Persistence and volatility of Beveridge cycles

Florian Sniekers[1]

University of Amsterdam, VU University Amsterdam and Tinbergen Institute

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Abstract

This paper aims to explain the magnitude and cyclical behavior of the fluctuations in unemployment and vacancies. Adding demand externalities to an otherwise standard search and matching model reduces the need for exogenous shocks in explaining these fluctuations. Under plausible parameter values, the equilibrium dynamics include a stable limit cycle that resembles the empirically observed counterclockwise cycles around the Beveridge curve. Quantitatively, these endogenous 'Beveridge cycles' can explain half of the volatility and almost all persistence of unemployment without any exogenous forces, avoiding the amplification and propagation problems of the standard model.

Keywords: Search and matching; Endogenous cycles; Dynamics of unemployment and vacancies; Demand externalities.

JEL Classification: E24; E32; J23; J63; J64.

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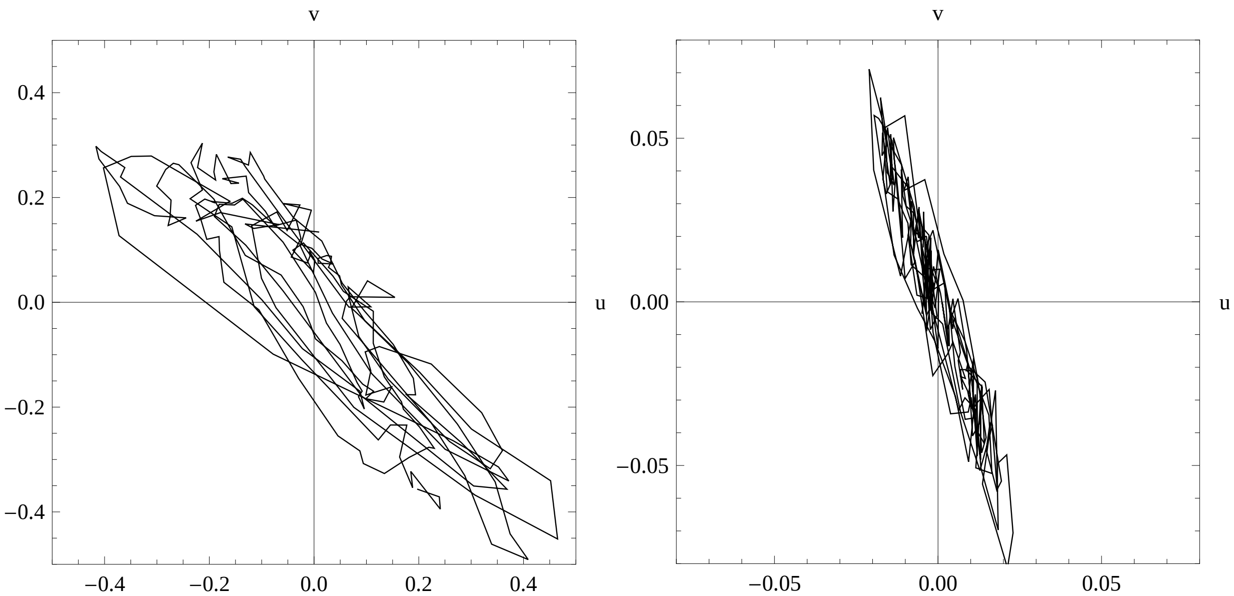
Email: f.j.t.sniekers@uva.nl

1 Introduction

The dynamic relation between vacancies and unemployment is characterized by counterclockwise cycles in the *unemployment, vacancy rate*-plane. Pissarides (2000, p. 36) considers these counterclockwise cycles to be a stylized fact of business cycles. After removing any long-term trends with an HP-filter, the cycles for the United States are presented in Figure 1(a).



(a) United States, 1951-2003.

(b) Simulation of Shimer (2005).

Figure 1: Cyclical component of the Beveridge curve.

Notes: Data are quarterly averages of a monthly series, taken from Shimer (2005). Vacancies are help-wanted advertisements as constructed by the Conference Board. Unemployment data are the seasonally adjusted series from the CPS. The trend is an HP-filter with smoothing parameter 10^5. The simulation is a representative realization with productivity shocks only.

The current paper explains these cycles with a limit cycle driven by demand externalities, based on Mortensen (1999) and explained below. Demand externalities generate feedback from aggregate employment to the (expected) revenue per worker. Such feedback can give rise to endogenous rational expectations cycles in vacancies and unemployment, which I call Beveridge cycles. I investigate whether these Beveridge cycles can match the empirically observed standard deviation and autocorrelation of unemployment. Calibrating the cycles to the median duration of the business cycle, 50 percent of the volatility in unemployment can be explained without any exogenous shocks. Since one-third to one-half of unemployment volatility is driven by fluctuations in the job separation rate - the rate at which employed workers flow into unemployment that is constant in this model - 50 percent amounts to almost all volatility in unemployment that is driven by job creation. In addition, without exogenously specifying a persistent stochastic process for productivity, the simulated autocorrelation of unemployment is close to the observed correlation.

In the canonical search and matching model of Pissarides (1985) unemployment is a state variable, while vacancies respond to shocks immediately. As a result, the model captures the counterclockwise direction of the cycle along the Beveridge curve. However, the model lacks cyclical responses, as can be seen comparing the panels of Figure 1. Panel 1(b) plots a realization of a detrended Beveridge curve as simulated by Shimer (2005), in his calibration with productivity shocks only. The simulation not only lacks unemployment volatility (note the scaling differences across the panels!), but it also mainly features near vertical dynamics, in contrast to the data in Panel 1(a).[2] Indeed, Shimer shows that both the amplification *and* the propagation of the standard search and matching model are too weak to generate the observed fluctuations in unemployment and vacancies in response to realistic exogenous shocks in productivity and/or the separation rate. While a large literature has arisen that adjusted some of the parameters or assumptions of the model to increase the amplification, the lack of propagation has received less attention.[3] Fujita and Ramey (2007) are the first to focus on the lack of propagation. They show that cyclical responses can be generated by the introduction of sunk costs of vacancy creation. However, spreading out the impact of a shock in such a way results in a counterfactually high cross correlation between labor market variables and productivity across time. Other articles addressing propagation, such as Petrosky-Nadeau (2014), follow Fujita and Ramey in focusing on the costs of vacancy creation.[4]

[2]Besides, although not entirely independent, the slope of the curve differs. I return to this issue below.

[3]With regard to amplification, Hall (2005) investigates the effect of wage rigidity, and Hall and Milgrom (2008) study the effect of an alternating offer bargaining model. Hagedorn and Manovskii (2008) argue that the outside option of the worker should be calibrated differently, while Pissarides (2009) argues that the solution to the Shimer puzzle should neither be found in wage stickiness nor in the outside option, but in the introduction of fixed matching costs. See also Mortensen and Nagypal (2007) for a discussion of several assumptions in this literature and how a particular combination of them can account for two-thirds of the volatility in the vacancy-unemployment ratio.

[4]Due to credit frictions, in Petrosky-Nadeau (2014) the costs of filing a vacancy are less procyclical than in the standard model, similar to Pissarides (2009). Also Yashiv (2006) and Hertweck (2013) tie the costs of recruiting more to actual hiring than to posting vacancies. Depending on the assumptions on wage setting, the costs of vacancy creation may also affect the benefits of vacancy creation via wages. Only Gertler and Trigari (2009) achieve persistence from wages per se, by staggered bargaining.

Alternatively, cycles can result from a self-reinforcing effect on the *benefits* of vacancy creation, in particular from feedback from aggregate employment to the (expected) revenue per worker. In Shimer (2005) the stochastic process in productivity accounts for 100 percent of the contemporaneous variation in labor market tightness.[5] I present a model at the other end of the spectrum, in which dynamics can be fully explained by an endogenous mechanism. This approach is not subject to problems of amplification and propagation, simply because exogenous shocks play no role. Within the endogenous business cycle literature that I am aware of, only Mortensen (1999) explains the counterclockwise cycles along the Beveridge curve. I present this model in the standard search and matching variables - unemployment and labor market tightness - to investigate whether it can quantitatively explain the labor market facts of the business cycle, while featuring counterclockwise cycles in unemployment and vacancies. Along the way I provide a global stability analysis of the standard model extended with variable search intensity and feedback from employment and show that it features a Bogdanov-Takens bifurcation.[6] As a result, a limit cycle exists for a range of parameter values, and it is stable. I include a positive value of leisure that is important for the empirical performance, but for very different reasons than in Hagedorn and Manovskii (2008). While the outside option of the unemployed in the standard (linear) search and matching model is simply a scale variable, in my nonlinear model it has a non-trivial impact on the existence and characteristics of equilibria. To the best of my knowledge no other paper calibrates a deterministic model to quantify its performance in explaining labor market dynamics.[7]

Several interpretations can be given to the feedback from aggregate employment to the (expected) revenue per worker. In Mortensen (1999), the production function of a single worker-employer match exhibits increasing returns in *aggregate* employment. These externalities are accounted for by external economies across sectors as in Hall (1988) and Caballero and Lyons (1989), maintaining the competitive nature of the goods market. Mortensen (1999, p. 890) also suggests an interpretation

[5] A univariate regression of labor market tightness (or the job finding rate) on productivity results in an R^2 of 1.00, as follows from his Table 3. Based on their Table 4, in Fujita and Ramey (2007) the R^2 would be 0.92. Table 2 of Petrosky-Nadeau (2014) (with cyclical recovery rates) implies an R^2 of 0.79.

[6] In a Bogdanov-Takens bifurcation, limit cycles emerge at the same time that two steady states do so.

[7] There are other papers that quantify the performance of a labor market model with strategic complementarities and multiple equilibria, but those that I know all rely on sunspot shocks to induce switches between different equilibrium paths. This idea is developed in Howitt and McAfee (1992) and Fanizza (1996), and Kaplan and Menzio (2013) perform a calibration.

based on Diamond (1982). The latter assumes that producers need to exchange their product for it to have value, and that the likelihood that they meet a trading partner is increasing in the total number of producers. As a result, the expected value of output is increasing in aggregate employment.[8] Another formulation of specialization in production relative to consumption that results in endogenous rational expectations cycles is Drazen (1988). He exploits spillovers across a monopolistic goods market and a labor market with search frictions, in which unemployment feeds back to demand for output.[9] I shall refer to these spillovers as demand externalities. A very interesting recent variant of such spillovers can be found in Kaplan and Menzio (2013).

In a reduced form, however, the three interpretations of the feedback are equivalent, since the (expected) value of the product is always an increasing function of aggregate employment. Together the interpretations capture the three examples of Cooper and John (1988) that can result in strategic complementarities: production technology, matching technology, and agents' demands. Since high employment results in a higher revenue per worker, it stimulates vacancy creation, which results in more employment, leading to new vacancies, and so on. If an increase in employment results in more vacancy creation via the demand externalities than the number of vacancies that is necessary for the original increase in employment via the matching process, employers overshoot the steady state level of vacancies. However, with more and more vacancies it takes a long time to fill an individual vacancy. Foreseeing an end to high employment and high revenue per worker, employers decrease their costly recruitment activity. At some point, fewer matches are made than jobs are destroyed, and with an increase in unemployment, revenue per worker falls. The above process in reverse completes the Beveridge cycle.

The next section presents the model. In Section 3 I analyze its steady states, and I continue with the characterization of the Beveridge cycle in Section 4. Section 5 summarizes the data, explains the parameter choices, and shows that the calibrated Beveridge cycle can account for most of the hiring driven volatility of unemployment and its autocorrelation.

[8]Diamond and Fudenberg (1989) show that such increasing returns in a matching function for the goods market can result in endogenous rational expectations cycles in reservation costs and employment. However, they have no explicit vacancies in their model.

[9]This model does contain vacancies, and is closest to Mortensen (1999) with a cycle in firm match value and unemployment. However, Drazen assumes an equal fixed number of firms and workers, which equates the number of vacancies and unemployed and thus rules out cycles in the two.

2 A model of unemployment and tightness

This section presents a Pissarides (2000) equilibrium search and matching model with variable search intensity and feedback from employment to the revenue per worker, but without aggregate uncertainty. It consists of risk-neutral representative agents of two types: employers and workers. Each type is homogenous with respect to skills and preferences. Matches of a single worker and employer produce a single consumption good, and the (expected) value of this product is increasing in aggregate employment. The next subsection presents the law of motion for unemployment.

2.1 Dynamics of unemployment

Workers can be either employed or unemployed. The labor force is normalized to one, so that the total number of employed workers, n, is equal to one minus the unemployment rate u. If a worker is unemployed, he can search for a job with a certain intensity s. The discouraged worker effect - unemployed workers stop looking for a job if the prospect of finding one is very bad - is modeled by s, because the labor force is fixed while search intensity will increase with labor market tightness. The representative employer can open vacancies. The total number of unemployed workers that finds a job in a certain period is given by the common constant returns Cobb-Douglas matching function $m(v, su) = m_0 v^{\eta}(su)^{1-\eta}$, with $0 < \eta < 1$ and $m_0 > 0$. Inputs of this function are aggregate recruiting activity represented by the vacancy rate v (scaled by the labor force), and aggregate search effort given by the unemployment rate times the search intensity of the unemployed. Define labor market tightness θ as the ratio of the inputs of the matching function: $\theta = \frac{v}{su}$. An individual unemployed worker finds work at the Poisson rate $\frac{m_0 v^{\eta}(su)^{1-\eta}}{u} = sm_0\theta^{\eta} \equiv s\lambda(\theta)$. Similarly, individual vacancies are filled at a rate $\frac{m_0 v^{\eta}(su)^{1-\eta}}{v} = \frac{\lambda(\theta)}{\theta}$. $\lambda(\theta)$ is increasing and concave. Jobs are destroyed at an exogenous rate $\delta \in (0, 1]$. These assumptions give the change in unemployment in terms of unemployment and labor market tightness

$$\dot{u} = \delta(1 - u) - su\lambda(\theta). \tag{1}$$

The next subsection describes the asset values of different labor market states for both workers and employers.

2.2 Values of different labor market states

Employers face a constant cost of opening a vacancy per unit of time, denoted by $k > 0$. Unemployed workers encounter periodical search costs $c(s)$, increasing and strictly convex in the intensity of search, with $c(0) = c'(0) = 0$. In addition, they receive a gross value of leisure $z > 0$, so that the net flow benefit of the unemployed is $z - c(s)$. The parameter z is assumed to be independent of labor market conditions and captures the combination of the unemployment benefit, the stigma of unemployment, the value of home production and the pure value of leisure that comes with unemployment. Note that the gross value of leisure does not affect the search costs, which seems a reasonable assumption for risk-neutral agents. This assumption highlights the effect that the value of leisure has on vacancy creation rather than labor supply.

All agents have utility that is linear in consumption, and discount the future with the same constant and exogenous rate r. These assumptions result in the following asset price equations for the value of holding a vacancy V for an employer

$$rV = -k + \frac{\lambda(\theta)}{\theta}(J^e - V) + \dot{V}, \tag{2}$$

and of being unemployed U for a worker

$$rU = z - c(s) + s\lambda(\theta)(W^e - U) + \dot{U}, \tag{3}$$

where J^e and W^e are the expected values of a match to an employer and worker respectively.

The stock of vacancies follows from a free entry condition. The competitive behavior of the employers ensures that for interior solutions the value of a vacancy in equilibrium is always zero. However, the value of a vacancy can be negative if employers cannot reduce vacancies because they did not open any at all, so that $V \leq 0$ with $V = 0$ if $\theta > 0$. Using the asset price equation in (2), for all $\theta > 0$ the expected value of a filled vacancy can then be expressed in terms of the costs and benefits of a vacancy, where the latter depends on labor market tightness,

$$J^e = \frac{k\theta}{\lambda(\theta)} \text{ for } \theta > 0. \tag{4}$$

At $\theta = 0$, J^e may be negative, but $\frac{k\theta}{\lambda(\theta)}$ is always bigger than or equal to zero. In the remainder of this section I focus on equilibria that have a positive level of labor market tightness.

The demand externalities are modeled by a production function of a single worker-employer match that is increasing in the *aggregate* level of employment. The expected periodical flow benefit of a match can thus be denoted by $\phi(1-u)$, with $\phi'(1-u) > 0$. The per period flows to an employer are then the expected value of output $\phi(1-u)$ minus the wage w. This wage is the per period flow income to the worker. The asset price equations of a job J and W, to an employer and a worker respectively, are then

$$rJ = \phi(1-u) - w - \delta(J - V) + \dot{J}, \tag{5}$$

and

$$rW = w - \delta(W - U) + \dot{W}.$$

Without aggregate shocks the representative worker and employer face no uncertainty, so that rational expectations and perfect foresight are equivalent. Employers are therefore correct in their expectations of the value of a filled vacancy, and thus the number of vacancies opened is actually maximizing the expected utility of employers. Using the free entry condition (4) correspondingly, and acknowledging that $V = 0$ for all interior equilibria, (5) can be rearranged to a law of motion of the value of a job in w, u and θ only

$$\dot{J} = (r + \delta)\frac{k\theta}{\lambda(\theta)} - \phi(1-u) + w. \tag{6}$$

2.3 Bargaining, wages, and search intensity

The wage is determined by Nash bargaining over the surplus of a match $p = J + W - V - U$, with worker's bargaining power equal to $\beta \in (0,1)$ and separation (U, V) as threat point. This assumption about the distribution implies that the worker's rent is equal to his share of the surplus and thus that

$$W - U = \frac{\beta}{1-\beta}(J - V), \text{ and } \dot{W} - \dot{U} = \frac{\beta}{1-\beta}(\dot{J} - \dot{V}),$$

where the latter follows because wages are continuously renegotiated.[10]

[10] Pissarides (2009) shows that the crucial assumption for job creation is that wages of *new* matches are given by this rule. How rents in ongoing jobs are split is inconsequential for job creation, and thus for the dynamics in

With rational expectations and Nash bargaining, and the condition that for interior equilibria $V = 0$ and thus also $\dot{V} = 0$, the asset price equations can be simplified to yield the wage w

$$w = \beta\phi(1-u) + \beta sk\theta + (1-\beta)\left[z - c(s)\right]. \tag{7}$$

The search intensity s in this wage equation is optimally chosen by the unemployed worker, knowing that the surplus of a match will be shared by Nash bargaining. From (3), the worker's net expected income from search activity g is given by $s\lambda(\theta)(W^e - U) - c(s)$. With the expression for J^e in (4), net expected income from search activity can be expressed in terms of labor market tightness

$$g(\theta) = \max_s \left[\frac{\beta}{1-\beta} sk\theta - c(s)\right]. \tag{8}$$

This expression can be recognized as an element of the wage. Using (8), the equation for the wage in (7) can be simplified to

$$w = g(\theta) + z + \beta\left[\phi(1-u) - g(\theta) - z\right]. \tag{9}$$

As usual, the wage is a linear combination of the net expected income from search and the gross value of leisure - together the outside option of the worker - and the expected value of output of the match.

In (8) the worker balances the benefits of search with the costs, such that

$$\frac{\beta}{1-\beta} k\theta = c'(s). \tag{10}$$

Given an increasing and strictly convex search cost function with $c'(0) = 0$, an optimal intensity exists and is unique. Moreover, since the benefits of search increase in labor market tightness, while the cost function is independent of it, the optimal intensity increases in tightness. It is zero

this model. Coles and Wright (1998) have shown that outside a steady state, Nash bargaining no longer necessarily corresponds to the outcome of strategic bargaining with appropriately defined threat points as the time between offers goes to zero, as it would in a stationary environment (Binmore et al., 1986). Consequently, the division rule in this paper should not be interpreted as the outcome of strategic bargaining, but as an axiomatic solution.

if tightness is zero, since without vacancies there is no payoff to search.

For the empirical application I choose a constant elasticity specification for the cost function, so that $c(s) = c_0 s^\gamma$ with $\gamma > 1$. This functional form results in the following closed-form solution for $s(\theta)$[11,12]

$$s(\theta) = \left(\frac{\beta}{1-\beta}\frac{k}{c_0\gamma}\theta\right)^{\frac{1}{\gamma-1}}.$$

2.4 Dynamics of tightness

In this subsection, I complete the model by deriving the law of motion for labor market tightness. The derivative of (4) with respect to time yields a differential equation for the expected value of a job which holds for all interior solutions

$$\dot{J}^e = \frac{k\dot{\theta}\lambda(\theta) - k\theta\lambda'(\theta)\dot{\theta}}{\lambda(\theta)^2} = \frac{k\dot{\theta}}{\lambda(\theta)}(1 - \frac{\theta\lambda'(\theta)}{\lambda(\theta)}). \tag{11}$$

Since the matching function is Cobb-Douglas, the elasticity $\frac{\theta\lambda'(\theta)}{\lambda(\theta)}$ is simply η. Therefore, equation (11) implies the following law of motion for labor market tightness

$$\dot{\theta} = \dot{J}^e \frac{\lambda(\theta)}{k(1-\eta)}.$$

By opening or closing vacancies, employers translate changes in expectations about the surplus of a filled vacancy in changes in labor market tightness.

Substituting the actual law of motion of J in (6) for the expected law of motion, and using the wage from (9), yields the following second differential equation in θ and u, valid for all interior solutions

$$\dot{\theta} = (r+\delta)\frac{\theta}{1-\eta} + (1-\beta)\frac{\lambda(\theta)}{k(1-\eta)}\left[g(\theta) + z - \phi(1-u)\right]. \tag{12}$$

[11]Note that s, just as u, is part of the definition of θ, but that aggregate tightness is given for the representative unemployed worker. Consequently, $s(\theta)$ can be expressed in terms of θ.

[12]This expression also allows for elimination of s in the expression of net expected income from search activity (8):

$$g(\theta) = \left(\frac{\beta}{1-\beta}k\theta\right)^{\frac{\gamma}{\gamma-1}}\left(\frac{1}{c_0\gamma}\right)^{\frac{1}{\gamma-1}}\left[1-\frac{1}{\gamma}\right].$$

Note that $g(\theta)$ is nonnegative given that $\gamma > 1$, as it should be. Secondly, $g(\theta)$ is increasing in θ, since it increases the likelihood of a match for a worker, and is zero if there are no vacancies.

Together with the law of motion for unemployment in (1), (12) describes rational expectations equilibria. High employment expectations make employers open vacancies immediately, because revenue per worker is expected to be higher in the future, but then hiring will be more costly too. Since more vacancies bring about higher employment, expectations are self-fulfilling. The next section shows that these self-fulfilling expectations may result in multiple equilibria. For an equilibrium, however, the laws of motion are restricted to a certain space. Unemployment can never be negative or exceed one, and labor market tightness can never be negative either. An equilibrium path starts at the given initial unemployment rate, and satisfies the transversality condition that tightness goes to zero if time goes to infinity, since then no surplus of a filled vacancy can be expected anymore.

In the presence of search frictions, an equilibrium is not necessarily efficient. Positive externalities of search and recruiting activity occur for the trading partners, for whom matching is *more* likely because of the availability of more (effective) trading partners. Negative externalities of search and recruiting activity occur for searchers of the same type, for whom matching is *less* likely because of increased congestion for trading partners. These externalities only cancel once the net private returns from search and recruiting activity equal the net social returns. As shown in Appendix A, this happens if the familiar Hosios (1990) condition is satisfied. That is, if and only if the bargaining power of employers $1 - \beta$ is equal to the elasticity of the matching function η, search intensity s and labor market tightness θ (by employers' adjustment of vacancies) is efficient. Note that the Hosios condition only concerns the search externalities, not the demand externalities. In fact, Mortensen (1999) shows that the multiple equilibria presented in the next section can be Pareto-ranked. The Beveridge cycle is one of these equilibria.

3 Steady state unemployment and tightness

I define a Beveridge cycle as a limit cycle in labor market tightness and unemployment, which results in enduring endogenous fluctuations in vacancies and unemployment.[13] A limit cycle is a periodic orbit such that it is the attractor of at least one other orbit as time approaches positive or negative infinity. Every limit cycle encloses a steady state. For that reason, this section presents the steady states of the dynamical system in labor market tightness θ and unemployment u, as

[13] That is, the fluctuations are not due to fluctuations in unemployment and search intensity only.

given by the differential equations in (12) and (1), and studies their stability. I focus on the plausible case with a positive gross value of leisure z, which is new relative to Mortensen (1999). The value of leisure has a qualitative impact on the theoretical results, and is quantitatively important for the empirical performance of the model. I show that if there exists a steady state with a positive employment level for $z > 0$, then there are generically multiple of them. Knowledge of the stability of these steady states helps to understand the Beveridge cycle that ultimately explains the data.

3.1 A no-trade steady state

Setting (1) equal to zero results in steady state unemployment in terms of labor market tightness, the $\dot{u} = 0$-locus or unemployment nullcline:

$$u = \frac{\delta}{\delta + s(\theta)\lambda(\theta)} = \frac{\delta}{\delta + m_0 \left(\frac{\beta}{1-\beta}\frac{k}{c_0\gamma}\right)^{\frac{1}{\gamma-1}} \theta^{\eta + \frac{1}{\gamma-1}}}, \tag{13}$$

where the last equality uses the Cobb-Douglas matching function and the optimal search intensity derived from the constant elasticity cost of search function. One can see that all workers are unemployed if tightness is zero, and that unemployment decreases in tightness via the job finding rate.

Specifying a production function, the tightness nullcline can also be expressed as unemployment in terms of tightness. Following Mortensen (1999) in choosing a constant elasticity function, production per match will be given by $\phi_0(1-u)^\alpha$, where $\alpha > 0$ represents the elasticity of the external effects. Besides, for empirical applications, α can be assumed to be smaller than 1. For all interior equilibria, the tightness nullcline derived from (12) can then be written as

$$u = 1 - \left[(r+\delta)\frac{k\theta}{\phi_0(1-\beta)\lambda(\theta)} + \frac{g(\theta)+z}{\phi_0}\right]^{\frac{1}{\alpha}}. \tag{14}$$

Again unemployment decreases in tightness: at a lower level of unemployment, the expected value of output will be higher, and therefore in equilibrium employers open more vacancies. At some level of unemployment, the nullcline crosses the $\theta = 0$-axis, and employers do not want to open any vacancies. From (14) with $\theta = 0$ we can see that this happens at

$$u_0 = 1 - (z/\phi_0)^{1/\alpha}. \tag{15}$$

This u_0 is the upper limit on the unemployment level, and thus the lower limit on the expected value of output, to have any vacancy creation at all. For smaller values of leisure, wages are lower. As a result, there is more surplus of a match, and employers open more vacancies. Due to a complementary slackness condition, the tightness nullcline consists of two sections: the combinations of u and θ as given by (14), and a linear section with $\theta = 0$ and $u_0 \leq u \leq 1$. Figure 2 shows the typical shapes of the unemployment and tightness nullclines, and indicates the location of u_0 for a positive gross value of leisure.[14]

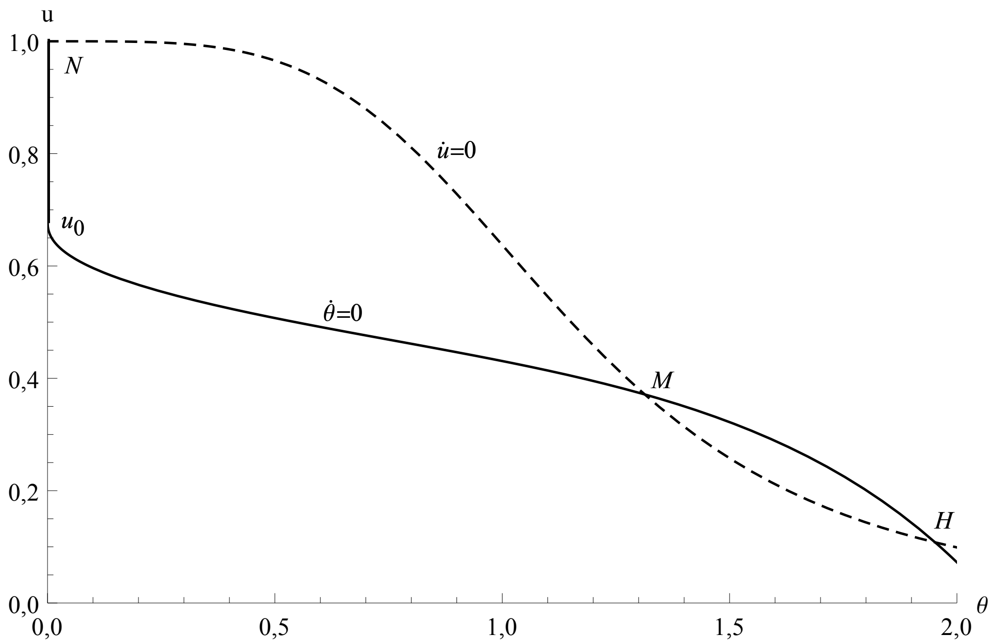


Figure 2: Nullclines of unemployment (dashed) and labor market tightness, resulting in the three steady states N, M, and H.

Notes: Except for $z > 0$, parameters are as much as possible from Mortensen (1999). They are $k = 0.3$, $\delta = 0.15$, $r = 0.01$, $\eta = 0.6$, $\gamma = 1.29$, $c_0 = 0.3$, $\phi_0 = 0.12$, $\alpha = 0.3$, $\beta = 0.29$, $m_0 = 4.3$ and $z = 0.71\phi_0$.

A steady state occurs if the two downward-sloping nullclines intersect. It is not generally

[14] Since unemployment can be expressed more easily as a function of tightness than the reverse, I plot unemployment on the vertical axis and tightness on the horizontal. Once I later plot vacancies to unemployment, unemployment will be on the horizontal axis as is common in the literature. With these conventions, counterclockwise cycles in unemployment and vacancies thus correspond to clockwise cycles in unemployment and tightness.

possible to give an explicit solution for a steady state. However, for $z = 0$ it can be seen that (14) and (13) intersect at $\theta = 0$ and $u = 1$, so that a no-trade steady state exists for $z = 0$, as in Mortensen (1999). For $z > 0$, u_0 will be smaller than 1, but the no-trade equilibrium will still exist, due to the linear section of the tightness nullcline on the $\theta = 0$-axis that intersects with (13) at $\theta = 0$ and $u = 1$. The example of Figure 2 results in three steady states: the no-trade steady state N at zero vacancies and zero employment, a steady state M with a relatively low but positive employment and labor market tightness, and a steady state H with high employment and tightness. The existence of steady states strictly in the positive quadrant (positive steady states for short) is discussed in the next subsection.

3.2 Two steady states in the positive quadrant

In Lemma 1 below I state that the unemployment nullcline can have two different shapes, depending on the elasticities of the matching function η and search cost function γ. The proof is in Appendix B. Since unemployment decreases for a higher tightness via the job finding rate, but never becomes zero unless tightness becomes infinite, the end of the nullcline is always convex. It is possible for the entire nullcline to be convex (as is the case for a fixed search intensity), but the nullcline has the shape of a negative logistic function if the cost function of search intensity is not too convex, so that search intensity varies considerably with tightness.

Lemma 1. *For $\eta + \frac{1}{\gamma-1} \leq 1$ the unemployment nullcline is convex, and for $\eta + \frac{1}{\gamma-1} > 1$ the nullcline has the shape of a negative logistic function.*

Mortensen (1999) refers to Burdett et al. (1984) to argue that $\gamma < 2$. Since $0 < \eta < 1$, the empirically relevant unemployment nullcline has the shape of a negative logistic function, and this is the case that is depicted in Figure 2. However, for $z > 0$ the qualitative results hold for both cases, and do not rely on the unemployment nullcline being either convex or negative logistic (or on search intensity being variable).

In particular, for both shapes of the unemployment nullcline and for a given $z > 0$, there are only steady states strictly in the positive quadrant if the scale parameter of the revenue per worker ϕ_0 is large enough. If not, the unemployment nullcline will lie entirely above the tightness nullcline, and the no-trade steady state is the only steady state.[15] As soon as the scale parameter

[15] If $z = 0$, at least one positive steady state always exists if $\eta + \frac{1}{\gamma-1} < 1$, independent of any other parameters.

of the revenue per worker becomes large enough, the two nullclines touch and a saddle-node bifurcation occurs. Bifurcations are of interest because in a bifurcation the qualitative properties of a dynamical system change as the result of a change in one or more parameters, ϕ_0 in this case.[16] Regions in the parameter space delimited by bifurcations are therefore called structurally stable: the qualitative dynamics are invariant to small perturbations of the parameters. In a saddle-node bifurcation two steady states emerge: a saddlepoint (with eigenvalues of opposite signs) and a node or focus (an antisaddle for short, for which the real parts of the eigenvalues are of the same sign). Depending on the shape of the nullclines, saddle-node bifurcations can happen multiple times. As a result, if any positive steady states exist, there is generically an even number of them. Focusing on the empirically relevant case in which positive steady states exist, and in which $\alpha \leq 1$ and $z > 0$, Proposition 2 states a sufficient condition for the existence of exactly two positive steady states. The proof is in Appendix C.

Proposition 2. *Suppose ϕ_0 is large enough to guarantee the existence of a steady state strictly in the positive quadrant. Then if $\alpha \leq 1$, a sufficient condition for the existence of exactly two positive steady states is* $z > \phi_0 \left(1 - \frac{\delta}{\delta+(r+\delta)(1-\eta)\eta(\gamma-1)}\right)^{\alpha}$.

The sufficient condition of Proposition 2 for the existence of exactly two positive steady states is satisfied by my final calibration as presented in Subsection 5.3, but is by no means necessary. For all parameter values that I experimented with, the interior tightness nullcline has the shape of the inverse of a negative logistic function (the shape of a negative logit function) for a positive value of leisure, as in Figure 2. Moreover, for $z > 0$ the tightness nullcline enters the positive quadrant below the unemployment nullcline. Given that the unemployment nullcline is convex or negative logistic by Lemma 1, and treating the tightness nullcline as inversely logistic or concave on the relevant segment, a saddle-node bifurcation occurs only once, resulting in exactly two positive steady states. The no-trade steady state always continues to exist. In the next subsection I study the stability of the steady states.

[16] Alternatively, for a given ϕ_0, z, α, the efficiency of matching parameter m_0 must be large enough, or vacancy costs k low enough. In Section 5 I will subsequently vary α and z to their respective saddle-node bifurcation values, given the other parameters. See e.g. Kuznetsov (2004) for more on bifurcation theory.

3.3 Stability of steady states

Equation (1) shows that unemployment is always decreasing in unemployment and tightness. Since revenue per worker is increasing in employment, (12) shows that tightness is globally increasing in unemployment. However, it is not clear how tightness globally responds to tightness itself. For that reason, I only study the stability of tightness on the tightness nullcline. Equalizing (12) to zero gives $(r+\delta)\frac{\eta}{1-\eta} = (1-\beta)\frac{\lambda(\theta)}{\theta}\frac{\eta}{k(1-\eta)}\left[\phi(1-u) - g(\theta) - z\right]$. With the Cobb-Douglas matching function, $\lambda'(\theta) = \eta\frac{\lambda(\theta)}{\theta}$. In a positive steady state, the Jacobian matrix is then

$$(16) \qquad \left.\begin{pmatrix} \frac{\partial\dot{\theta}}{\partial\theta} & \frac{\partial\dot{\theta}}{\partial u} \\ \frac{\partial\dot{u}}{\partial\theta} & \frac{\partial\dot{u}}{\partial u} \end{pmatrix}\right|_{\dot{\theta}=0} = \begin{pmatrix} r+\delta+s(\theta)\lambda(\theta)\frac{\beta}{(1-\eta)} & (1-\beta)\frac{\lambda(\theta)}{k(1-\eta)}\phi'(1-u) \\ -s'(\theta)\lambda(\theta)u - s(\theta)\lambda'(\theta)u & -\delta - s(\theta)\lambda(\theta) \end{pmatrix}.$$

Since the upper left entry is positive, tightness is increasing in itself on the $\dot{\theta}=0$-locus. We know that one of the steady states is a saddlepoint, and the other is a node or focus. Only a focus features surrounding oscillatory dynamics that can result in endogenous fluctuations. The Jacobian matrix allows me to distinguish which steady state is the saddlepoint, and which is the antisaddle. Proposition 3 therefore states a necessary condition for the Beveridge cycle.

Proposition 3. *A steady state in the positive quadrant is an antisaddle if and only if the unemployment nullcline crosses the tightness nullcline from above.*

Proof. We know from the Jacobian matrix in (16) that $\frac{\partial\dot{\theta}}{\partial\theta} > 0$, $\frac{\partial\dot{\theta}}{\partial u} > 0$, $\frac{\partial\dot{u}}{\partial\theta} < 0$, and $\frac{\partial\dot{u}}{\partial u} < 0$ in steady state. Depending on whether the product of the diagonal elements $\frac{\partial\dot{\theta}}{\partial\theta}\frac{\partial\dot{u}}{\partial u}$ or the product of the cross-diagonal elements $\frac{\partial\dot{\theta}}{\partial u}\frac{\partial\dot{u}}{\partial\theta}$ is more negative, the determinant is negative or positive respectively. If and only if the determinant of the Jacobian matrix at a steady state is negative, it has eigenvalues of different signs and thus saddle path dynamics (see e.g. Kuznetsov (2004, p. 49). If and only if the determinant is positive, the eigenvalues are of equal sign and the steady state is an antisaddle.

The slopes of the nullclines in any of the positive steady states are given by $\frac{du}{d\theta}|_{\dot{u}=0} = -\frac{\partial\dot{u}}{\partial\theta}/\frac{\partial\dot{u}}{\partial u}$ and $\frac{du}{d\theta}|_{\dot{\theta}=0} = -\frac{\partial\dot{\theta}}{\partial\theta}/\frac{\partial\dot{\theta}}{\partial u}$. Now we see that $\frac{\partial\dot{\theta}}{\partial\theta}\frac{\partial\dot{u}}{\partial u} > \frac{\partial\dot{\theta}}{\partial u}\frac{\partial\dot{u}}{\partial\theta}$ if and only if $\frac{du}{d\theta}|_{\dot{u}=0}$ is steeper than $\frac{du}{d\theta}|_{\dot{\theta}=0}$ in the steady state under study. Since both terms are negative by the sign restrictions, this corresponds to the unemployment nullcline crossing the tightness nullcline from above. □

For the unemployment nullcline to cross the tightness nullcline from above, unemployment must decrease more from an increase in labor market tightness than the decrease in unemployment (and thus the increase in the revenue per worker) that is necessary to motivate such an increase in tightness. Tightness must thus result in many matches, and employers must react strongly to profit opportunities. This interpretation of Proposition 3 helps to understand the Beveridge cycle. Imagine a recovery from a trough, in which tightness increases and unemployment decreases via the matching process. Unemployment decreases more from an increase in labor market tightness than the decrease in unemployment (and thus the increase in the revenue per worker) that is necessary to motivate such an increase in tightness. As a result, employers want to open even more vacancies. As soon as employers open more vacancies, it becomes more attractive for workers to search, which makes it more attractive for employers to open vacancies, and so on and so forth. Employers overshoot the steady state tightness for the alluringly high revenue per worker at high aggregate employment levels. However, with so many vacancies and such a low unemployment rate it takes a long time to fill an individual vacancy. Consequently, while unemployment still decreases but employers foresee an end to the boom, they do not want to spend valuable resources on vacancies that are hard to fill. Expecting higher unemployment in the future and thus smaller benefits of a filled vacancy, employers reduce labor market tightness. With tightness decreasing, workers decrease their search intensity, and the benefits of opening a vacancy for an employer decrease even more. As a result, at some point fewer matches are made than jobs are destroyed, and unemployment increases. Higher unemployment feeds back to a lower revenue per worker, so that employers overshoot the steady state level of tightness in the trough as well. However, with so few vacancies, a single vacancy is filled very fast. Consequently, while unemployment still increases but employers foresee an end to the trough, they are willing to spend some resources on vacancies that will be filled very soon and pay off at the higher employment levels of the future. As a result, search intensity recovers, and vacancies are filled even easier. Job matching takes over from job destruction again, and unemployment decreases, completing the cycle.

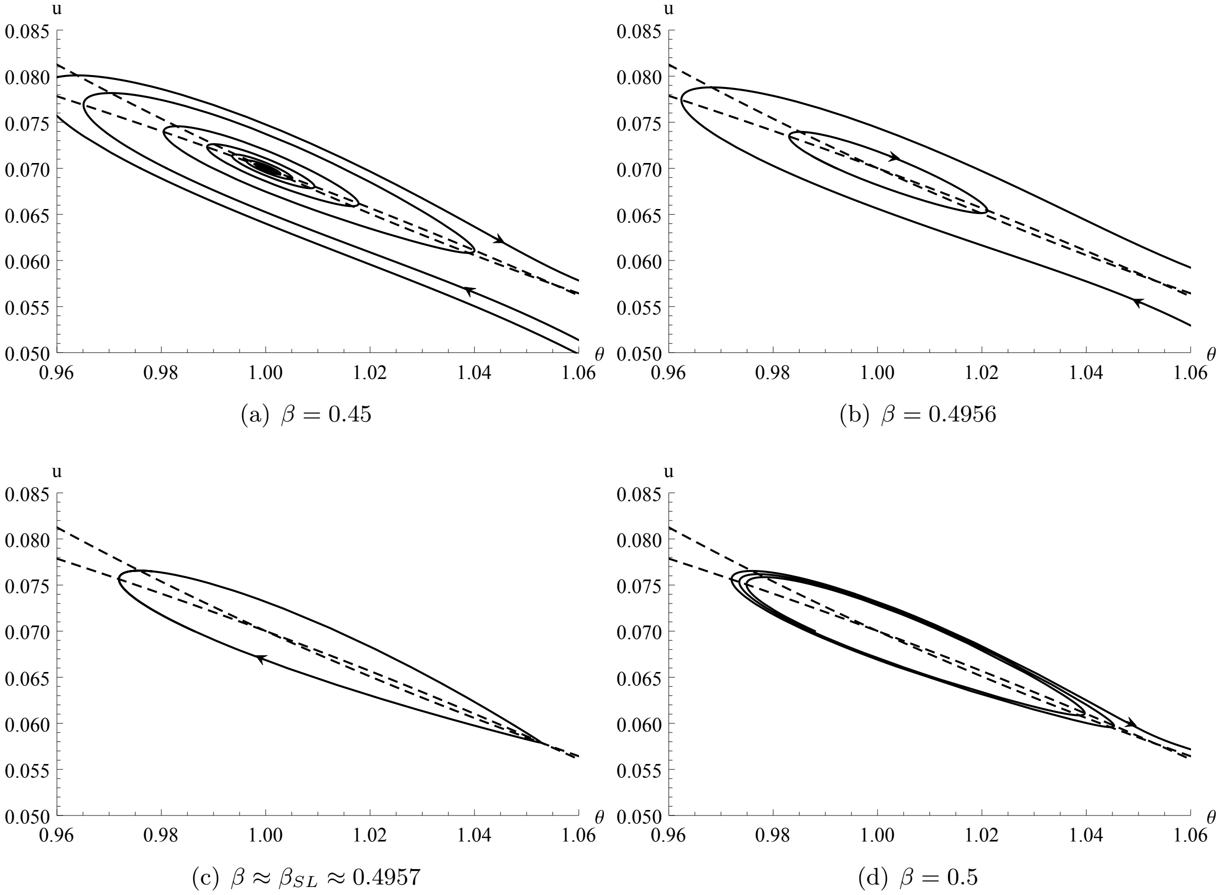
As can be seen in Figure 2, steady state M is the intersection of the unemployment nullcline crossing the tightness nullcline from above. As a result, only this steady state can feature surrounding oscillatory dynamics such as a Beveridge cycle. Steady state H is characterized by saddlepath dynamics, just as the no-trade steady state.

4 The Beveridge cycle

The previous section shows that if any positive steady states exist, there are generically two of them. The one with the higher tightness and employment (H) is a saddlepoint, while the other (M) is an antisaddle. In this section I show that the Beveridge cycle enclosing the antisaddle exists for a range of values for workers' bargaining power, and that it is stable. This conclusion follows from the occurrence of a Bogdanov-Takens bifurcation, which is a theoretical result that is new relative to Mortensen (1999). A Bogdanov-Takens bifurcation generically occurs in a system of two or more parameters, in which a Hopf bifurcation, a saddle-loop bifurcation, and a saddle-node bifurcation come together in a single point in the parameter space. This Bogdanov-Takens point is important because it is an organizing center for the global dynamics: it fully characterizes the different kinds of qualitative dynamics of a system. The next subsection presents the relevant kinds of qualitative dynamics of the search and matching model with feedback from unemployment, demarcated by a Hopf and a saddle-loop bifurcation. Afterwards, I show the occurrence of these bifurcations more formally. Finally, I show the existence of Bogdanov-Takens singularity.

4.1 Examples of oscillatory dynamics

Figure 3 plots phase diagrams for four different values of the workers' bargaining power β. These phase diagrams zoom in on the positive steady states at the parameter values of the final calibration as presented in Subsection 5.3 (except, of course, for β), and the nullclines are dashed. Starting off from a small β in Panel 3(a), the antisaddle is a stable focus, so that it attracts oscillating orbits from initial conditions for unemployment outside itself. Increasing β, at some critical value the eigenvalues of the Jacobian matrix at the focus become purely imaginary, and a Hopf bifurcation occurs. In a Hopf bifurcation, a periodic orbit emerges out of a focus, and inherits its stability. Because the focus of Panel 3(a) is stable, in this Hopf bifurcation it becomes unstable and gives rise to a stable limit cycle. Panel 3(b) presents this limit cycle. The limit cycle grows for a larger and larger workers' bargaining power until it coincides with the stable and unstable manifolds of the saddlepoint. When this happens, a saddle-loop bifurcation occurs, which is depicted in Panel 3(c). At this bifurcation the periodic orbit connects the saddlepoint with itself, and is therefore called a homoclinic orbit. At the saddle-loop bifurcation the basin of attraction outside the periodic orbit has disappeared, except for the remaining saddlepath below the saddlepoint. For larger values of β as in Panel 3(d), periodic orbits do not exist any longer. The focus remains unstable, but now the orbits originating from its neighborhood will no longer be bounded, except for the stable manifolds of the saddlepoints.



(a) $\beta = 0.45$

(b) $\beta = 0.4956$

(c) $\beta \approx \beta_{SL} \approx 0.4957$

(d) $\beta = 0.5$

Figure 3: Representative phase diagrams for $\alpha = 0.185$, varying β: (a) Stable steady state; (b) Stable limit cycle; (c) Homoclinic orbit of the saddle-loop bifurcation; (d) No closed orbits at efficient bargaining.

Notes: Nullclines are dashed, intersecting twice. k and c_0 used to normalize steady state M tightness to 1. Parameter values other than β from final calibration as presented in Subsection 5.2. $\beta_{Hopf} \approx 0.49557$.

4.2 A Hopf and saddle-loop bifurcation

The phase diagrams suggest the occurrence of a Hopf and a saddle-loop bifurcation. In this subsection I confirm the occurrence of these bifurcations. In a Hopf bifurcation the eigenvalues of the Jacobian matrix at the antisaddle cross the imaginary axis, so that the trace becomes zero.

The trace is given by the sum of the diagonal elements of the Jacobian matrix (16), and is thus

$$\frac{\partial \dot{\theta}}{\partial \theta} + \frac{\partial \dot{u}}{\partial u} = r + s(\theta)\lambda(\theta)\left(\frac{\beta}{(1-\eta)} - 1\right). \tag{17}$$

The trace indicates whether the dynamics locally converge to the antisaddle (the trace is negative and the antisaddle is a sink), diverge from it (positive trace; source), or neither (zero trace; center). Equation 17 shows that if the Hosios condition is satisfied, thus if $\beta = 1 - \eta$, the trace is just r in either steady state. However, it follows from Proposition 4 that for both steady states the trace can be zero or negative even under positive discounting.

Proposition 4. *If the job finding rate in the positive steady states exceeds the positive discount rate, there is a $\beta_{Hopf} \in (0, 1-\eta)$ for which the antisaddle undergoes a Hopf bifurcation, and there exists a limit cycle for a range of values for workers' bargaining power on one side but close to β_{Hopf}.*

Proof. In any antisaddle the real parts of the eigenvalues are of the same sign, so that if the trace has a simple zero in the antisaddle, the eigenvalues cross the imaginary axis and a Hopf bifurcation occurs. From (17) we see that the trace has a simple zero at $\beta_{Hopf} = (1-\eta)\left(1 - \frac{r}{s(\theta)\lambda(\theta)}\right) \in (0, 1-\eta)$ if $s(\theta)\lambda(\theta) > r > 0$. Therefore, for $s(\theta)\lambda(\theta) > r > 0$ in the antisaddle there exists a $0 < \beta_{Hopf} < 1 - \eta$ for which a Hopf bifurcation occurs. As a result, a limit cycle exists for values of β on one side of but close to β_{Hopf}. □

Consequently, there exists a limit cycle in unemployment and labor market tightness. Because search intensity increases with labor market tightness, a rise in tightness is not the result of a fall in search intensity but the consequence of an increase in vacancies, giving rise to enduring endogenous fluctuations in vacancies and unemployment: a Beveridge cycle.

While in the Hopf bifurcation the Beveridge cycle coincides with a steady state (focus M), in a so-called saddle-loop bifurcation it assumes its maximal size. A saddle-loop bifurcation occurs when the stable and the unstable manifolds of a saddlepoint connect to form a so-called homoclinic orbit. In Hamiltonian systems (where all orbits are level curves) homoclinic orbits are a generic phenomenon, but in systems where the trace is generically nonzero the existence of a homoclinic orbit is not robust to a small perturbation of a single parameter. In such systems the existence of a homoclinic orbit can be proven by perturbing a Hamiltonian system, and then the Andronov-Leontovich theorem (see e.g. Kuznetsov (2004, p. 200)) states that a limit cycle bifurcates on one side of the homoclinic orbit.

Mortensen (1999) shows that his model is characterized by Hamiltonian dynamics if the discount rate r is zero and the sharing rule is efficient, and that in this case a homoclinic orbit generically exists. Using Melnikov perturbation, he also shows that for a small distortion of the conditions - a positive r combined with a smaller than efficient β - the existence of a homoclinic orbit can still be proven. For that reason, in Lemma 5 I first prove equivalence between the dynamical system of Mortensen and the one presented here. Secondly, in Proposition 6 I extend his result on the existence of a homoclinic orbit to the case of a positive value of leisure.

Mortensen (1999) models a system of differential equations in the surplus of a match p and employment n, and allows the worker's bargaining power β to depend on p. To assess the empirical performance of this model and to present a global analysis of the extended standard search-and-matching model, I have modeled it in labor market terms for a fixed β. However, apart from the introduction of a positive value of leisure in my model, the two systems are smoothly equivalent for $\beta(p) = \beta$, as stated in Lemma 5. In this case, the two systems can be seen as the same system written in different coordinates, retaining the same eigenvalues of the corresponding equilibria and the same periods of the corresponding limit cycles (Kuznetsov, 2004, p. 42). It is proven by the recognition that there is a smooth one-to-one correspondence between employment and unemployment, and surplus and tightness respectively. The proof is in Appendix D.

Lemma 5. *The dynamical system in unemployment u and labor market tightness θ and Mortensen (1999)'s dynamical system in employment n and surplus p for $\beta(\theta) = \beta$ and a positive value of leisure z are smoothly equivalent for all interior equilibria.*

Since the systems are smoothly equivalent, there must be a homoclinic orbit in u and θ under the same conditions for which Mortensen (1999) finds a homoclinic orbit in n and p. Proposition 6 simply extends this result to the case of a positive value of leisure, and switches to β as bifurcation parameter. As a corollary, the Andronov-Leontovich theorem establishes the existence of a family of limit cycles. The proof is in Appendix E.

Proposition 6. *Suppose that parameters are such that two positive steady states exist for $z > 0$, $r = 0$, and $\beta = 1 - \eta$. Then there exist a $\beta_{SL} < 1 - \eta$ such that for a sufficiently small $r > 0$*

a homoclinic orbit in u and θ exists. Moreover, there exists a family of stable limit cycles for a range of values for the workers' bargaining power on one side of but close to β_{SL}.

4.3 Stable Beveridge cycles

The previous subsection shows that there is a limit cycle on one side of β_{Hopf}, and that there is a stable limit cycle on one side of β_{SL}. To show that a stable limit cycle exists for a range of values for the workers' bargaining power $\beta \in (\beta_{Hopf}, \beta_{SL})$, I present the bifurcation diagram of Figure 4. The figure shows combinations of parameters α and β for which bifurcations occur. The regions bounded by these bifurcations can be represented by qualitatively similar phase diagrams, where (a), (b), and (d) correspond to the respective panels of Figure 3 but which are reprinted for convenience. The (thin) solid line represents combinations of α and β for which a saddle-node bifurcation occurs, so that above the line (in region (0)) no positive steady states exists while below it there are two of them. The bold (solid) curve depicts the occurrence of a saddle-loop bifurcation as in Figure 3(c). Right of this curve is the familiar region (d) in which all equilibria are either steady states or saddlepaths, and in which the dots indicate efficient bargaining. The dashed line represents combinations of α and β for which a Hopf bifurcation occurs, so that left of this curve (in region (a)) there is a continuum of equilibria spiraling towards focus M. Region (b), bounded by the Hopf and the saddle-loop bifurcations, features the Beveridge cycle. Finally, there is a Bogdanov-Takens point where the the Hopf bifurcation, saddle-loop bifurcation, and saddle-node bifurcations come together.

While I have established the occurrence of these three bifurcations separately, the finding of a Bogdanov-Takens bifurcation guarantees that these bifurcations fully describe the behavior of the dynamical system. For instance, it rules out that the limit cycle born at the Hopf bifurcation is unstable and annihilates the stable limit cycle born at the saddle-loop bifurcation upon collision. To show more formally that a Bogdanov-Takens bifurcation occurs, Proposition 7 proves Bogdanov-Takens singularity. This condition for a Bogdanov-Takens bifurcation requires that both eigenvalues are zero in a steady state, and is proven in Appendix F.[17]

[17]Under certain genericity conditions that require the construction of a normal form and that are too involved to present here, this singularity is sufficient for the occurrence of a Bogdanov-Takens bifurcation, see e.g. Kuznetsov (2004, p. 322).

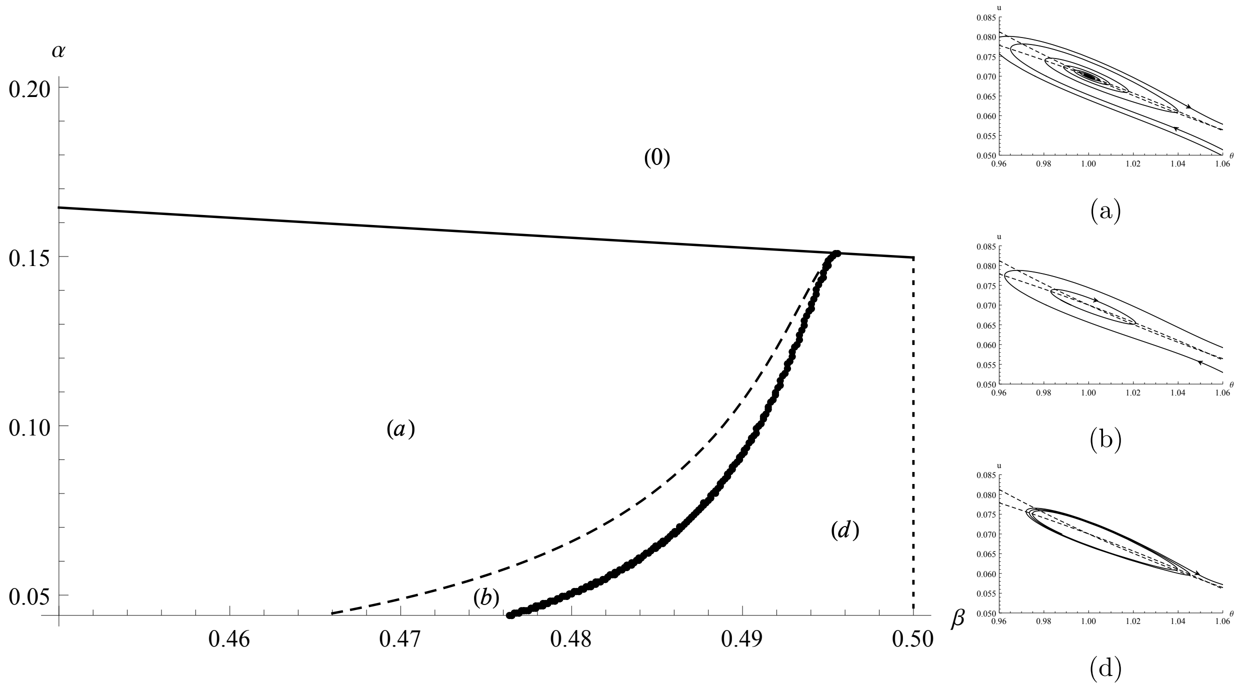


Figure 4: Bifurcation diagram of the Bogdanov-Takens bifurcation, with α and β as bifurcation parameters. *Notes:* Other parameters at the values of Table 3, so that $\alpha_{BT} \approx 0.151$ and $\beta_{BT} \approx 0.496$. The solid line corresponds to the saddle-node bifurcation, the bold line to the saddle-loop bifurcation, and the dashed line to the Hopf bifurcation. Efficient bargaining at $\beta = 0.5$ (dotted line). Regions (a), (b), and (d) refer to the panels of Figure 3, reprinted for convenience. No positive steady states in region (0). Homoclinic orbit hits u-axis for $\alpha \approx 0.044$.

Proposition 7. *Define Bogdanov-Takens singularity as a steady state with two zero eigenvalues but a nonzero Jacobian matrix. There exists a point in parameter space that satisfies this singularity, and it is unique for* $\alpha < 1$.

Consequently, for a fixed α sufficiently low to guarantee the existence of two positive steady states, but sufficiently high for the homoclinic orbit to remain in the positive quadrant (it leaves the quadrant at $\alpha \approx 0.044$), there exists a family of limit cycles enclosed by a Hopf and saddle-node bifurcation. Moreover, these limit cycles are all either attracting or repelling. Because Proposition 6 shows that the limit cycle born at the saddle-loop bifurcation is stable, the same holds for the Hopf bifurcation. This implies that $\beta_{Hopf} < \beta_{SL}$, because the trace in focus M must be positive in the presence of a stable limit cycle. As can be seen in (17), a positive trace in M only occurs for $\beta > \beta_{Hopf}$. Consequently, a stable Beveridge cycle exists for $\beta \in (\beta_{Hopf}, \beta_{SL})$.

I take the dynamics of the Beveridge cycle to be the relevant dynamics to explain the actual data, because of the evidence on the cyclical dynamics as presented in Figure 1(a), and its status as a stylized fact. Moreover, the stability of the limit cycle further supports its plausibility as a data-generating process, although its basin of attraction may be small. For these reasons, I pay only very limited attention to other equilibria in the remainder of this paper. Since for the Beveridge cycle to exist β must be smaller than efficient, I will assume this market failure in my calibration. This choice is based on the observed counterclockwise cycles rather than evidence on the bargaining power itself. In addition, Mortensen (1999) shows that saddlepoint H Pareto-dominates all other equilibria, so that my focus on the Beveridge cycle assumes yet another coordination failure.

Figure 4 shows that the set of β's that give rise to a Beveridge cycle has a positive but small measure. Although the values of β that do result in a Beveridge cycle seem not implausible as a value for the worker's bargaining power, my proposed data-generating process is not very robust to changes in β. On the other hand, the range for which orbits oscillate is much bigger, containing almost all β's smaller than β_{SL}.[18] Indeed, there exists a large set of β's smaller than β_{Hopf} for which the dynamics oscillate around the antisaddle, eventually settling down in it as in Figure 3(a). Especially if β is smaller than but close to β_{Hopf}, it may take a very long time to reach the steady state. In such a case, many business cycles could be explained with one exogenous shock in fundamentals or beliefs. Consequently, these kind of spirals could also be the data-generating process of the counterclockwise cycles in unemployment and vacancies, but I will focus on the limit cycles and therefore not exploit the additional degrees of freedom that exogenous shocks provide. The next section presents the numerical part of this paper.

5 Calibrating the Beveridge cycle

In this section I calibrate the Beveridge cycle. First I summarize the data on unemployment, vacancies and the duration of the business cycle. I subsequently explain my parameter choices, and show the calibration results. Finally, I discuss the robustness of the calibration.

5.1 Data on the business cycle

I take information on the volatility and persistence of unemployment and other statistics from Shimer (2005), who reported summary statistics of quarterly US data from 1951 to 2003. The

[18] Only for $\beta < 0.1$ the eigenvalues may stop to be complex (depending on the value for α), thus far beyond region (a) as depicted in Figure 4.

standard deviation and autocorrelation of unemployment u are presented in Table 1, together with those of other relevant variables in the model and their cross-correlations. These variables are vacancies v, the ratio of vacancies to unemployment v/u, the job finding rate f, the job separation rate δ, and labor productivity y.[19]

		u	v	v/u	f	δ	y
Standard deviation		0.190	0.202	0.382	0.118	0.075	0.020
Quarterly autocorrelation		0.936	0.940	0.941	0.908	0.733	0.878
	u	1	-0.894	-0.971	-0.949	0.709	-0.408
	v		1	0.975	0.897	-0.684	0.364
Correlation matrix	v/u			1	0.948	-0.715	0.396
	h				1	-0.574	0.396
	δ					1	-0.524
	y						1

Table 1: Summary statistics of table 1 of Shimer (2005), quarterly US data, 1951-2003.

Notes: f stands for the job finding rate, y for the revenue per worker. Standard deviations and correlation coefficients in logs as deviations from an HP trend with smoothing parameter 10^5.

My model of the Beveridge cycle has a constant job separation rate. However, since the reported standard deviation of the job separation rate in Table 1 is not zero, in reality some of the variance of unemployment is driven by job separations rather than fluctuations in hiring. Indeed, Pissarides (2009, p. 1344) argues that one-third to one-half of the volatility in unemployment is driven by fluctuations in the inflow into unemployment. As a result, a model with constant job separation rate should not explain all volatility in unemployment.

Other relevant statistics are the average unemployment, job finding, and separation rates, which are 0.0567, 1.355, and 0.102 respectively. Remarkably, the mean unemployment rate over time lies substantially lower than the steady state value u^{SS} consistent with the observed job finding and separation rates: $u^{SS} = 0.102/(0.102 + 1.355) = 0.0700$. Also in my model, the steady state and mean unemployment rates need not coincide. From the perspective of this model, the economy spends more time on the lower than on the upper part of the Beveridge cycle, or negative deviations exceed positive deviations.

The duration of the business cycle provides another natural statistic for an endogenous cycle.

[19] Statistics on vacancies are based on the help-wanted advertising index (HWI), so that an average is uninformative. On the other hand, vacancies are likely to be underreported anyway. The index offers a more reliable picture of their cyclical properties, as correspondence with the shorter JOLTS data confirms (Shimer, 2005, p. 29).

Only for an AR(1)-process the autocorrelation coefficient is the single most informative statistic for persistence. Unlike the autocorrelation coefficient, the duration of the business cycle is invariant to logarithmic transformations and HP-filtering of the (simulated) data, so that it is a robust calibration target. Figure 5 shows the duration of the business cycles falling entirely in the sample period from 1951 to 2003. The duration seems skewed to the right, and depending on whether the cycle is measured from peak to peak or from trough to trough, the median cycle last 20.5 or 19.3 quarters respectively. The average is 23.8 quarters irrespective of measurement from peak or trough. The next subsection presents the parameter choices.

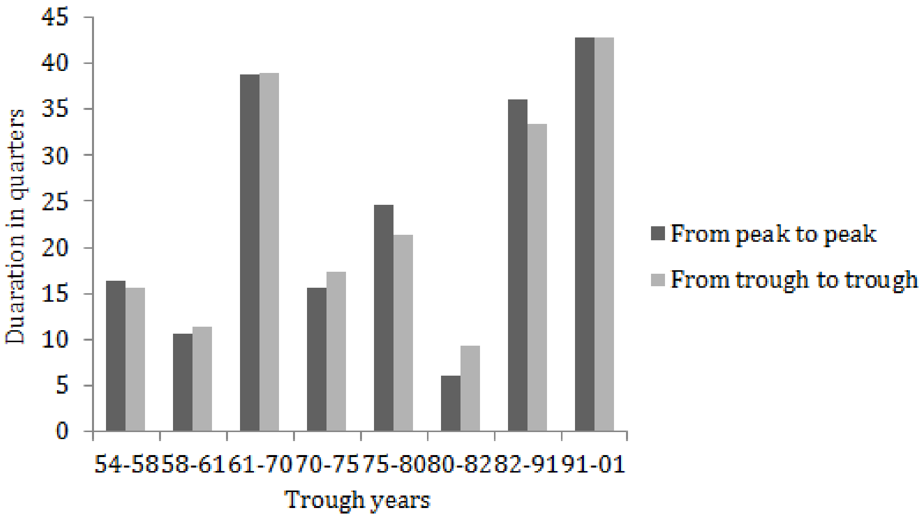


Figure 5: Duration in quarters of all completed NBER business cycles between 1951 and 2003.
Source: http://www.nber.org/cycles.html, collected at May 27, 2013.

5.2 Parameter choices

As made clear in the previous sections, the elasticity of the externalities α, workers' bargaining power β, and the gross value of leisure z are the three most important parameters for calibrating the Beveridge cycle. I choose these parameters to target a particular steady state unemployment rate, duration of the business cycle, and relative value of leisure. First, I use the remaining parameters to normalize steady state tightness, or fix them exogenously. More specifically, I follow Mortensen (1999, p. 909-910) in choosing γ equal to 1.29, which is based on empirical evidence in Burdett et al. (1984). I set the elasticity of the matching function η to 0.5. Parameter choices for the elasticity of the matching function have varied considerably across the literature, ranging from 0.28 (Shimer, 2005) to 0.6 (Blanchard and Diamond, 1989). I follow the choice of Hall and Milgrom (2008), which is consistent with the evidence on the United States in Petrongolo and Pissarides (2001). Finally, I set the constant separation rate δ equal to its empirical quarterly average of 0.102, fix the quarterly discount rate r at a common 0.012, and normalize the scale parameter for output ϕ_0 to 1.

The model allows for two more normalizations, because the level of the ratio of vacancies to unemployment $\frac{v}{u}$ and the value of search intensity s are intrinsically meaningless.[20] Consequently, recruiting and search parameters k and c_0 can be chosen to target $\frac{v}{u}$ and s respectively, and thus labor market tightness θ, in steady state M. Following Shimer (2005), I normalize the steady state ratio of vacancies to unemployment to 1, just as search intensity, so that steady state tightness $\theta \equiv \frac{v}{su}$ is 1 as well. Since the job finding rate is $s(\theta)m_0\theta^\eta$, these choices force matching efficiency m_0 to the observed quarterly rate of 1.355. The exogenously fixed parameters are summarized in Table 2.

Parameter		Value	Source
Discount rate	r	0.012	Standard
Scale revenue per worker	ϕ_0	1	Normalization
Separation rate	δ	0.102	Shimer (2005)
Scale matching function	m_0	1.355	Shimer (2005)
Elasticity matching function	η	0.5	Petrongolo and Pissarides (2001)
Elasticity search cost	γ	1.29	Burdett et al. (1984)

Table 2: Exogenously fixed parameters.

Turning to the three remaining parameters α, β and z, I use the elasticity of the externalities α and the gross value of leisure z to match the location of steady state M to the 7 percent steady state unemployment rate consistent with the observed average job finding and separation rates. This calibration target requires a high gross value of leisure or substantial demand externalities. As argued in Section 3, an increase in the gross value of leisure z shifts the tightness nullcline

[20] Doubling the cost of vacancies k and multiplying the scale parameter of the matching function m_0 by 2^η, halves θ by reducing the number of vacancies opened, but leaves the search intensity, job finding rate, and unemployment rates unchanged. This holds globally, as can be checked in the laws of motion of (1) and (12). Similarly, doubling the scale parameter of the search cost function c_0, changes search intensity s by a factor $2^{-\frac{1}{\gamma}}$ and thus θ by $2^{\frac{1}{\gamma}}$, while leaving search costs $c(s)$ unchanged. Combined with a multiplication of matching efficiency m_0 by a factor of $2^{\frac{1-\eta}{\gamma}}$, however, the job finding and unemployment rates are unaffected.

down to $u_0 = 1 - (z/\phi)^{1/\alpha}$ at $\theta = 0$. This means that the revenue per worker (which increases in the employment rate) must be higher to have any vacancy creation at all. Since such an increase leaves the unemployment nullcline unaffected, a higher z lowers the unemployment rate in steady state M as long as it continues to exist. The magnitude of this coordinating effect is mediated by the size of the externalities α.

As discussed in the introduction, the reduced form of the feedback from employment to (expected) revenue per worker is mathematically equivalent across the three cases of Cooper and John (1988): matching technology, production technology, and agents' demands. Concerning matching technology, I am not aware of any empirical studies on thick-market externalities in the goods market. Increasing returns in production have been studied more often, but are equally controversial.[21] Finally, there is not much empirical evidence of demand spillovers between the labor and goods market, except for the shopping externalities of Kaplan and Menzio (2013). Based on the idea that unemployed people spend less *and* force down markups, their calibration shows a decrease of 3.8 percent in nominal labor productivity as the result of unemployment rising from 5.3 to 9 percent. With my constant elasticity functional form - $\phi(1-u)^\alpha$ - such a small decrease in labor productivity corresponds to substantial externalities: $\alpha = 0.97$.

Concerning the flow benefit of leisure, authors explaining the volatility of unemployment vary considerably in their parameter choices.[22] In contrast to these calibrations, my model includes both a gross value of leisure z and variable search costs $c(s)$, together making up the net value of leisure. On top of that, while productivity in other models fluctuates around an exogenous average

[21] For instance, Caballero and Lyons (1989) estimate aggregate returns to scale of 1.3, so that Mortensen sets α equal to 0.3. However, the methodology and corresponding high estimates of this earlier literature have been questioned by Basu and Fernald (1995) and Burnside (1996), who are unable to reject constant returns to scale. Responding to this critique, Harrison (2001) allows for sector-specific external effects. While she rejects increasing returns in the consumption goods sector, her two-standard-error confidence intervals (across different specifications) of externalities in the investment goods sector suggest values between 0.021 and 0.172.

[22] Shimer (2005) assumes a relatively low 0.4. This value encourages Hagedorn and Manovskii (2008) to argue that Shimer's lack of amplification is due to his calibration, not to the model. Their own calibration strategy results in a relatively high value of 0.955. Hall and Milgrom (2008) and Pissarides (2009) argue that a flow benefit of 0.955 is too high. Hall and Milgrom themselves come up with a calibration that gives a value equal to 0.71, which is close to the 0.73 of Mortensen and Nagypal (2007). They argue that this value is much more realistic because of its small elastic labor supply. In the calibration of Hagedorn and Manovskii, unemployment would be very responsive to changes in the value of leisure, such as a policy change in the unemployment benefit.

of 1, productivity in my model is endogenous. For that reason, the relevant calibration target of my model is not z, but the net value of leisure relative to output

$$\zeta = \frac{z - c(s)}{\phi(1-u)} = \frac{z - c_0 \left(\frac{\beta}{1-\beta} \frac{k}{c_0 \gamma} \theta \right)^{\frac{\gamma}{\gamma-1}}}{\phi_0 (1-u)^{\alpha}}.$$

In matching the steady state unemployment rate to 7 percent, I do not allow the relative value of leisure to exceed the 0.955 of Hagedorn and Manovskii (2008). Moreover, I also present an alternative static calibration in which I target a relative value of leisure of 0.71. The size of the externalities α is subsequently used to obtain an unemployment rate of 7 percent at steady state M. Table 3 presents the trade-off between high externalities and a high relative value of leisure in explaining a 7 percent unemployment rate. It shows the calibration targets u and ζ, the required α and z, and also the vacancy and search cost parameters k and c_0 required to normalize the steady state search intensity and tightness to 1.[23] For the sake of illustrating the static trade-off between α and ζ only, I use the steady state at the Bogdanov-Takens bifurcation.[24] Indeed, at this bifurcation the externalities required for a Beveridge cycle around a 7 percent unemployment rate are the smallest.

In case one allows for $\zeta = 0.955$, this *static* calibration results in required externalities of $\alpha = 0.151$ to explain a 7 percent steady state unemployment. Given the evidence on increasing returns in production above, this magnitude of the external effect seems reasonable. If I target $\zeta = 0.71$, 7 percent unemployment is obtained for $\alpha = 0.973$. Such a value clearly falls outside the range of estimates of increasing returns, but is consistent with the calibration of Kaplan and Menzio (2013). As a last alternative, one can exploit the normalized labor force of my model, and argue that not unemployment, but non-employment should be the calibration target. CPS data show an average employment rate of 60 percent, implying a job finding rate of 0.153.[25] Table 3 shows that matching a non-employment rate of 40 percent, while keeping the value of leisure ζ at 0.71, only requires modest externalities: $\alpha = 0.127$.

[23] If other parameters (such as large externalities) make opening vacancies and searching for jobs very attractive, the normalization requires bigger search and vacancy costs.

[24] At the Bogdanov-Takens bifurcation the nullclines just touch and the two positive steady states overlap. This single steady state will not be enclosed by a limit cycle, but two steady states and a limit cycle exist for infinitesimally lower values for bifurcation parameters α and β, keeping other parameters fixed.

[25] As a result of this lower rate, also β_{BT} decreases to approximately 0.46. The other parameters remain the same.

Calibration		High leisure	High externalities	Non-employment
St.st. unemployment	u^{SS}	0.07	0.07	**0.40**
St.st. net value of leisure	ζ^{SS}	**0.955**	0.71	0.71
Elasticity externalities	α	0.151	**0.973**	0.127
Gross value of leisure	z	0.974	0.841	0.746
Vacancy cost	k	0.039	0.235	0.122
Scale search cost	c_0	0.029	0.179	0.081

Table 3: Parameters to target a given unemployment rate u and relative net value of leisure ζ at the Bogdanov-Takens bifurcation steady state.

Notes: Trade-off unaffected by scale parameter of search costs c_0 and costs of vacancy k, but chosen to equalize search intensity s and labor market tightness θ to 1 in steady state. $\beta_{BT} = 0.495572$ and other parameters from Table 2, except for last column where $m_0 = 0.153$ (the empirical job finding rate for $u = 0.4$) and $\beta_{BT} = 0.460784$.

In my subsequent *dynamic* calibration I only continue with a 7 percent unemployment rate and a 0.955 relative value of leisure as calibration targets for steady state M. For a slightly smaller α than at the Bogdanov-Takens bifurcation, two positive steady states exist, and the endogenous Beveridge cycle surrounding M can be used to explain the volatility and persistence of unemployment. The bifurcation analysis of the last section makes clear that the bargaining power of workers β is important for the existence of the Beveridge cycle. I choose the β that is closest to efficient bargaining but still gives rise to a limit cycle with a basin of attraction outside its orbit. This limit cycle approaches the homoclinic orbit, the largest closed orbit possible taking the other parameters as given. By moving β further away from efficient bargaining in the direction of the Hopf bifurcation, the limit cycle can be made arbitrarily small, but then the Beveridge cycle can explain little volatility.

Simultaneously increasing the elasticity of the externalities α and the gross value of leisure z drives the two positive steady states apart, while M can be kept at its unemployment calibration target by matching efficiency parameter m_0 and while the relative net value of leisure remains $\zeta = 0.955$. I choose α such that the duration of the Beveridge cycle, given the other parameters, corresponds to the median duration of the NBER business cycle over the sample period. For that reason I choose α to target a Beveridge cycle duration of 20 quarters. Together with the parameters in Table 2, the calibration strategy described here results in the parameters of Table 4. Note that $\alpha = 0.185$, slightly above the upper bound of estimates on increasing returns in production, but substantially smaller than $\alpha = 0.97$ that is implicit in Kaplan and Menzio (2013).

The next subsection presents the results of my calibration.

Parameter		Value	Calibration target	
Vacancy cost	k	0.0386158	St.st. vacancy ratio	$v/u = 1$
Scale search cost	c_0	0.0294244	St.st. search intensity	$s = 1$
Gross value of leisure	z	0.9716870	Relative net value of leisure	$\zeta = 0.955$
Workers' bargaining power	β	0.4957012	Closest to efficient	$\eta = 0.5$
Elasticity externalities	α	0.185	Duration business cycle	20 quarters

Table 4: Calibrated parameters, with their respective calibration targets.

5.3 Calibration results

In this subsection I present the results of a calibration of the Beveridge cycle. I draw 212 quarterly observations from the simulated Beveridge cycle, and compute time series for unemployment, vacancies, the vacancy-unemployment ratio, the job finding rate, and revenue per worker. More specifically, for each variable I sample 20 time series for 20 different starting points all around the cycle. I report the average statistics over the cycle, although differences are small. As for the original data, I take logs and deviations from an HP trend with smoothing parameter 10^5. Table 5 is the simulated counterpart to Table 1, presenting the standard deviation, autocorrelation and cross-correlations of the endogenous variables of the model.

		u	v	v/u	f	y
Standard deviation		0.095	0.037	0.119	0.106	0.001
Quarterly autocorr.		0.915	0.855	0.910	0.910	0.910
	u	1	-0.543	-0.966	-0.966	-0.999
	v		1	0.742	0.742	0.543
Correlation matrix	v/u			1	1.000	0.965
	f				1	0.965
	y					1

Table 5: Summary statistics of the calibrated Beveridge cycle over 212 quarters.

Notes: f stands for the job finding rate, y for revenue per worker. Statistics are averages from samples across the 20 quarter cycle. All variables are in logs as deviations from an HP trend with smoothing parameter 10^5.

Table 5 shows that the calibrated Beveridge cycle is able to account for half of the standard deviation of unemployment, of which the total is 0.190. Since one-third to one-half of the volatility in unemployment is driven by fluctuations in the job separation rate that is constant in this model, the Beveridge cycle can explain at least three quarters of the hiring driven volatility, and may even explain all of it. In addition, the simulated autocorrelation is 0.915, close to the empirical estimate of 0.936. The autocorrelation coefficients of all other endogenous variables are also similar to their observed counterparts.

The calibrated Beveridge cycle fails to explain sufficient volatility in vacancies and in the ratio of vacancies to unemployment. However, the standard deviation of the job finding rate is close to the observed standard deviation. Most importantly, the 0.001 standard deviation of the expected revenue per worker is smaller than the actual 0.020, so that the Shimer puzzle is not reintroduced in disguise: I do not need a high volatility in my endogenously changing revenue per worker to explain high volatility in unemployment, whereas the standard search and matching model as calibrated by Shimer (2005) would require large shocks in productivity to have sufficient volatility in unemployment. Moreover, the volatility of revenue per worker is so low that the volatility of vacancies could be increased by introducing productivity shocks.

Unemployment and the expected revenue per worker are almost perfectly correlated in my model by construction, so that the simulated correlation coefficient overstates the actual negative correlation between these two variables. In addition, my calibration understates the correlation between unemployment and vacancies. As a result, vacancies are not as closely associated with their ratio to unemployment and the job finding rate as in the data. On the other hand, this lack of correlation breaks the extremely tight counterfactual link between vacancies and revenue per worker in the standard search and matching model. The high counterfactual correlation with revenue per worker survives to a much larger extent for the ratio of vacancies to unemployment and the job finding rate, because of the choice to model the demand externalities as a function of unemployment. However, the assumed almost perfect correlation between unemployment and the expected revenue per worker can be broken by additional productivity shocks as well.

As the result of the combination of a not very negative correlation between unemployment and vacancies and a lack of volatility in vacancies, my model understates the regression coefficient from regressing v on u. The coefficient implicit in Table 5 is -0.210, whereas the empirically observed equivalent based on Table 1 is -0.950. However, the standard search and matching model overstates the coefficient, as the value implicit in Shimer (2005) is -2.78. Again, a combination of my endogenous mechanism and exogenous productivity shocks may produce the actual almost one-to-one inverse relationship between vacancies and unemployment.

To show graphically that my model explains the counterclockwise cycles in the *unemployment,vacancy rate*-plane, I present Figure 6. Panel 6(a) contains 21 simulated quarterly observations that complete a cycle, HP-filtered and connected by straight lines. Although the Beveridge cycle is not as steep as the actual data, its cycles are similar to the observed ones in Figure 1(a), and rotate counterclockwise. Compared to the dynamics of the standard search and matching model in Figure 1(b), the Beveridge cycle introduces cyclical dynamics, and increases unemployment volatility substantially. One can argue that the comparison with Shimer (2005) is unfair, as the value of leisure in my calibration is much higher. For that reason, Panel 6(b) plots a simulation of the standard search and matching model with productivity shocks only, following the procedure of Shimer (2005), but with my parameter values.[26] The panel shows that unemployment volatility increases for a higher value of leisure, but that the dynamics still feature nearly vertical dynamics and triangular responses. The corresponding summary statistics are presented in Table 6. The table shows that a high value of leisure somewhat decreases cross-correlations and does increase volatility, but the effects are small relative to the original results of Shimer (2005).

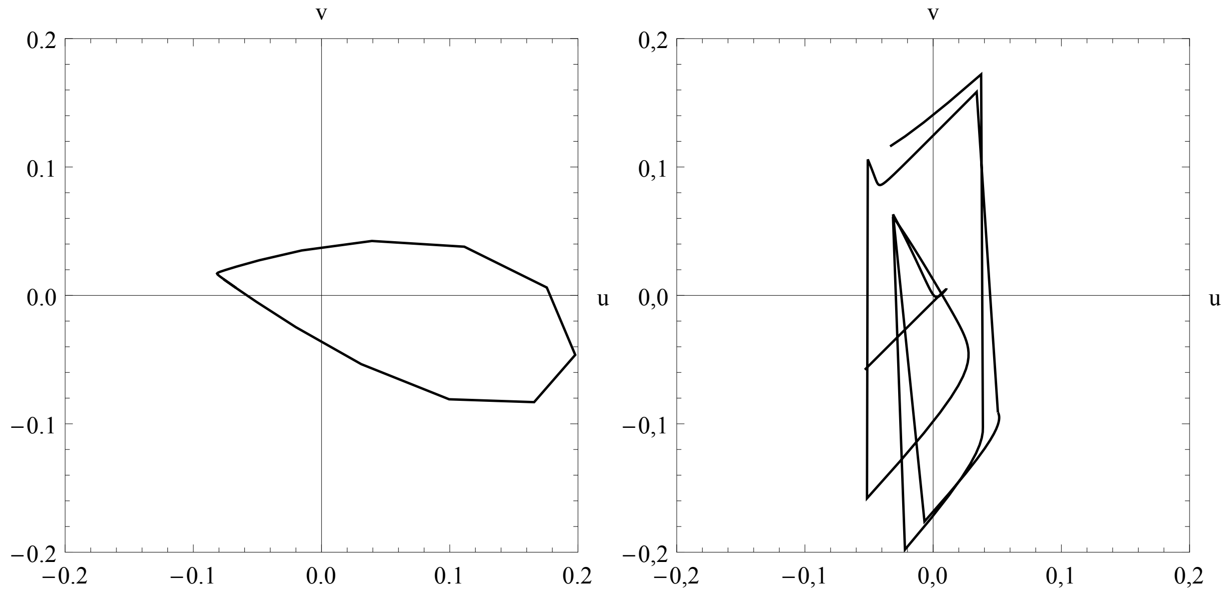
		u	v	v/u	f	y
Standard deviation		0.029	0.059	0.086	0.043	0.02
Quarterly autocorr.		0.932	0.822	0.878	0.878	0.878
	u	1	-0.818	-0.886	-0.886	-0.882
	v		1	0.984	0.984	0.972
Correlation matrix	v/u			1	1.	0.989
	f				1	0.989
	y					1

Table 6: Summary statistics of the standard model for parameters from Tables 2 and 4.

Notes: Procedure of Shimer (2005), with Ornstein-Uhlenbeck parameters $\sigma = 0.035$ and $\gamma = 0.00003$.

Finally, Figure 7 shows a simulated time series of unemployment over 212 quarters, connected by straight lines. The time series is very regular, much more so than actual data. However, exogenous shocks in fundamentals or beliefs can cause variations in amplitude and period of the cycle, without altering its driving mechanism. The next subsection discusses the robustness of the calibration.

[26] Except, of course, for the parameters of the stochastic process, which are recalibrated to match the observed volatility and autocorrelation of labor productivity.



(a) The Beveridge cycle

(b) The standard model with $z = 0.955$

Figure 6: Simulated counterclockwise cycles in unemployment and vacancies.

Notes: Simulated data are in logs as deviations from an HP trend with smoothing parameter 10^5, and are connected by straight lines. Panel (a) plots 21 quarterly datapoints from the calibrated Beveridge cycle. Panel (b) plots one simulation of 212 quarters based on Shimer (2005), but with parameters from Tables 2 and 4. Matching the standard deviation and autocorrelation of labor productivity in this case requires $\sigma = 0.035$ and $\gamma = 0.00003$ for the parameters of the Ornstein-Uhlenbeck process.

5.4 Robustness and unemployment benefits

I claim that the calibrated Beveridge is able to explain most hiring-driven volatility of unemployment, unlike the standard search and matching model as calibrated by Shimer (2005). However, Hagedorn and Manovskii (2008) argue that the low value of leisure in Shimer's calibration is the reason for his lack of amplification. Since I use Hagedorn and Manovskii's high net relative value of leisure to explain the *level* of unemployment, I stress that my results on the *volatility* of unemployment are not caused by this value of leisure. Table 6 shows already that my parameter choices do not deliver much volatility for the standard search and matching model. To show that the volatility of the Beveridge cycle is not driven by the high value of leisure either, I adopt the calibration targets of column 3 of Table 3 as well. I explain 40 percent nonemployment and a 0.71 net relative value of leisure, keeping the elasticity of the externalities equal to the previous calibration so that $\alpha = 0.185$. Adopting the workers' bargaining power approaching the homoclinic

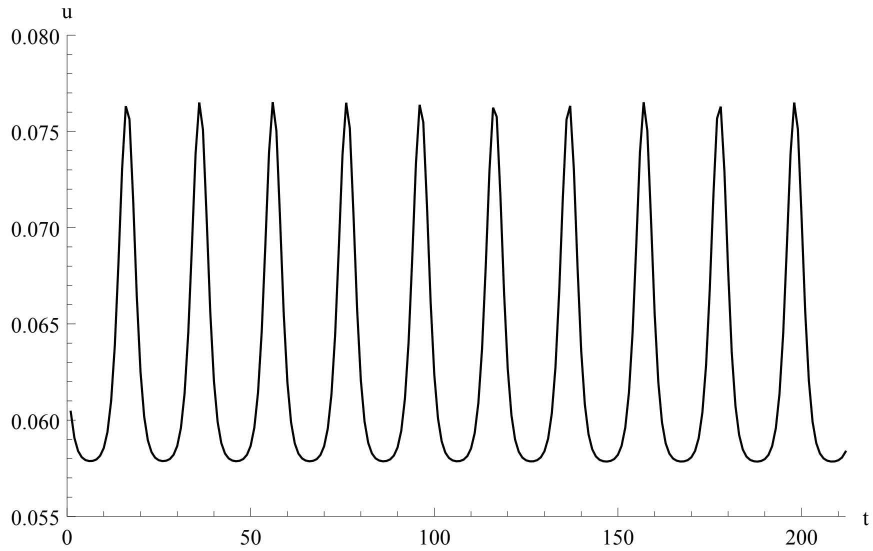


Figure 7: Simulated time series of unemployment.

Notes: 212 quarterly datapoints from the calibrated Beveridge cycle, connected by straight lines.

orbit ($\beta \approx 0.4641$) gives a standard deviation of unemployment of 0.132, more than two-thirds of the observed volatility. I conclude that the value of leisure does not drive my results on the volatility of unemployment.[27]

As another robustness exercise I calibrate the elasticity of the externalities to the average duration of the business cycle, instead of the median. For 23 quarters as a calibration target and worker's bargaining power approaching its new saddle-loop bifurcation value, α and β are 0.177 and approximately 0.49567, respectively. Not surprisingly, the 23-quarter Beveridge cycle results in a larger autocorrelation coefficient than the 20-quarter cycle, and smaller elasticities result in a smaller standard deviation. Still, Table 7 shows that the 23-quarters cycle results in qualitatively similar statistics as the 20-quarter cycle.

Finally, in Section 3 I show that a higher gross value of leisure z lowers the steady state M unemployment rate. A high gross value of leisure serves like a cue that communicates to all labor market participants that all possible positive steady states require a high expected revenue per worker and thus a low unemployment rate. Wages rise with the value of leisure, so that no positive

[27] The volatility of unemployment is mainly driven by the elasticity of the externalities. The fact that the standard deviation of unemployment *increases* for a lower net relative value of leisure, keeping the elasticity of the externalities fixed, is because in this case a lower α is required for the saddle-node bifurcation, as can be seen in Table 3. Consequently, the same $\alpha = 0.185$ is larger relative to the smallest required externalities and drives the two steady states further apart, allowing for a larger volatility.

		u	v	v/u	f	y
Standard deviation		0.074	0.026	0.091	0.081	0.001
Quarterly autocorr.		0.938	0.893	0.935	0.935	0.935
	u	1	-0.597	-0.974	-0.974	-1.
	v		1	0.763	0.763	0.596
Correlation matrix	v/u			1	1.	0.973
	f				1	0.973
	y					1

Table 7: Summary statistics of the 23-quarter Beveridge cycle.

steady state with a low revenue per worker can exist. Consequently, employers expect that revenue per worker will be high in the future, and open vacancies accordingly. Via the matching process, more vacancies result in lower unemployment, and thus the expectation of a high revenue per worker is realized. A higher unemployment benefit - a component of the gross value of leisure - can therefore shift coordination to a low unemployment steady state. Although these comparative statics might seem counterintuitive, I show that the dynamics of the Beveridge cycle move in the opposite 'intuitive' direction.

Increasing the value of leisure to $z = 0.9718$ while keeping all other parameters fixed, steady state unemployment decreases from 0.070 to 0.068. This is the comparative statics effect described above. However, I claim that the dynamical system of the Beveridge cycle is the data-generating process. Sampling from the slightly displaced Beveridge cycle, the average unemployment rate over the cycle increases from 0.063 to 0.068. Consequently, my model predicts a positive effect of the unemployment benefit on observed unemployment, as most economists would expect. I stress that this argument is not based on adjustment dynamics, but compares datapoints on two different Beveridge cycles. As can be seen in Figure 7, the Beveridge cycle for the calibrated value of leisure spends most of its time on the segment of the cycle with low unemployment rates. A Beveridge cycle for a higher value of leisure spends its time more evenly over the cycle. This effect dominates the displacement of steady state M and its enclosing Beveridge cycle. As a result, a Beveridge cycle with a high value of leisure produces a lower average unemployment rate than a cycle with a high value of leisure, even though steady state M moves in the opposite direction. Only a nonlinear model features such divergence of comparative statics and 'comparative cycles'.

6 Conclusion

Mortensen (1999) presents a parsimonious model to show that multiple Pareto-ranked cycles and steady states can coexist, and that different expectations can be self-fulfilling and result in each of these equilibria. By presenting a Bogdanov-Takens bifurcation, I show that a stable limit cycle - the Beveridge cycle - exists for a range of values for the workers' bargaining power enclosed by a Hopf and a saddle-loop bifurcation. I calibrate this Beveridge cycle to the median duration of the business cycle. For plausible parameter values, the calibrated Beveridge cycle can account for most of the hiring-driven volatility and almost all persistence of unemployment. In addition, this Beveridge cycle looks qualitatively similar to the counterclockwise cycles in the *unemployment, vacancy rate*-plane.

The calibration requires a substantial value of leisure to explain a Beveridge cycle around a sufficiently low unemployment rate. However, I stress again that a substantial value of leisure does not drive my results on the volatility of unemployment, as it does in Hagedorn and Manovskii (2008). A limitation of my study is that the range of values for the workers' bargaining power that results in a limit cycle is small. Although my calibration focuses on this purely deterministic Beveridge cycle, for a much bigger set of bargaining power parameter values a single shock can result in counterclockwise fluctuations that are able to explain many business cycles but eventually settle down into a steady state.

The model of this paper ignores other relevant sources of unemployment volatility. Most importantly, job destruction is constant. Besides, productivity shocks could increase the volatility of vacancies and decrease the (negative) correlation between vacancies and unemployment. Shocks to these and other parameters provide a natural complement to the endogenous mechanism of this paper. On top of that, the indeterminacy of equilibrium allows for belief shocks, which directly result in another level of labor market tightness by the opening or closing of vacancies by employers and the adjusted search intensity of the unemployed. However, while additional exogenous shocks can result in more irregular time series than those generated by my calibration, they also provide additional degrees of freedom.

My calibration shows that exogenous shocks are not required to explain almost all persistence of unemployment, and most of the volatility that is driven by job creation. Rather than providing another solution to the amplification and propagation problems of the standard search and matching model, I present the quantitative performance of a deterministic model in explaining the volatility and persistence of unemployment over the business cycle. Understanding endogenous unemployment fluctuations can help in designing policies that efficiently reduce the level or volatility of unemployment, and guide future empirical studies on the effect of the unemployment benefit.

Appendix

A Hosios condition

In this appendix I show that $1-\beta=\eta$ is the efficient sharing rule for a social planner that takes the demand externalities as given, just as employers and workers do, but does internalize search externalities. The proof extends the standard efficiency results of Pissarides (2000) to out-of-steady state dynamics. For the social planner to ignore the feedback from employment to revenue per worker, let ϕ denote the fixed productivity of a match. Social welfare is then given by

$$\int_0^\infty \mathrm{e}^{-rt}\left[(1-u)\phi+u(z-c(s))-k\theta su\right]. \tag{18}$$

The social planner maximizes this function by choosing both the socially efficient level of labor market tightness and search intensity, subject to the law of motion of unemployment given in (1). First-order conditions for the optimal θ and s, given the constraint on the dynamics of u, are

$$-\mathrm{e}^{-rt}\left[\phi-z+c(s)+k\theta s\right]+\mu\left[\delta+s\lambda(\theta)\right]-\dot{\mu}=0, \tag{19}$$

$$-\mathrm{e}^{-rt}ksu+\mu su\lambda'(\theta)=0, \tag{20}$$

$$-\mathrm{e}^{-rt}\left[c'(s)u+k\theta u\right]+\mu u\lambda(\theta)=0. \tag{21}$$

Both (20) and (21) can be rewritten to yield μ, so that

$$\mu=\frac{\mathrm{e}^{-rt}k}{\lambda'(\theta)}=\frac{\mathrm{e}^{-rt}k\theta}{\eta\lambda(\theta)} \tag{22}$$

$$=\frac{\mathrm{e}^{-rt}\left[c'(s)+k\theta\right]}{\lambda(\theta)}. \tag{23}$$

The efficient search intensity s is therefore given by

$$\frac{1-\eta}{\eta}k\theta = c'(s).$$

Comparing this expression with the privately chosen intensity in (10), search intensity is efficient if and only if $\beta = 1-\eta$, the Hosios condition.

The expression for μ in (22) can be used to derive

$$\dot{mu} = \frac{\mathrm{e}^{-rt}k\dot{\theta} - r\mathrm{e}^{-rt}k\theta}{\eta\lambda(\theta)} - \frac{\mathrm{e}^{-rt}k\theta\eta\lambda'(\theta)\dot{\theta}}{\eta^2(\lambda(\theta))^2} = \frac{\mathrm{e}^{-rt}k[(1-\eta)\dot{\theta} - r\theta]}{\eta\lambda(\theta)}. \tag{24}$$

Substituting (22) and (24) into (19) and rearranging, yields

$$\begin{aligned}
\frac{(1-\eta)k\dot{\theta} - rk\theta}{\eta\lambda(\theta)} &= \frac{k\theta\left[\delta + s\lambda(\theta)\right]}{\eta\lambda(\theta)} - \left[\phi - z + c(s) + k\theta s\right], \\
\Leftrightarrow \frac{(1-\eta)k\dot{\theta}}{\lambda(\theta)} &= \frac{k\theta\left[\delta + r\right]}{\lambda(\theta)} - \eta\left[\phi - z + c(s) - \frac{1-\eta}{\eta}k\theta s\right], \\
\Leftrightarrow \dot{\theta} &= \frac{\theta}{1-\eta}\left[\delta + r\right] - \frac{\eta\lambda(\theta)}{(1-\eta)k}\left[\phi - z + c(s) - \frac{1-\eta}{\eta}k\theta s\right].
\end{aligned}$$

Comparing this expression with the privately chosen tightness in (12), taking into account the definition of $g(\theta)$ in (8), labor market tightness is efficient if and only if $\beta = 1-\eta$, the Hosios condition.

B Proof of Lemma 1

Proof. Define $\chi \equiv \eta + \frac{1}{\gamma-1}$. Differentiate (13) twice with respect to θ, to obtain

$$\frac{d^2u}{d\theta^2} = \frac{\delta m_0\chi\left(\frac{\beta}{1-\beta}\frac{k}{c_0\gamma}\right)^{\frac{1}{\gamma-1}}\theta^{\chi-2}\left[\delta\left(1-\chi\right) + \left(1+\chi\right)m_0\left(\frac{\beta}{1-\beta}\frac{k}{c_0\gamma}\right)^{\frac{1}{\gamma-1}}\theta^{\chi}\right]}{\left[\delta + m_0\left(\frac{\beta}{1-\beta}\frac{k}{c_0\gamma}\right)^{\frac{1}{\gamma-1}}\theta^{\chi}\right]^3}.$$

For $\chi \leq 1$ the second derivative is positive for all $\theta > 0$, so that the unemployment nullcline is convex. For $\chi > 1$, the second derivative can be positive or negative, depending on θ. More specifically, for $\chi > 1$ there exists a unique inflection point at the positive labor market tightness given by

$$\theta^* = \left[\frac{\delta(\chi - 1)}{(1+\chi) m_0 \left(\frac{\beta}{1-\beta} \frac{k}{c_0 \gamma} \right)^{\frac{1}{\gamma - 1}}} \right]^{\frac{1}{\chi}}.$$

Consequently, for $\chi > 1$ the unemployment nullcline is concave for $0 < \theta < \theta^*$, and convex for all $\theta > \theta^*$, so that as a whole it has the shape of a negative logistic function. □

C Proof of Proposition 2

Proof. The second derivative with respect to θ of the tightness nullcline in (14) is

$$\frac{d^2 u}{d\theta^2} = -\frac{1}{\alpha} \left[\frac{(r+\delta)k\theta}{\phi_0(1-\beta)\lambda(\theta)} + \frac{g(\theta)+z}{\phi_0} \right]^{\frac{1-\alpha}{\alpha}} \left[\frac{k\beta s(\theta)}{(\gamma-1)\phi_0(1-\beta)\theta} - \frac{(r+\delta)k(1-\eta)\eta}{\phi_0(1-\beta)\lambda(\theta)\theta} \right]$$
$$- \frac{1-\alpha}{\alpha^2} \left[\frac{(r+\delta)k\theta}{\phi_0(1-\beta)\lambda(\theta)} + \frac{g(\theta)+z}{\phi_0} \right]^{\frac{1}{\alpha}-2} \left[\frac{(r+\delta)k(1-\eta)}{\phi_0(1-\beta)\lambda(\theta)} + \frac{k\beta s(\theta)}{\phi_0(1-\beta)} \right]^2.$$

One can see that for $\alpha \leq 1$, the tightness nullcline is concave for sure on the segment of the nullcline for which

$$s(\theta)\lambda(\theta) > (r+\delta)(1-\eta)\eta(\gamma-1).$$

Define ξ as the job finding rate equal to $(r+\delta)(1-\eta)\eta(\gamma-1)$. Given that the unemployment nullcline is convex or negative logistic by Lemma 1, if any positive steady state exist, generically exactly two positive steady states exist if the tightness nullcline lies below the unemployment nullcline for any potential non-concave segment of the former. In that case, the concave segment of the tightness nullcline intersects at most twice with the unemployment nullcline. A sufficient condition for any non-concave segment of the tightness nullcline to lie below the unemployment nullcline is the maximum unemployment rate giving rise to any vacancy creation (u_0 as given by (15)) to be lower than the unemployment rate consistent with the job finding rate ξ. Consequently, assuming the existence of a steady state in the positive quadrant, for $\alpha \leq 1$ generically exactly two steady states exist if

$$z > \phi_0 \left(1 - \frac{\delta}{\delta + \xi} \right)^{\alpha}.$$

□

D Proof of Lemma 5

Following the definition of Kuznetsov (2004, p. 42), two smooth systems $\dot{x} = \mu(x)$, $x \in \mathbb{R}^n$ and $\dot{y} = \nu(y)$, $y \in \mathbb{R}^n$ are not only topologically equivalent, but also smoothly equivalent if (1) an invertible map $f : \mathbb{R}^n \to \mathbb{R}^n$ exists such that $y = f(x)$, if (2) this map is smooth together with its inverse, and if (3) f can be used to change coordinates such that holds that $\mu(x) = M^{-1}(x)\nu(f(x))$, where $M(x) = \frac{df(x)}{dx}$ is the Jacobian matrix of $f(x)$ at x. As a result, f is not only a homeomorphism, but also a diffeomorphism.

Proof. My dynamical system in u and θ is given for all interior equilibria by the two smooth differential equations in (12) and (1), for convenience reprinted below

$$\begin{aligned} \dot{\theta} &= (r+\delta)\frac{\theta}{1-\eta} + (1-\beta)\frac{\lambda(\theta)}{k(1-\eta)}\left[g(\theta) + z - \phi(1-u)\right], \\ \dot{u} &= \delta(1-u) - s(\theta)u\lambda(\theta), \end{aligned}$$

with $u \in [0,1]$ and $\theta \in (0, \bar{\theta}]$ where the upper bound $\bar{\theta}$ follows from the transversality condition. The dynamical system of Mortensen (1999) extended with z is for all interior equilibria given by the following two smooth differential equations

$$\dot{p} = (r+\delta)p + g(p) + z - \phi(n), \tag{25}$$

$$\dot{n} = h(p)(1-n) - \delta n, \tag{26}$$

with $p \in (0, \bar{p}]$ and $n \in [0,1]$, and where $h(p) = s(\theta)\lambda(\theta)$, $g(p) = g(\theta)$, and $\bar{\theta}$ and $\bar{p}$ coincide, for the invertible map defined by

$$p = \frac{k\theta}{(1-\beta)\lambda(\theta)} = \frac{k}{(1-\beta)m_0}\theta^{1-\eta}, \tag{27}$$

$$n = 1-u. \tag{28}$$

Recognizing that rational Nash bargaining implies $J^e = (1-\beta)p$, the first equation follows from the free-entry condition in (4), while the second is true by definition. Both equations are smooth together with their inverses, so that they satisfy the second requirement as well. The Jacobian matrix of this diffeomorphism is given by

$$M(x) = \begin{pmatrix} \frac{k(1-\eta)}{(1-\beta)\lambda(\theta)} & 0 \\ 0 & -1 \end{pmatrix}.$$

If we apply the map in (27) and (28), then indeed

$$\begin{pmatrix} (r+\delta)\frac{\theta}{1-\eta} + (1-\beta)\frac{\lambda(\theta)}{k(1-\eta)}\left[g(\theta) + z - \phi(1-u)\right] \\ \delta(1-u) - s(\theta)u\lambda(\theta) \end{pmatrix} = \begin{pmatrix} \frac{(1-\beta)\lambda(\theta)}{k(1-\eta)} & 0 \\ 0 & -1 \end{pmatrix} \times$$
$$\begin{pmatrix} (r+\delta)\frac{k\theta}{(1-\beta)\lambda(\theta)} + g(\theta) + z - \phi(1-u), \\ s(\theta)u\lambda(\theta) - \delta(1-u) \end{pmatrix},$$

so that the two systems also satisfy the last of the three requirements. As a result, they are smoothly equivalent as long as $p > 0$, or equivalently $\theta > 0$. □

E Proof of Proposition 6

Proof. Mortensen (1999)'s Hamiltonian function in p and n (for $r = 0$ and $\beta = 1 - \eta$) extended for a positive value of leisure is

$$H(p, n) = \int_0^n \phi(x)dx + (1-n)[g(p) + z] - \delta pn, \tag{29}$$

and therefore the differential vector system allowing for a small distortion such that $r > 0$ and $\beta < 1 - \eta$ is defined by

$$\dot{x} = F(x) + \varepsilon G(x) \text{ with } x = \begin{pmatrix} p \\ n \end{pmatrix},$$

$$F(x) = \begin{bmatrix} F_1(x) \\ F_2(x) \end{bmatrix} = \begin{bmatrix} -\frac{\partial H}{\partial n} \\ \frac{\partial H}{\partial p} \end{bmatrix} = \begin{bmatrix} \delta p + \int_0^p h(q)dq - f(n) + z \\ h(p)[1-n] - \delta n \end{bmatrix},$$

$$G(x) = \begin{bmatrix} G_1(x) \\ G_2(x) \end{bmatrix} = \begin{bmatrix} rp + g(p) - \int_0^p h(q)dq \\ 0 \end{bmatrix},$$

where ε is a small positive number. $F(x)$ is the Hamiltonian vector field, and $\varepsilon G(x)$ is a (time independent) perturbation attributable to positive discounting and a smaller than efficient bargaining power. Indeed, remember that $h(p) = s(\theta)\lambda(\theta)$ and $g(p) = g(\theta)$ for the map in (27), so that can be checked that $g(p) = \frac{\beta}{1-\eta}\int_0^p h(q)dq$. Although a homoclinic orbit generically exists in Mortensen's Hamiltonian system, for $z > 0$ part of this homoclinic orbit may fall outside the positive quadrant. Suppose that two positive steady states exist under efficient bargaining and without discounting. Define p_H and n_H as the values of surplus and employment at saddlepoint H respectively, and $n_0 \equiv 1 - u_0$ as the employment level at the intersection of the tightness nullcline with the unemployment axis (thus with u_0 as defined in (15)). If and only if $H(p_H, n_H) \leq H(0, n_0)$, the homoclinic orbit is entirely situated in the positive quadrant. Because in a Hamiltonian system all equilibrium paths are level curves, combinations of u and θ on the homoclinic orbit have the same value of the Hamiltonian as the saddlepoint on it. The laws of motion in (D) show that the antisaddle M is a local minimum in the system. Moving along the continuum of surrounding closed orbits, the largest possible closed orbit in the positive quadrant lies on n_0. Consequently, if $H(p_H, n_H) \leq H(0, n_0)$, a homoclinic orbit connecting H to itself lies entirely in the positive quadrant.[28] Since I have no explicit solution for p_H, I computed the value of the Hamiltonian function numerically. For all parameters experimented with in the calibration, but maintaining $r = 0$ and $\beta = 1 - \eta$, this test verifies the existence of a homoclinic orbit in the Hamiltonian system with $z > 0$. For a small perturbation towards positive discounting and a smaller than efficient β the steady states continue to exist. The question is whether the same holds for the homoclinic orbit.

Because z does not enter the trace, the Melnikov function $M(p, n)$ does not change relative to Mortensen (1999). Adapting it for a fixed β yields

$$M(p, n) = \int_\Gamma \left[r + h(p)(\frac{\beta}{1-\eta} - 1) \right] dpdn,$$

where Γ is the area enclosed by the homoclinic orbit in the Hamiltonian system. Note that the

[28] Substituting (15) into (E) it can be checked for $z > 0$ and $\alpha > 0$ that $H(0, n_0) < H(0, 0) = z$, so that $H(p_H, n_H) \leq H(0, n_0)$ implies $H(p_H, n_H) < H(0, 0)$. The latter ensures that the saddlepoint on the homoclinic orbit is steady state H rather than the no-trade steady state, for which no homoclinic orbit in the positive quadrant exists.

Melnikov function is independent of ε. Now $\beta_{SL} < 1 - \eta$ can be chosen to target any sufficiently small $r = \hat{r} > 0$ with

$$\hat{r} = \frac{\int_\Gamma \left[h(p)\left(1 - \frac{\beta_{SL}}{1-\eta}\right)\right] dpdn}{\int_\Gamma dpdn}.$$

For $r = \hat{r}$ the Melnikov function has a simple zero at β_{SL}, so that for a sufficiently small distortion a homoclinic orbit in p and n continues to exist for $z > 0$ and remains in the positive quadrant. By Lemma 5 the same must hold for the system in θ and u.

According to the Andronov-Leontovich theorem, a limit cycle bifurcates on one side of this homoclinic orbit, and it is stable if the trace of the Jacobian matrix at saddlepoint H is negative. Because the homoclinic orbit is proven for a perturbed Hamiltonian system, the trace is only based on the distortion, and is simply equal to ε times the integrand of the Melnikov function at H:

$$\operatorname{tr}(H) = \varepsilon \left[r + h(p_H)\left(\frac{\beta}{1-\eta} - 1\right)\right].$$

Given that $\beta < 1 - \eta$, the integrand of the Melnikov function is monotonically decreasing in p. Consequently, for the Melnikov function to be zero $\operatorname{tr}(H)$ is negative for β on either side but close to β_{SL}, so that the limit cycle is stable. □

F Proof of Proposition 7

Proof. Because the proof is more concisely written in surplus than in tightness, I present it for Mortensen (1999)'s system extended with $z > 0$. Remember that by Lemma 5 the two systems are smoothly equivalent so that the eigenvalues are the same. The nullclines of the dynamical system in (D) are

$$(r + \delta)p + g(p) + z = \phi(n) \tag{30}$$

$$n = \frac{h(p)}{h(p) + \delta}, \tag{31}$$

and its nonzero Jacobian matrix is

$$J = \begin{pmatrix} r + \delta + g'(p) & -\phi'(n) \\ h'(p)(1-n) & -h(p) - \delta \end{pmatrix}.$$

Both eigenvalues are zero if and only if both the determinant and the trace are zero, so that

$$\text{tr} = r + g'(p) - h(p) = 0, \tag{32}$$

and

$$\det = \phi'(n)h'(p)(1-n) - (r+\delta+g'(p))(h(p)+\delta) = 0. \tag{33}$$

Remember that $\frac{\phi'(n)n}{\phi(n)} = \alpha$, that $g(p) = \frac{\beta}{1-\eta}\int_0^p h(q)dq$, and moreover that $h(p) = s(\theta)\lambda(\theta)$ so that using the map in (27) $\frac{h'(p)p}{h(p)} = \frac{1-\eta+\eta\gamma}{(1-\eta)(\gamma-1)} \equiv \kappa$. Substituting (32) and the elasticities into (33) yields

$$\alpha\frac{\phi(n)}{n}\kappa\frac{h(p)}{p}(1-n) = (h(p)+\delta)^2.$$

Substituting the nullclines of (31)

$$\alpha\kappa\delta\frac{(r+\delta)p + g(p) + z}{p} = (h(p)+\delta)^2.$$

Consequently, both eigenvalues are non-degenerately zero in steady state if the function

$$B(p) = (h(p)+\delta)^2 - \alpha\kappa\delta\left[r+\delta+\frac{h(p)-r}{1+\kappa}+\frac{z}{p}\right], \tag{34}$$

has a simple zero. Because $\lim_{p\to\infty} B(p) = \infty$, $\lim_{p\to 0} B(p) = -\infty$, and $B(p)$ is continuous, this condition is satisfied by the Intermediate Value Theorem. Moreover, for $\alpha < 1$ the condition is satisfied only once, because $B'(p) > 0$ if $2 > \alpha\frac{\kappa}{1+\kappa}$.[29] □

[29]The same holds if $z = 0$, but then $\lim_{p\to 0} B(p) = \delta^2 - \alpha\delta\kappa\left(\delta + \frac{\kappa r}{\kappa+1}\right)$, so that (34) can only be zero for a sufficiently large α and κ. A sufficient condition met by Mortensen's numerical example and my calibration is $\alpha\kappa > 1$.

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