ (e.g. $\partial \epsilon_1^{SP}/\partial \epsilon_2^{SP} = -\rho, \partial \epsilon_2^{SP}/\partial \epsilon_1^{SP} = -\sigma$), this immediately gives us:

\textbf{Proposition 3. The effort of agents are always substitutes, the increase (decrease) in one agent’s effort always discourages (encourages) the effort by the other agent.}
We can also get some comparatives by taking derivative of $p,q,m,n$ with respect to $w, \gamma_1, \gamma_2$:

\[
\frac{\partial m}{\partial \gamma_1} = \frac{2(-1 + \gamma_1)^2 + w^3(\gamma_1 - \gamma_2)^2 - w (3 + \gamma_1^2 - 2\gamma_2 - 2\gamma_1\gamma_2) + w^2 (-2\gamma_1 + \gamma_1^2 - (-2 + \gamma_2)\gamma_2)}{(3 + (-1 + w^2)\gamma_1^2 + w^2\gamma_2^2 - 2\gamma_1 (1 + w^2\gamma_2))^2} < 0
\]

\[
\frac{\partial m}{\partial \gamma_2} = -\frac{2w(-3 + (1 + w)^2\gamma_1^2 + 2w\gamma_2 + w^2\gamma_2^2 - 2\gamma_1 (-1 + w + w\gamma_2 + w^2\gamma_2))}{(3 + (-1 + w^2)\gamma_1^2 + w^2\gamma_2^2 - 2\gamma_1 (1 + w^2\gamma_2))^2} > 0
\]

\[
\frac{\partial m}{\partial w} = \frac{2(\gamma_1 - \gamma_2)(-3 + (1 + w)^2\gamma_1^2 + 2w\gamma_2 + w^2\gamma_2^2 - 2\gamma_1 (-1 + w + w\gamma_2 + w^2\gamma_2))}{(3 + (-1 + w^2)\gamma_1^2 + w^2\gamma_2^2 - 2\gamma_1 (1 + w^2\gamma_2))^2} > 0
\]

\[
\frac{\partial p}{\partial \gamma_1} = \frac{w(-1 - w(\gamma_1 - \gamma_2)^2 + w^2(\gamma_1 - \gamma_2)^2 + \gamma_2)}{(1 + (-1 + w)w\gamma_1^2 - w\gamma_2 + w^2\gamma_2^2 + \gamma_1 (-1 + w + w\gamma_2 - 2w^2\gamma_2))^2} < 0
\]

\[
\frac{\partial p}{\partial \gamma_2} = -\frac{w(-1 + \gamma_1 + w^2\gamma_1^2 - 2w^2\gamma_1\gamma_2 + w^2\gamma_2^2)}{(1 + (-1 + w)w\gamma_1^2 - w\gamma_2 + w^2\gamma_2^2 + \gamma_1 (-1 + w + w\gamma_2 - 2w^2\gamma_2))^2} > 0
\]

\[
\frac{\partial p}{\partial w} = \frac{(\gamma_1 - \gamma_2)(-1 + \gamma_1 + w^2\gamma_1^2 - 2w^2\gamma_1\gamma_2 + w^2\gamma_2^2)^2}{(1 + (-1 + w)w\gamma_1^2 - w\gamma_2 + w^2\gamma_2^2 + \gamma_1 (-1 + w + w\gamma_2 - 2w^2\gamma_2))^2} > 0
\]

\[
\frac{\partial n}{\partial \gamma_1} = \frac{-2((-1 + \gamma_1)^2 + w^3(\gamma_1 - \gamma_2)^2 - w (3 + \gamma_1^2 - 2\gamma_2 - 2\gamma_1\gamma_2) + w^2 (2\gamma_1 - \gamma_1^2 + (-2 + \gamma_2)\gamma_2))}{(3 + (-1 + w^2)\gamma_1^2 + w^2\gamma_2^2 - 2\gamma_1 (1 + w^2\gamma_2))^2} < 0
\]

\[
\frac{\partial n}{\partial \gamma_2} = \frac{2w(-3 + (1 + w)^2\gamma_1^2 - 2w\gamma_2 + w^2\gamma_2^2 + 2\gamma_1 (1 + w + w\gamma_2 - w^2\gamma_2))}{(3 + (-1 + w^2)\gamma_1^2 + w^2\gamma_2^2 - 2\gamma_1 (1 + w^2\gamma_2))^2} < 0
\]

\[
\frac{\partial n}{\partial w} = \frac{-2(\gamma_1 - \gamma_2)(-3 + (1 + w)^2\gamma_1^2 - 2w\gamma_2 + w^2\gamma_2^2 + 2\gamma_1 (1 + w + w\gamma_2 - w^2\gamma_2))}{(3 + (-1 + w^2)\gamma_1^2 + w^2\gamma_2^2 - 2\gamma_1 (1 + w^2\gamma_2))^2} < 0
\]

\[
\frac{\partial q}{\partial \gamma_1} = \frac{-w((-1 + w)^2\gamma_1^2 - 2(-1 + w)\gamma_1(-1 + w\gamma_2) + \gamma_2 (1 - 2w + w^2\gamma_2))}{(1 + (-1 + w)w\gamma_1^2 - w\gamma_2 + w^2\gamma_2^2 + \gamma_1 (-1 + w + w\gamma_2 - 2w^2\gamma_2))^2} < 0
\]

\[
\frac{\partial q}{\partial \gamma_2} = \frac{-w((-1 + w)^2\gamma_1^2 - 2(-1 + w)\gamma_1(-1 + w\gamma_2) + \gamma_2 (1 - 2w + w^2\gamma_2))}{(1 + (-1 + w)w\gamma_1^2 - w\gamma_2 + w^2\gamma_2^2 + \gamma_1 (-1 + w + w\gamma_2 - 2w^2\gamma_2))^2} < 0
\]

\[
\frac{\partial q}{\partial w} = \frac{-(\gamma_1 - \gamma_2)((-1 + w)^2\gamma_1^2 + w\gamma_2(-2 + w\gamma_2) + \gamma_1 (-1 - 2w^2\gamma_2 + 2w(1 + \gamma_2)))}{(1 + (-1 + w)w\gamma_1^2 - w\gamma_2 + w^2\gamma_2^2 + \gamma_1 (-1 + w + w\gamma_2 - 2w^2\gamma_2))^2} < 0
\]

\[
\frac{\partial n}{\partial \gamma_1} \text{ and } \frac{\partial q}{\partial \gamma_2} \text{ can be positive or negative, the graph below shows the domain in the space where these derivatives are Negative.}
\]

The welfare for principal under these cases are:
\[
\Pi_P^{SP} = 2[n(1 - m)\alpha_2 + m\alpha_1]B \\
= -4 \left( \left( -1 + w^2 \right) \gamma 1^3 - 2w^2\gamma 1^2(1 + \gamma 2) - 2(1 + w^2\gamma 2^2) + \gamma 1 \left( 3 + w^2\gamma 2(4 + \gamma 2) \right) \right) \left( 3 + (-1 + w^2) \gamma 1^2 + w^2\gamma 2^2 - 2\gamma 1(1 + w^2\gamma 2) \right)^2 B
\]

\[
\Pi_P^{UP} = 2\alpha_1 B = B \\
\Pi_P^{DOWN} = 2[q(1 - p)\alpha_2 + p\alpha_1]B \\
= -4 \left( \left( -1 + w^2 \right) \gamma 1^3 - 2w^2\gamma 1^2(1 + \gamma 2) - 2(1 + w^2\gamma 2^2) + \gamma 1 \left( 3 + w^2\gamma 2(4 + \gamma 2) \right) \right) \left( 3 + (-1 + w^2) \gamma 1^2 + w^2\gamma 2^2 - 2\gamma 1(1 + w^2\gamma 2) \right)^2 B
\]

It might be better to start with a symmetric case, where \(\alpha_1 = \alpha_2\). This means there is no difference between the two agents’ productivity and loyalty. We can set \(\alpha_1 = \alpha_2 = 0.5\), so that \(\Pi^{UP}\) can be normalized to 1.

6 The Spill-over Effect between Agents

6.1 The indifference frontier

In this section we can consider two kinds of spill-over effect: (1) the efficiency enhancing and destroying effect due to integration, as introduced by Riyanto (2000) and (2) the change of interest congruence in the bargaining.

We can think of the efficiency enhancing effect of integration where Principal’s every possible benefit \(B_i\) becomes \((1 + \Delta)B_i\) where \(\Delta > 0\). This happens when integration promotes synergy \(^3\) between the departments. Or efficiency destroying effect of integration where \(\Delta < 0\), where maybe the effort by one agent has some negative externality on the other. \(\Delta = 0\) when there is no externality due to integration.

The other kind of spill over effect is when the project is chosen through bargaining between informed \(A_1\) and \(A_2\), the Principal’s conditional expected pay-off of the “\(A_1\)’s favorite project” (given \(A_2\) is also informed about this project) may be different from that of unconditional mean of “\(A_1\)’s favorite project”. So this kind of project might actually has a different interest congruence \(\alpha_3\) to the Principal. And it could be that \(\alpha_3 > \alpha_1\), meaning

\(^3\)Or complementary gains as in Olsen (1996).
the bargaining between the agents betrays the principal’s interest, or also could be $\alpha_3 < \alpha_1$, meaning the bargaining respects the principal’s interest better than $A_1$’s individual decision.

Moreover, we can assume there is a fixed cost $C$ for separation (e.g. new offices, new equipment, or just communication cost). $C > 0$ means it is costly to set up two departments, and $C < 0$ means running two small departments is more cost saving than running one big department.

Substitute these into (17), (18) and (19) we can get:

$$\Pi_{SP} = 2n\alpha_2 + 2m\alpha_1 + mn(\alpha_3 - \alpha_1 - 2\alpha_2)B - C$$

(23)

$$\Pi_{UP} = 2\alpha_1(1 + \Delta)B$$

(24)

$$\Pi_{DOWN} = 2[q\alpha_2 + p\alpha_1 + pq(\alpha_3 - \alpha_1 - \alpha_2)](1 + \Delta)B$$

(25)

Intuitively, the larger the $\Delta$, the more we prefer integration to separation. The larger the $C$ the more we want to integrate. The larger the $\alpha_3$ the more we prefer the cases with bargaining (separation and $A_2$ control) to the cases without bargaining ($A_1$ control). Especially, if $\Delta = C = 0$, $\alpha_3 > \alpha_1$, $\alpha_1 \gg \alpha_3 - \alpha_1$, $\alpha_2 \to 0$, $m \gg p$, and $m \to 1$ we can have $\Pi_{SP} > 2\alpha_1B = \Pi_{UP}$, $\Pi_{SP} > \Pi_{DOWN}$, meaning separation dominates integration. To sum up, we have:

**Proposition 4.** When spill-over effect of merge is taken into account, both separation and integration can be optimal. It is better for the principal to have integration of control when synergy effect and large set-up cost of new division are large, and separation when they are small. The proper scope of separation and integration also depends on the interest congruence parameters before and after bargaining.

### 6.2 Numerical Simulations

Because we have many parameters in this model, it is really difficult to solve analytically the frontier of the optimality for different control allocations. We can do pick some values and do numerical simulation instead.

**With respect to synergy and set-up cost**

A rough illustration on the effect of synergy $\Delta$ and set up cost $C$: large synergy and cost to set up separate departments make it more favorable for the principal to take integration, and vise versa. So there is a frontier in the graph on which the principal is indifferent to
have integration or separation. She prefers to have integration when the combination lies
to the northeast of the frontier, and vice versa.

Figure 3: The frontier of the dominance area of separation and integration with respect to
$\Delta$ and $C$.

**With respect to interest congruence parameter $\alpha_3$**

We let $C = \Delta = 0$, $\alpha_1 = 0.5$, so $\Pi_{UP}^{IP}$ is normalized to 1. We use $\alpha_2$ and $\alpha_3$ as the axis
in the $xy$-plane. $\alpha_2$ represents the relative productivity of $A_2$, $\alpha_3$ represents the interest
congruence enhancing (destroying) effect of bargaining. So it is intuitive to expect the result
will favor $A_2$ control for large $\alpha_2$ values, and favor separation for large $\alpha_3$ values. We can
start from a very simple combination of parameters: $\gamma_1 = \gamma_2 = w = 0.5$. Which can be
shown in the following figure:

Figure 4: The principal’s benefit when $\gamma_1 = \gamma_2 = w = 0.5$. The yellow surface with black
grid is the utility under agent 2 control, the white surface with yellow grid is under agent
1 control and the orange one is her utility under separation.

The $z$-axis represents the principal’s benefit under these divisional structures. So the
optimal control right is the highest surface in the graph. This graph shows integration
under either agent 1 or agent 2 control can be optimal. The 2-D graph will be like:

![2-D graph showing A1 and A2 control dominance]

Figure 5: The case when only integration can be optimal.

Starting from this, we may continue to investigate qualitatively how the optimal control area changes with the three parameters. First we may want to see the case when $\gamma_1$ increases or decreases.

![Graph showing the utility level of the principal]

Figure 6: The principal’s benefit when $\gamma_1 = 0.1, \gamma_2 = w = 0.5$ (left) and $\gamma_1 = 0.9, \gamma_2 = w = 0.5$ (right). The pink surface with black grid is the utility under agent 2 control, the white surface with pink grid is under agent 1 control and the purple one is her utility under separation.

In this case all three control allocations can be optimal. There is a frontier on which the principal is indifferent between each two control allocations. The 2-D graph will be like.

We tried many other combinations of parameters and found the dominance region is either like figure 5 or figure 7. But unfortunately it seems there is no clear pattern how the area of dominance region is associated with the parameter values.
7 Conclusion and Discussion

We build an incomplete contract model where setup (communication) cost, synergy effect, incentive induced by control structure and change in the interest congruence between the principal and agent matter for the decision on internal integration (separation) of divisions at the same time. The first two factor are basically very similar to the adaptation-coordination trade off in communication literature before, and the incentive induced by control gives very similar result as Grossman and Hart (1986), where it is usually better to assign the control right to the people whose incentive is more important to the performance measure we concern (joint payoff in Grossman and Hart (1986), and the principal’s benefit in our model). We show that the “integration dominance” argument usually still applies when there is no change of interest congruence, because intuitively, separation is like a linear combination of both kind of integrations ($A_1$ control in $D_1$ and $A_2$ control in $D_2$ is equivalent to half $A_1$ control on both and half $A_2$ control on both when $D_1$ and $D_2$ are assumed to be identical), which is usually dominated by corner solution in this kind of problems. The change of interest congruence during bargaining is a key factor that differentiate internal integration problem with external integration, which may lead to a considerable area of separation dominance not seen before. The rationale for the principal to choose separation is integration may either harms the productive person too much (when she gives control to the unproductive one, whose favorite project happens not to maximize the joint payoff of the two agents), or erase the room for interest congruence enhancing bargaining (when she gives control to the productive one, whose favorite project happens not to maximize the joint payoff of the two agents). Then separation comes as a “compromise” of the two and therefore is better of both extremes. The optimal divisional structure may be a balance of all four forces discussed above. When the incentive problem and synergy effect

Figure 7: The case when both integration and separation can be optimal.
is more important, and there is positive setup cost for new division, it is better to have integration with productive agent control. When the enhancement of interest congruence is more important, and there is positive cost to merge divisions, it is better to have separation.

The principal plays a central role in the choice of control right allocation in this paper. We can also note several differences between her role in this model and that in the past literatures. First, the principal in our work is not a “searching (productive) principal” in Aghion and Tirole (1997), because she does not search, and therefore does not make decision on each specific projects. We restrict our attention on the principal’s decision on the divisional structure, and leave out her role in daily operation. We can think of many real life examples of this assumption. For instance, it is not necessary for the president of a hospital to treat a lot of patients everyday, or the president of a university to do a lot of research. It is more important for them to make decisions on the general management level and allocate the decision right to the divisions properly to promote the overall efficiency of the organization. Secondly, she is not a “bargaining principal” in the models on the principal’s choice between in-house production and out-sourcing (Hart et al, 1997). The principal in our model does not bargain directly with her agents, but plays a central role in the bargaining between the agents, because her decision on the control allocation determines their bargaining position. Finally, she is not a principal making their decision based on the functional features of the divisions (Bolton and Dewatripont, 1994, Hart and Moore, 2005), because the two departments perform the same kind of task.

We are aware that in complete contracting literature, there are also papers that compares the efficiency of contracts under three kinds of organizational structures: separation, integration and nested departments (Baron and Besanko, 1992, Melumad et al, 1995, Mookherjee and Tsumagari, 2004, Severinov, 2008). It should be noticed that we use “separation” in complete and incomplete contract for the same meaning, but the “integration” in this paper is more like the “nested departments” or “subcontracting” in complete contract. “Integration” is used in complete contract only for the case when the departments are merged, one agent is made the manager while the other agent is removed from the game. The current model can be extended and enriched in many ways. We leave the development of a model with more than two agents, and analytical solution of the frontier of the dominance control area for future work.
References


