The Size of Stable International Environmental Agreements in the case of Stock Pollution *

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29th March 2011

Abstract

Most of the literature on stability of International Environmental Agreements is essentially static and can therefore not identify changes in the size of the stable coalition in connection with changes in the stock of pollutants. This is a relevant issue because most global pollution problems are dynamic with stock externalities. This paper shows that the incentives for membership indeed change with changes in the stock of pollutants and that technology choice is an important factor when considering the consequences.

JEL codes: Q20, C70, F42

Keywords: IEAs, coalitional stability, stock externalities, dynamics

*This research has been supported by the Netherlands Organization for Scientific Research (NWO) under a MaGW-VIDI grant.

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1 Introduction

The literature on stability of International Environmental Agreements (IEAs) usually employs static models and identifies a fixed size for the stable coalition (see Barrett (2003) and Finus (2003) for good surveys). However, many of the underlying environmental problems (such as climate change) involve stock externalities and are based on dynamic accumulation of pollution. The question rises whether the size of the stable coalition may change with changes in the stock of pollutants and, if yes, what the consequences are.

This paper formulates a standard international pollution control model and follows the main part of the literature by using a stability concept developed for cartel analysis by d'Aspremont *et al.* (1983), introduced into the literature on IEAs by Hoel (1992), Carraro & Siniscalco (1993) and Barrett (1994). This concept requires that a signatory does not have an incentive to leave the agreement (internal stability) and that a non-signatory does not have an incentive to join the agreement (external stability). This paper differs from the standard literature in two respects. Firstly, the choice to become a member of the coalition is not a one-shot decision but may change over time. Since the stock of pollutants changes over time, the incentives to join or to leave the agreement may change as well and therefore it may happen that countries join or leave the agreement at a later time. Secondly, in order to increase the possible spectrum of stable coalition sizes an extra positive externality for members is introduced. The standard model only allows for very small coalition sizes, which implies that a dynamic analysis cannot show much variation either. Therefore it is assumed that signatories can share R&D costs for the development of technology that is needed to lower emission levels. This will increase the incentives to join the coalition, so that a wider range of stable coalitions becomes feasible (see Buchner & Carraro 2005, and Ruis & de Zeeuw 2010).

Rubio & Ulph (2007) is one of the few papers in which a model is developed where membership of an IEA changes with the level of the stock but that model differs in two respects. Firstly, the basic model is different and leads, for example, to non-signatories emitting some maximum level of emissions. Secondly, when the size of the stable coalition is determined at a later time, membership is randomly assigned so that all the countries have the same expected present value of future net benefits. In our paper the basic model is standard and membership is not randomly assigned. We focus in that context on the question how the size of the stable coalition changes with changes in the stock of pollutants and how technology choice affects the results. First we show for average levels of R&D and technology that the size of the stable coalition decreases if the stock of pollutants increases. This implies that the countries should not wait too long with negotiations, not only because the stock of pollutants becomes high but also because they lose opportunities for cooperation. Next we show what happens for high levels of R&D with high effectiveness of abatement and for low levels of R&D with low effectiveness of abatement, respectively. In these cases we assume that the countries do not take account of possible changes in the size of the coalition in the future but we will show that if such an agreement is regularly renegotiated, the size will actually change. More specifically, if R&D levels for clean technology are low so that the effectiveness of abatement is low as well, the size of the stable coalition will decrease over time and the steady-state stock of pollutants will be high. On the other hand, if R&D levels are high so that the effectiveness of abatement is high as well, the grand coalition can eventually be sustained: for a high initial stock of pollutants, the size of the stable coalition will increase over time and the stock of pollutants will decrease and converge to a low steady state. It is left for further research to consider what happens if the countries take account of expected future changes in the size of the stable coalition.

Besides the contribution to the analysis of IEAs, this paper also provides an extension to the theory of differential games. The countries are assumed to be symmetric but asymmetries arise due to the choice to become a signatory or a non-signatory. If this is a once-and-for-all decision, the resulting differential game is still standard (see Rubio & Casino 2005). However, in this paper the asymmetries may change over time and such a differential game has, to our knowledge, not been considered before.

Section 2 presents the basic pollution control model. In section 3 the value functions of signatories and non-signatories are derived for a fixed coalition size. In section 4 the size of the stable coalition is given as a function of the stock of pollutants and this result is used to discuss what happens if the agreement is regularly renegotiated. Section 5 concludes the paper.

2 The model

There are n countries, $n > 2$. Without concern for the environment, each country will emit a level E of pollutants and we assume that this level is constant over time. These pollutants are added to a global stock of pollutants s that decays at a rate β. This stock of pollutants is damaging and we assume that damage costs are given by the quadratic form $\frac{1}{2}s^2$. Countries can choose to become a member of the coalition or to stay out. The size of the coalition is denoted by k and the number of outsiders by $\ell = n - k$. Each outsider j can reduce emissions down to $E - \gamma a_j(t)$, where the parameter γ denotes the effectiveness of abatement a at time t. The coalition can reduce emissions down to $kE - k\gamma a_0(t)$ where a_0 denotes the abatement level the coalition agrees upon for each member. We assume that abatement costs are given by the quadratic form $\frac{1}{2}a^2$. The total costs can simply be written as the sum of these quadratic forms by choosing the appropriate scale for abatement a: the scale parameter can be incorporated into the parameter γ. Finally, the effectiveness of abatement is related to the choice of technology. This technology has to be developed at a fixed cost c. We assume that the coalition members can share this cost to reflect the positive externalities of R&D among cooperating countries. We do not postulate a functional relationship between c and γ but we will consider cases where both c and γ are either high or low or have intermediate values. The total costs of the coalition are denoted by kC_0, where C_0 are the total costs of an individual coalition member, and the total costs of outsider j are denoted by C_j. The discount rate is denoted by ρ. This leads to the following model: the coalition of size k minimises the costs of each individual coalition member, assuming that all members perform the same action a_0:

$$C_0 = \int_0^\infty \left(\frac{1}{2}s(t)^2 + \frac{1}{2}a_0(t)^2 + \frac{c}{k} \right) \mathrm{e}^{-\rho t} \, \mathrm{d}t \tag{1}$$

whereas the outsiders minimise

$$C_j = \int_0^\infty \left(\frac{1}{2}s(t)^2 + \frac{1}{2}a_j(t)^2 + c \right) \mathrm{e}^{-\rho t} \, \mathrm{d}t, \qquad j = 1, 2, \cdots, \ell. \tag{2}$$

subject to the dynamics of the stock of pollutants, given as

$$\dot{s}(t) = nE - \beta s(t) - k\gamma a_0(t) - \sum_{j=1}^{\ell} \gamma a_j(t), \qquad s(0) = s_0. \tag{3}$$

The model (1)-(3) is a differential game (Başar & Olsder 1982) with the coalition of size k and the ℓ outsiders as $\ell + 1$ players. We assume that countries can monitor the stock of pollutants s and can therefore condition their abatement levels a at time t on the current level of that stock. This implies that we focus on Nash equilibria in feedback strategies $a(s)$.

3 Fixed coalition size

We start with solving the model of the previous section for a fixed coalition size k.

The current-value Pontryagin functions[1] are given respectively as

$$P_0(a_0, s, p_0) = \frac{1}{2}s^2 + \frac{1}{2}a_0^2 + \frac{c}{k} + p_0 \left(nE - \beta s - k\gamma a_0 - \gamma \sum_{j=1}^{\ell} a_j \right), \tag{4}$$

$$P_i(a_i, s, p_i) = \frac{1}{2}s^2 + \frac{1}{2}a_i^2 + c + p_i \left(nE - \beta s - k\gamma a_0 - \gamma \sum_{j=1}^{\ell} a_j \right), \quad i = 1, 2, \cdots, \ell. \tag{5}$$

Minimising these functions over a_0 and a_i respectively yields $a_0 = k\gamma p_0$ and $a_i = \gamma p_i$ for $i = 1, \cdots, \ell$.

The current-value Hamilton functions of the countries then read as

$$H_0(s, p_0) = \frac{1}{2}s^2 - \frac{1}{2}k^2\gamma^2 p_0^2 + \frac{c}{k} + p_0 \left(nE - \beta s - \gamma^2 \sum_{i=1}^{\ell} p_i(s) \right), \tag{6}$$

$$H_i(s, p_i) = \frac{1}{2}s^2 - \frac{1}{2}\gamma^2 p_i^2 + c + p_i \left(nE - \beta s - k^2\gamma^2 p_0(s) - \gamma^2 \sum_{j \neq i} p_j(s) \right), \quad i = 1, 2, \cdots, \ell. \tag{7}$$

The strategies $a_0 = k\gamma p_0$ and $a_i = \gamma p_i$, being feedback strategies, depend on the state s

[1] Also called pre-Hamilton functions or Hamilton functions.

and therefore the system of state and co-state equations is given as

$$\dot{s}(t) = nE - \beta s(t) - k\gamma^2 p_0(s(t)) - \gamma^2 \sum_{j=1}^{\ell} p_j(s(t)), \tag{8}$$

$$\dot{p}_0(t) = p_0(t)\left(\rho + \beta + \gamma^2 \sum_{j=1}^{\ell} p_j'(s(t))\right) - s(t), \tag{9}$$

$$\dot{p}_i(t) = p_i(t)\left(\rho + \beta + k^2\gamma^2 p_0'(s(t)) + \gamma^2 \sum_{j\neq i} p_j'(s(t))\right) - s(t), \qquad i = 1, 2, \cdots, \ell. \tag{10}$$

We restrict ourselves to the symmetric case in which all outsiders use the same abatement strategy $a_1 = \gamma p_1$ and we restrict ourselves to linear strategies, given by

$$a_0(s) = k\gamma p_0(s) = k\gamma(p_{00} + p_{01}s), \quad a_1(s) = \gamma p_1(s) = \gamma(p_{10} + p_{11}s). \tag{11}$$

Note that we abuse notation slightly by denoting both the time-varying co-state variables $p_i(t)$ as well as their state-dependent counterparts $p_i(s)$ with the same letter p.

The state dynamics (3) take the form

$$\dot{s}(t) = nE - \beta s(t) - \gamma^2(kp_{00} + \ell p_{10}) - \gamma^2(kp_{01} + \ell p_{11})s(t), \qquad s(0) = s_0. \tag{12}$$

Note that

$$\dot{p}_i(t) = p_i'(s(t))\dot{s}(t) = p_{i1}\dot{s}(t), \qquad i = 0, 1. \tag{13}$$

Substituting (12) in (13) and the result together with the definitions (11) in equations (9) and (10) leads to two linear equations in $s(t)$ that have to hold everywhere. This, in turn, leads to a system of equations in the coefficients p_{ij}, $i, j = 0, 1$. The resulting expressions are complex and we do not give them explicitly. [2]

Because we restricted ourselves to linear strategies, the cost functions are quadratic. The cost functions for the coalition and for an outsider are respectively denoted by $C_0(k, s)$ and $C_1(k, s)$, where k denotes the size of the coalition. Because $dC/ds = p$, the coefficients p_{i1}, $i = 0, 1$, are the coefficients of the quadratic terms in the value functions and must therefore be positive in this case of cost minimisation. It follows from the state dynamics (12) that the state converges to a unique equilibrium

$$\bar{s}(k) = \frac{nE - \gamma^2(kp_{00} + \ell p_{10})}{\beta + \gamma^2(kp_{01} + \ell p_{11})}. \tag{14}$$

The cost functions (or value functions) can be computed by first determining $C_i(k, \bar{s})$, $i = 0, 1$, and then setting

$$C_i(k, s) = C_i(k, \bar{s}) + \int_{\bar{s}}^{s} p_i(\tilde{s})\, \mathrm{d}\tilde{s}, \qquad i = 0, 1. \tag{15}$$

[2] These expressions are available from the authors upon request.

4 Stability

The largest coalition that satisfies internal and external stability (at initial state s_0 and after a once-and-for-all decision on membership) is found by starting at $k = n$ and lowering k step by step until for some $k > 1$

$$C_0(k, s_0) < C_1(k-1, s_0), \tag{16}$$

if such a k exists. In this case the costs of a member of a coalition of size k are smaller than the costs of an outsider to a coalition of size $k-1$, so that it does not pay for a coalition member to choose to be an outsider.

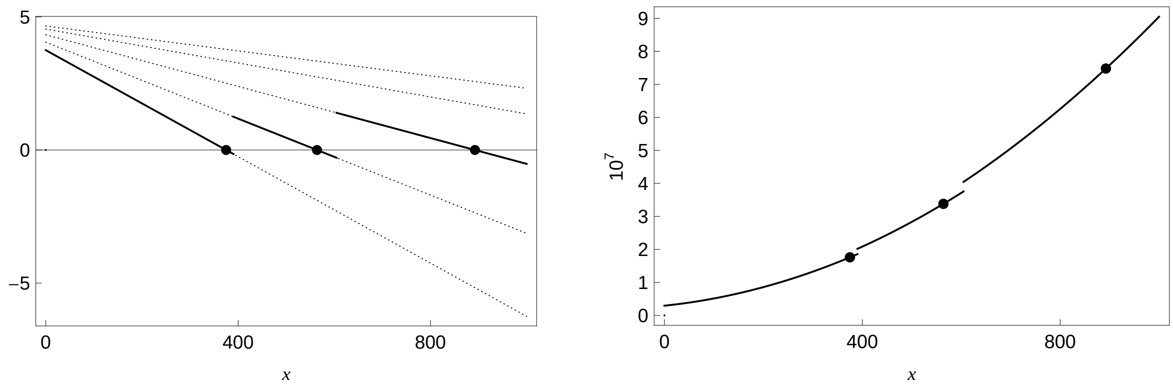
If the costs of developing a new technology c are equal to 0, we are back in the well-known case (see Rubio & Casino 2005) and the size of the stable coalition is small and equal to 2 for this specific model. However, if these costs are positive so that membership of the coalition is more beneficial in the sense that these R&D costs can be shared, a whole spectrum of stable sizes becomes possible. We present three interesting cases for different sets of parameter values.

Consider first the parameter values: $n = 5$, $E = 5$, $\beta = 0$, $\gamma = 0.004$, $c = 20000$ and $\rho = 0.03$. This means that the number of (blocks of) countries negotiating is equal to 5, business-as-usual emission levels are equal to 5, natural decay is 0, the effectiveness of abatement is 0.004, R&D costs are equal to 20000, and the discount rate is 0.03. The result is calculated with the help of Mathematica[3] and is depicted in Figure 1(a). Note that the state dynamics (12) depend on the size of the coalition: the dotted lines plot the right-hand side of the state dynamics (12) as a function of s for each size of the coalition $k = 1, 2, .., 5$. The intersection points with the s-axis are the steady-states $\bar{s}(k)$, and the lowest line corresponds to the dynamics in case of the grand coalition $k = 5$. The thick lines denote the dynamics for the largest stable coalition when we start in that region of s. For small values of s, the grand coalition is stable; it loses stability at $s \approx 390$. Between $s \approx 390$ and $s \approx 605$, the coalition of size 4 is stable and for larger starting values of s, the coalition of size 3 is stable. Total costs are determined as the sum of all value functions. Figure 1(b) depicts the total costs. At the values $s \approx 390$ and $s \approx 605$, the costs jump up because the size of the stable coalition decreases.

This result has interesting implications. It follows that if the stock of pollutants has risen above a jump point before environmental concerns are taken seriously, costs not only become higher because the stock is high but also because a large stable coalition cannot be sustained anymore. Furthermore, the steady-state of the stock will be higher than in case the agreement had been implemented before the stock passed the jump point. It follows that it matters to act quickly, not only because the stock of pollutants is still low but also because it is easier to establish cooperation.

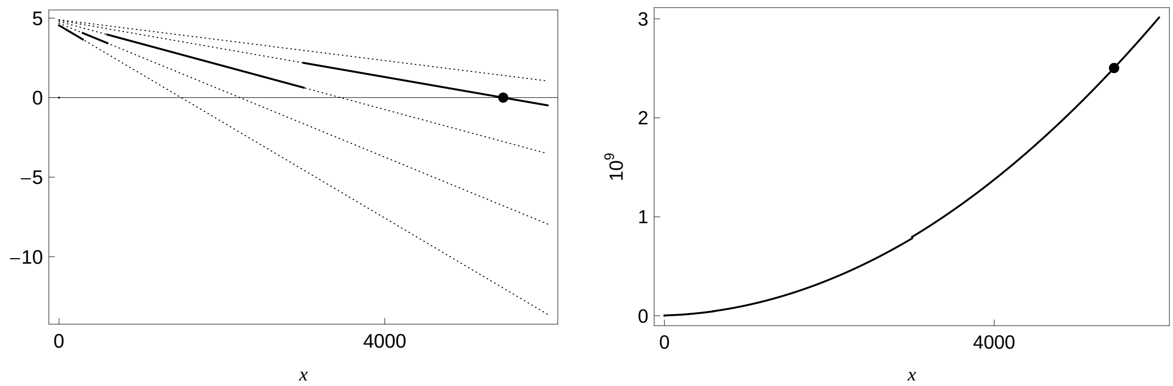
In Figure 1(a) the state remains in the region where it starts and therefore the size of the largest stable coalition does not change when the state converges to the steady state. However, this is not necessarily the case for all parameter settings. Consider the same parameter set as above but change the parameters (γ, c) first to $(0.002, 5000)$ and then to $(0.01, 40000)$. In the first case the countries invest in a cheaper and less effective technology than before, and in the second case the countries invest in a more expensive and more effective technology. The results are depicted in Figures 2 and 3, respectively.

[3]Details are available from the authors upon request.



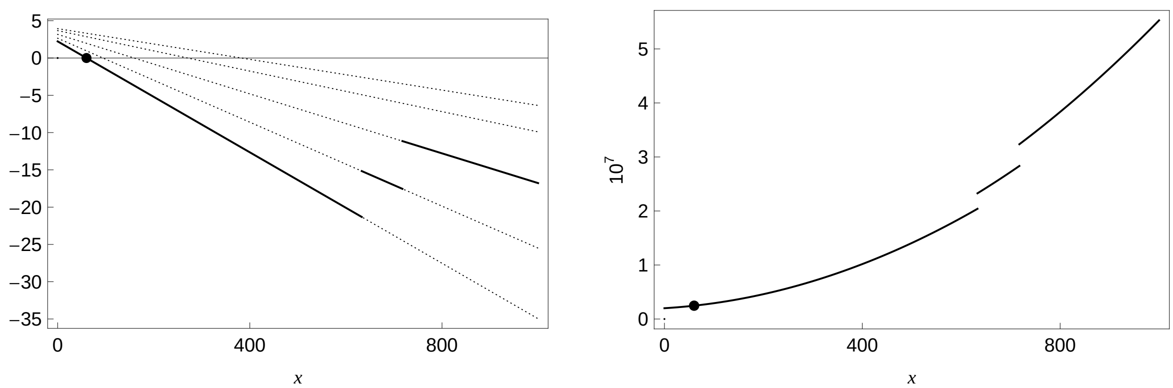
(a) Dynamics (b) Total costs

Figure 1: Pollution dynamics and total costs



(a) Dynamics (b) Total costs

Figure 2: Cheap and ineffective technology



(a) Dynamics (b) Total costs

Figure 3: Expensive and effective technology

In these cases the size of the largest stable coalition may change when the state converges to the steady state. In the first case, when we start at a very low stock, the size of the largest stable coalition is 5 but this size changes to 4 and then to 3 before the state has reached the steady state that results for a coalition of size 5. Similarly, when we start in the region where the size of the largest stable coalition is 3, this size changes to 2 before the state has reached the steady state. This implies that the size of the stable coalition will change if the agreement is renegotiated after these incentives have changed. In the second case, it is the other way around. When we start at a very high stock, the size of the largest stable coalition is 3 but this size increases to 4 and then to 5 before the state has reached the steady state that results for a coalition of size 3. The implications are the same as in the first case.

It follows that if the decision on membership is not taken once and for all, the size of the stable coalition may change with changes in the stock of pollutants. In the two cases above it is shown that if an International Environmental Agreement is regularly renegotiated, the size of the stable coalition will not stay the same. It is clear that if R&D costs are low, the extra incentive to join the coalition is low as well so that it will be harder to sustain a large stable coalition. Furthermore, it is clear that low effectiveness of abatement will lead to a high stock of pollutants. Similarly, high R&D costs and a high effectiveness of abatement have the opposite effects. It is remarkable, however, that the size of the stable coalition changes over time. In the first case, if we start at a very low stock of pollutants, the countries will first coordinate on the grand coalition but this coalition loses stability quickly. When the agreement is renegotiated, the size of the coalition decreases step by step until the size is only 2, and the stock of pollutants converges to a very high steady state. In the second case, the opposite occurs. If we start at a very high stock of pollutants, the size of the stable coalition is only 3 initially but it increases step by step until the grand coalition is reached, and the stock of pollutants converges to a very low steady state. Since these two cases only differ in technology choice, it follows that this matters a lot. In case of a cheap and ineffective technology, a large stable coalition cannot be sustained and the steady-state stock of pollutants is very high. In case of an expensive and effective technology, however, the grand coalition is stable or becomes stable over time, and the steady-state stock of pollutants is very low.

The model is vulnerable for an important type of critique: the countries are not forward looking with respect to the size of the stable coalition. Although the International Environmental Agreement is regularly renegotiated, it is assumed that the decision of the countries to join the coalition or not is not based on expected changes in the size of the stable coalition. It is left for further research to show what happens if the countries are also forward looking in this respect.

5 Conclusion

This paper connects the size of a stable International Environmental Agreement to the level of the stock of pollutants. The spectrum of stable coalition sizes is increased by introducing an extra positive externality of shared R&D costs within the agreement. First it is shown for average levels of R&D costs that the size of the stable coalition decreases with an increasing level of the stock of pollutants. It pays to implement an agreement early, not only because the stock of pollutants is still low but also because a large stable coalition is still possible. Next it is shown that in case of high and low levels of R&D costs, respectively, the size of the stable coalition may change over time if the agreement is regularly renegotiated. For a cheap and ineffective technology, a large stable coalition cannot be sustained and the steady-state stock of pollutants is very high. For an expensive and effective technology, however, the grand coalition is stable or becomes stable over time and the steady-state stock of pollutants is very low.

Further research will focus on three refinements of the model. First, we want to incorporate technology choice as one of the decision stages in the model. Second, we want to consider the time-inconsistency in the model and investigate what happens if countries incorporate the changes in the size of the coalition that are to be expected. Third, we want to consider non-linear dynamics with tipping points as this describes the most important environmental problems of our time such as climate change.

References

Barrett, S. 1994. Self-enforcing international environmental agreements. *Oxford Economic Papers*, **46**, 878–894.

Barrett, S. 2003. *Environment and Statecraft. The strategy of environmental treaty-making*. Oxford: Oxford University Press.

Başar, T., & Olsder, G.J. 1982. *Dynamic Noncooperative Game Theory*. New York: Academic Press.

Buchner, B., & Carraro, C. 2005. Economic and Environmental Effectiveness of a Technology-Based Climate Protocol. *Climate Policy*, **4**, 229–248.

Carraro, C., & Siniscalco, D. 1993. Strategies for the international protection of the environment. *Journal of Public Economics*, **52**(3), 309–328.

d'Aspremont, C., Jacquemin, A., Gabszewicz, J., & Weymark, J. 1983. On the stability of collusive price leadership. *Canadian Journal of Economics*, **16**(1), 17–25.

Finus, M. 2003. Stability and design of international environmental agreements: the case of transboundary pollution. *Pages 82–158 of:* Folmer, H., & Tietenberg, T. (eds), *The International Yearbook of Environmental and Resource Economics 2003/2004*. Cheltenham: Edward Elgar.

Hoel, M. 1992. International environmental conventions: the case of uniform reductions of emissions. *Environmental & Resource Economics*, **2**(2), 141–159.

Rubio, S., & Casino, B. 2005. Self-enforcing international environmental agreements with a stock pollutant. *Spanish Economic Review*, **7**(2), 89–109.

Rubio, S., & Ulph, A. 2007. An infinite-horizon model of dynamic membership of international environmental agreements. *Journal of Environmental Economics and Management*, **54**(3), 296–310.

Ruis, A., & de Zeeuw, A. 2010. International cooperation to combat climate change. *Public Finance and Management*, **10**(2), 379–404.