Equilibrium type of competition with horizontal product innovation*

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Abstract

Singh and Vives (1984) consider a game where duopolists first commit to a strategic variable, quantity or price, and then compete in selling horizontally differentiated products. Here product substitutability is endogenized by allowing firms to undertake R&D investments to increase differentiation. This has important consequences for the determination of the equilibrium type of competition. Whereas in the original model Cournot competition always ensued in equilibrium, horizontal product innovation allows all types of market competition to be an equilibrium, depending on model parameters. As market size increases, the game of choosing the strategic variable changes structure. For small market size it is a dominance solvable game with Cournot competition as unique outcome. For higher market size, the firms face a Prisoner's Dilemma where Bertrand competition would be Pareto optimal, but Cournot competition is the non-cooperative Nash Equilibrium. As market size further increases, the game of choosing market variables becomes a Hawk-Dove game where, in pure strategy equilibrium, one firm sets quantity and the other sets price. When market size increases even further, setting prices will be the strictly dominant strategy and Bertrand competition is the unique equilibrium outcome for a relatively small parameter-range. Finally, for sufficiently high market size all equilibria corresponding to differentiated duopoly abruptly dissappear and the market separates into two monopolies.

JEL: L13, D43.

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1 Introduction

In their seminal paper, Singh and Vives (1984) show that, when the nature of duopoly competition is the result of non-cooperative choices made by the firms, Cournot competition is the only outcome we can expect to find in equilibrium[1]. Their model considers a sequential two-stage game, where firms first commit to using one of two strategic variables, price or quantity, and subsequently compete by optimizing profits over their chosen strategic variable.

In Singh and Vives (1984), when solving the game by backward induction, the firms will realize that setting quantities strictly dominates setting prices as long as the goods are (imperfect) substitutes. This paper qualifies their results by introducing an intermediary stage in the game where the firms perform product R&D to increase product differentiation. When the substitutability of the two products is

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[1]From a social planner's point of view, this is a negative result, as Bertrand competition always dominates Cournot competition in terms of welfare.

thus endogenized, firms are no longer bound to end-up in Cournot competition. Instead, depending on the size of the market, all three possible modes of competition that are available in this setup - Cournot, Bertrand and PQ[2] - can be equilibria.

The results of Singh and Vives (1984) have already been under scrutiny for some time and, on several occasions, have been shown to be sensitive to specific model assumptions. Motta (1993) shows that in a vertical differentiation model with investment in quality, Bertrand competition can lead to higher equilibrium profits than Cournot competition, as stronger competition in the market stage creates incentives for the low quality firm to innovate less and for the high quality firm to innovate more, ultimately leading to a higher and mutually more profitable quality differential. Hackner (2000) extended the horizontally differentiated model to an n-firm oligopoly and introduced exogenous quality heterogeneity among firms. With strong heterogeneity, high quality firms can earn higher profits under Bertrand competition than under Cournot competition. However, Hackner (2000) and Motta (1993) do not endogenize the choice of strategic variables (i.e. they do not analyze PQ competition) so it is impossible to tell what the equilibrium type of competition would be if firms could choose their strategic variables before competing in the market. Correa-Lopez and Naylor (2004) extend the duopoly model by introducing production costs in the form of wages which are set through bargaining between the firm and unionized labor. Since wage increases have a higher impact on Cournot than on Bertrand profits[3], when unions are strong enough, Cournot profits can become smaller than Bertrand profits.

When there is also exogenous quality asymmetry, Correa-Lopez (2007) shows that either PQ or Cournot competition can be equilibria when strategic variables are endogenously selected. Matsumura and Ogawa (2012) find that in a mixed duopoly where one firm is profit maximizing and the other is a welfare maximizing state-owned firm, setting prices is a dominant strategy. Pal (2014) analyzes the choice of strategic variables in duopolies for network goods and finds that for strong network effects firms face a Prisoner's Dilemma ending up in Pareto sub-optimal Cournot competition. However, in his model, Bertrand and PQ competition cannot be equilibria.

In contrast to most of the previous reversal results cited above[4], the results presented here do not require any exogenous asymmetry. At the same time, despite its simplicity, the model accommodates all types of competition. A corollary result is that equilibrium product differentiation is not a smooth function of market size, exhibiting abrupt jumps at the points where the equilibrium type of competition changes. Furthermore, close to the threshold where the duopoly splits into two monopolies, the set of pure-strategy subgame perfect equilibria of the game suffers multiple qualitative transformations where the number of solutions changes. The following section introduces a formal model the results of which are presented in Section 3. A final section sums up the results and concludes.

2 Model

The model follows Singh and Vives (1984) in assuming there are two firms who compete in a market for differentiated products. Demand and inverse demand result from the utility maximization problem of a representative consumer with a taste for product diversity, as captured by the parameter, δ. The consumer solves:

$$q = \begin{array}{c} \arg\max_q \alpha \sum_{i=1}^2 q_i - \frac{1}{2} \sum_{i=1}^2 \left(q_i^2 + \delta q_i q_{-i}\right) + m \\ \text{s.t. } p \cdot q + m = y \end{array}.$$

The first order conditions yield:

$$p_i\left(q_i, q_{-i}\right) = \alpha - q_i - \delta q_{-i}. \tag{1}$$

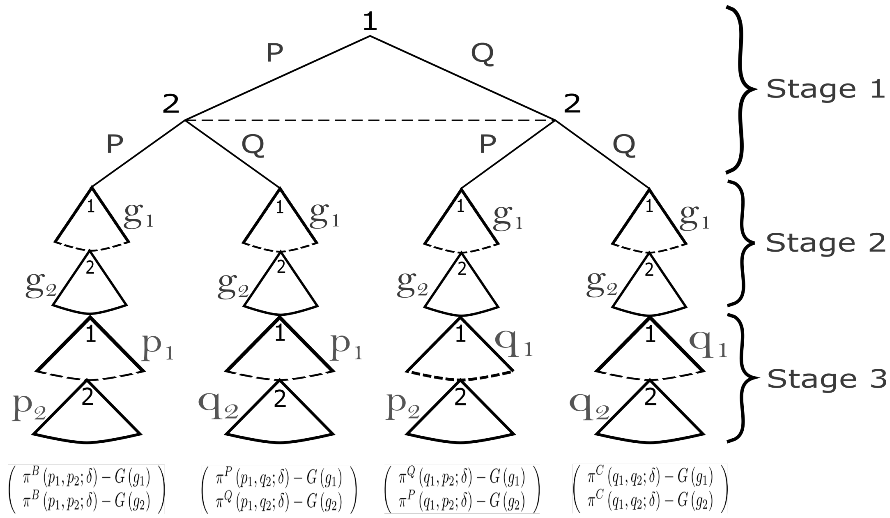
Direct demand can be computed as:

[2]PQ refers here to the asymmetric case where one firm sets price and the other sets quantity

[3]Intuitively, this follows from the fact that Cournot competitors sell relatively smaller quantities for higher prices.

[4]The exceptions are Correa-Lopez and Naylor (2004) and Pal (2014)

Figure 1: The game in extensive form



$$q_i\left(p_i, p_{-i}\right) = \frac{\alpha\left(1-\delta\right) - p_i + \delta p_{-i}}{1-\delta^2}. \tag{2}$$

For the asymmetric scenario where firm i sets quantity and the other firm sets price, we can readily derive from the above equations that:

$$\begin{aligned} p_i\left(q_i, p_{-i}\right) =& \alpha\left(1-\delta\right) - q_i\left(1-\delta^2\right) + \delta p_{-i} \\ q_{-i}\left(q_i, p_{-i}\right) =& \alpha - p_{-i} - \delta q_i. \end{aligned} \tag{3}$$

The two firms play the extensive form three-stage noncooperative game represented in Figure 1. At each stage, the firms act simultaneously but, when the stage is over, they observe the action taken by the other player.

In the first stage they simultaneously choose their strategic variable $v_i \in \{p, q\}$ which can be either price, p, or quantity, q.

During the second stage, each firm can reduce product substitutability, δ, by making costly R&D efforts. Denoting by g_i the R&D effort level of a firm during the second stage, we assume that given the efforts of both firms, product substitutability during the third and last stage of the game will be:

$$\delta = 1 - b\left(g_1 + g_2\right). \tag{4}$$

A reduction in product substitutability of $b\,g_i$ will cost firm i a monetary amount of $G\left(g_i\right) = \frac{g_i^2}{2}$. The action space at this stage is restricted to $g_i \in \left[0, \frac{1}{2b}\right]$. This means that, as in Lin and Saggi (2002), firms are allowed to contribute symmetrically to the total amount of product differentiation by setting a positive R&D effort g_i at quadratic investment cost, but cannot singlehandedly decrease substitutability below $\frac{1}{2}$. When both firms make no R&D effort, the products are perfect substitutes whereas, when efforts are at the upper bound, fully differentiated products are sold by two monopolists.

In the last stage of the game, firms compete on the market by setting their respective strategic variables (chosen in the first stage) such as to simultaneusly and competitively maximize their profits for a given the level of product substitutability, δ, that was achieved during the second stage. Formally, they will simultaneously solve:

$$\max_{v_i} \pi_i(v_i, v_{-i}) = \begin{cases} \max_{q_i} p_i(q_i, q_{-i}) q_i, & v_i = v_{-i} = q \\ \max_{p_i} p_i q_i(p_i, p_{-i}), & v_i = v_{-i} = p \\ \max_{p_i} p_i q_i(q_{-i}, p_i), & v_i = p,\ v_{-i} = q \\ \max_{q_i} p_i(q_i, p_{-i}) q_i & v_i = q,\ v_{-i} = p, \end{cases}$$

3 Results

The game is solved by backward induction and we focus on pure-strategy subgame perfect equilibria, therefore we proceed by describing optimal firm behavior from the last to the first stage.

3.1 Third stage: Market competition

In the third stage, firms take as given an observable realization of δ from the second stage and maximize their profits with respect to the strategic variables chosen in the first stage. As R&D investments are sunk costs at this stage we can disregard them.

Using (1) we obtain Cournot equilibrium quantities, prices and profits:

$$\begin{aligned} q_i^C &= p_i^C = \frac{\alpha}{2+\delta} \\ \pi_i^C &= \left(q_i^C\right)^2. \end{aligned} \tag{5}$$

Using (2) we obtain Bertrand equilibrium quantities, prices and profits:

$$\begin{aligned} q_i^B &= \frac{\alpha}{2+\delta-\delta^2} \\ p_i^B &= \frac{(1-\delta)\alpha}{2-\delta} \\ \pi_i^B &= \left(1-\delta^2\right)\left(q_i^B\right)^2. \end{aligned} \tag{6}$$

Using (3) we obtain PQ equilibrium quantities, prices and profits:

$$\begin{array}{llll} q^Q = & \frac{\alpha(2-\delta)}{4-3\delta^2} & q^P = & \frac{\left(2-\delta^2-\delta\right)\alpha}{4-3\delta^2} \\ p^Q = & \left(1-\delta^2\right)\frac{\alpha(2-\delta)}{4-3\delta^2} & p^P = & \frac{\left(2-\delta^2-\delta\right)\alpha}{4-3\delta^2} \\ \pi^Q = & \left(1-\delta^2\right)\left(q^Q\right)^2 & \pi^P = & \left(q^P\right)^2. \end{array} \tag{7}$$

Note that for any given amount of product substitutability, $\delta > 0$, the results above imply the following ranking of equilibrium prices, quantities and profits:

$$\begin{array}{ccccccc} \pi^C & > & \pi^Q & > & \pi^B & > & \pi^P \\ q^Q & > & q^B & > & q^C & > & q^P \\ p^C & > & p^P & > & p^Q & > & p^B. \end{array}$$

When products are fully differentiated, $\delta = 0$, all profits, quantities and prices are equal as the two firms behave as monopolies serving two distinct markets.

3.2 Second stage: R&D investment

In the second stage of the game, firms simultaneously solve:

$$\begin{array}{c} \max_{g_i} \pi_i^T(\delta) - G(g_i) \\ \text{s.t.} \delta = 1 - bg_i - bg_j \\ 0 \leq g_i \leq \frac{1}{2b} \end{array} \tag{8}$$

where π^T, $T \in \{C, B, P, Q\}$ stands for the profits obtained in the three distinct scenarios for the third stage determined in (5), (6), and (7).

Notice that the program in (8) depends only on two parameters, α and b. In Appendix A, I show that normalizing $b = 1$ is without loss of generality. The first order conditions[5] of the program in (8) do not always have analytical solutions[6]. By fixing parameter $b = 1$ and swiping over parameter α, commonly interpreted as market size, we can solve numerically for R&D efforts and it is possible to compute the corresponding optimal quantities, prices and profits as depicted in Figures 4 through 8.

The best-response functions corresponding to the three market scenarios can be computed numerically and are depicted in Figures 2 and 3. Inspecting the best-response functions we notice that equilibrium R&D efforts, as a function of market size, behave in qualitatively the same way. For small markets, $\alpha < \sqrt{2}$, the best response functions have a unique intersection, which I will call *main* and denote by $\left(g_i^m, g_j^m\right)$. However, at $\alpha = \sqrt{2}$, a second interesection of the best-response functions appears in the upper right corner of the economically relevant range for R&D efforts, $(g_i, g_j) \in \left[0, \frac{1}{2}\right]^2$. This happens simultaneusly for all three types of competition, Bertrand, PQ and Cournot[7] and in economic terms this second crossing represents full differentiation where the two firms are monopolies on two independent markets. These *secondary* solutions $\left(g_i^s, g_j^s\right)$ are paired to the *main* solutions in the sense that when α becomes large enough they become complex conjugates. Before the *secondary* and *main* solutions to the first order conditions merge and become complex, they gradually approach each other, the *main* one increasing and the *secondary* one decreasing for higher values of α. All the while, R&D efforts at the upper bound, $\left(\frac{1}{2}, \frac{1}{2}\right)$ remain a third intersection for the best response functions and are denoted by $\left(g_i^M, g_j^M\right)$. When the *main* and *secondary* become complex, the upper bound solutions, corresponding to two monopolies, remain the unique Nash equilibrium for the subgames starting at the second stage. The *main* and *secondary* solutions dissappear into the complex plane at $\alpha = 1.433$ for Bertrand competition, $\alpha = 1.441$ for PQ competition and at $\alpha = 1.462$ for Cournot competition.

The first order conditions are high order polinomial equations with cross terms and have multiple solutions in all three market scenarios. Under Cournot competition, the first order conditions have four symmetric solutions, two of which are always complex (for positive α). The other two solutions are the *main* and *secondary* solutions described above. Interestingly the *secondary* solution does not satisfy second order condition for profit maximization for most values of α, the second order derivatives becoming negative right before the solution enters $\left[0, \frac{1}{2}\right]^2$.[8] Under Bertrand competition, the first order conditions have six symmetric solutions. As side from our *main* and *secondary* solutions, there are two solutions which are always complex. Two more are real and, although they satisfy second order conditions for joint profit maximization, they lead to negative profits and investment far outside the boundaries we imposed on R&D efforts $g_i \in \left[0, \frac{1}{2}\right]$. Under PQ competition, there are five solutions to the first order conditions. Again we have two solutions that are always complex. There is also one solution that is always real, but has no economic meaning as it involves the price-setter minimizing profits by making negative investments in product differentiation; even-though the quantity setter is maximizing own profits, making a positive investment, both firms obtain negative profit.

By taking the profit ranking that obtains in the absence of product innovation, $\pi^C > \frac{\pi^Q + \pi^P}{2} > \pi^B$, as a proxy for the intensity of competition[9], we can make some observations regarding the relation between the intensity of market competition and innovation efforts as depicted in Figure 4 and 5, connecting our results with the two Schumpeterian, Mark I and Mark II, conjectures regarding the relation between competition and innovation[10]. Comparing investments between either Cournot and

[5]See Appendix B.

[6]Analytical solutions, although not necessarily tractable, can be obtained for Bertrand and Cournot competition. For PQ competition no closed form solutions exist.

[7]By plugging $g_i = \frac{1}{2}$ into the first order derivatives of the profit function, net of innovation costs, for any of the three market scenarios, Cournot, Bertrand or PQ we obtain $\frac{\alpha^2}{4} - \frac{1}{2}$. Which means that when first order conditions on R&D effort are satisfied with equality at the boundary, we have $\alpha = \sqrt{2}$, see also Appendix B.

[8]This is also the case under the other two types of competition, Bertrand and PQ.

[9]This interpretation follows Bonanno and Haworth (1998).

[10]Schumpeter's first conjecture, Schumpeter (1934), referred to in the literature as Mark I, argues that higher intensity

Figure 2: Best response functions in the R&D stage of the game for Bertrand, PQ and Cournot competition for R&D efforts $(g_i, g_j) \in \left[0, \frac{1}{2}\right]^2$. Each row corresponds to different values for α : (a) $\alpha = 0.3$; (b) $\alpha = 0.7$; (c) $\alpha = 1.2$; (d) $\alpha = \sqrt{2}$. At $\alpha = \sqrt{2}$, for all three market types, the secondary solutions, $\left(g_i^s, g_j^s\right)$, cross into the economically relevant plane segment, $\left[0, \frac{1}{2}\right]^2$ and coincide with the upper boundary solution that corresponds to fully differentiated products sold by two monopolists, $\left(g_i^M, g_j^M\right)$.

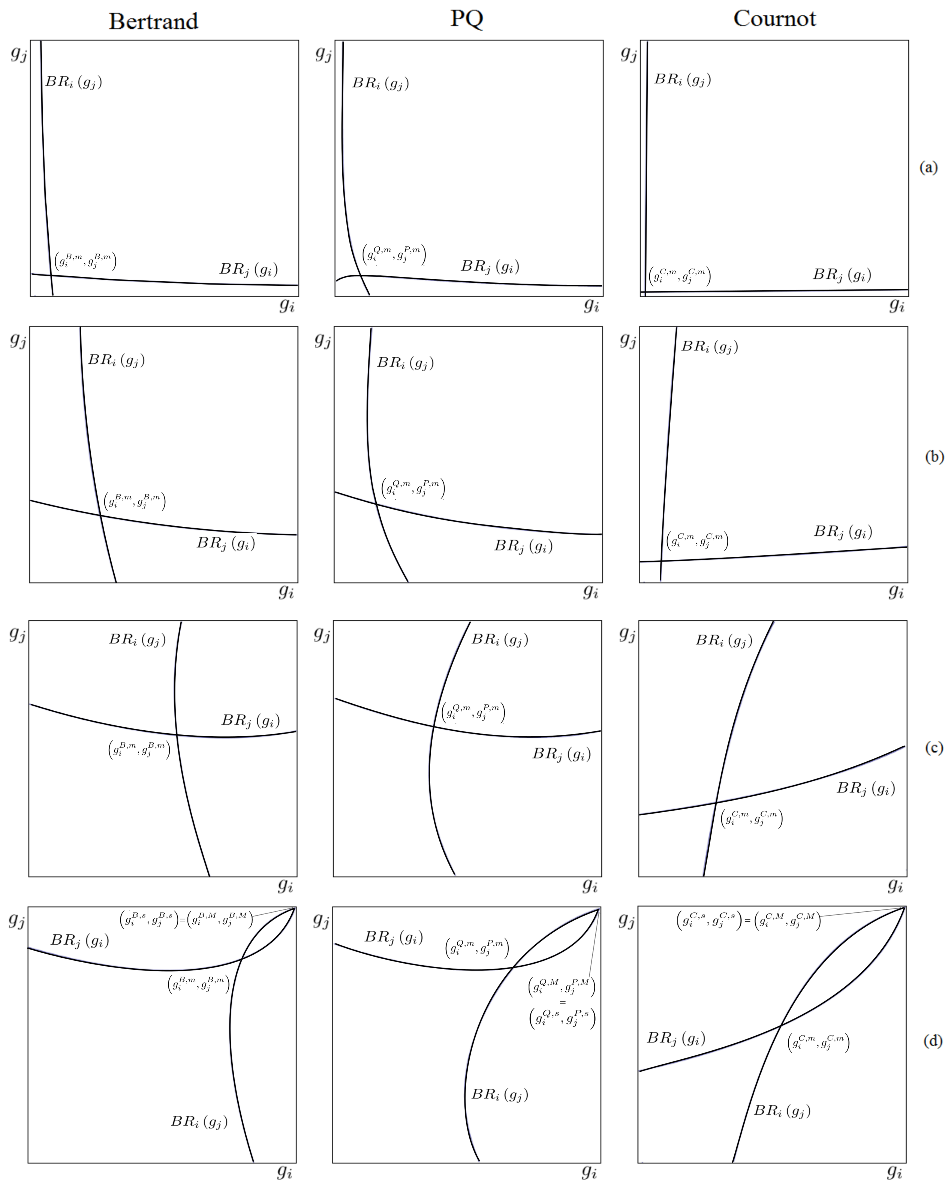


Figure 3: Best response functions in the R&D stage of the game for Bertrand, PQ and Cournot competition for R&D efforts $(g_i, g_j) \in \left[0, \frac{1}{2}\right]^2$. Each row corresponds to different values for α : (e) $\alpha = 1.433$; (f) $\alpha = 1.441$; (g) $\alpha = 1.462$. At $\alpha = 1.433$ the main and secondary solutions for Bertrand competition coincide, while for higher α the two solutions become complex and the best response functions only cross at the upper bound for R&D efforts. At $\alpha = 1.441$ the main and secondary solutions for PQ competition coincide, while for higher α the two solutions become complex and the best response functions only cross at the upper bound for R&D efforts. At $\alpha = 1.462$ the main and secondary solutions for Cournot competition coincide, while for higher α the two solutions become complex and the best response functions only cross at the upper bound for R&D efforts.

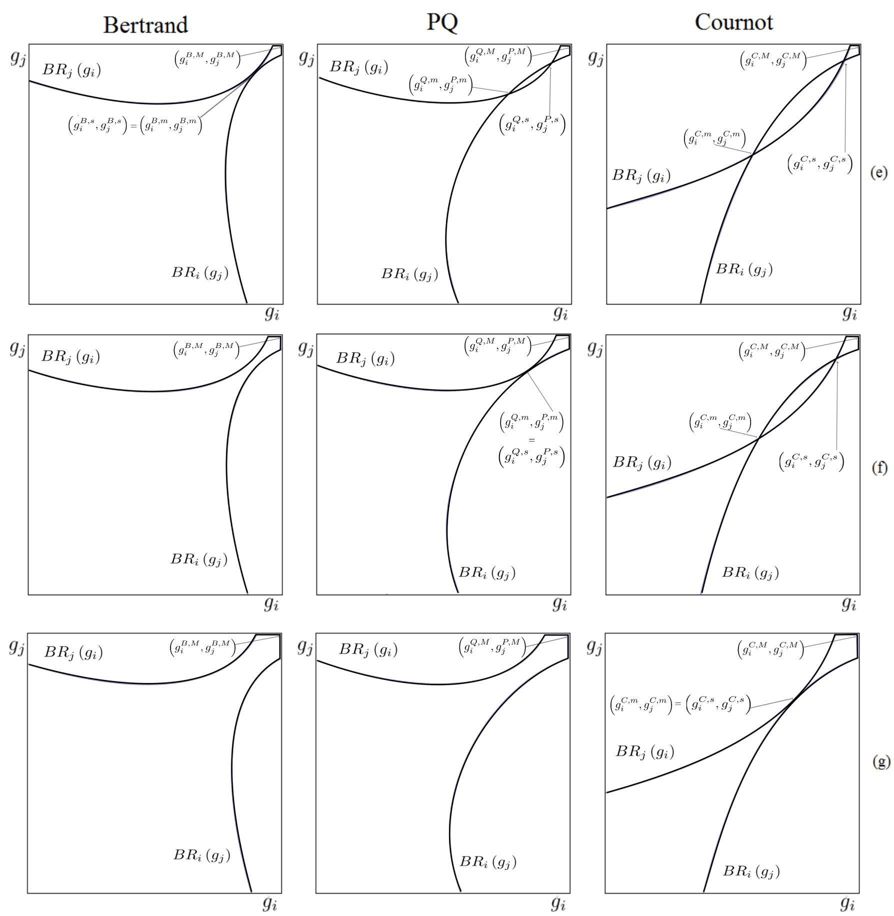
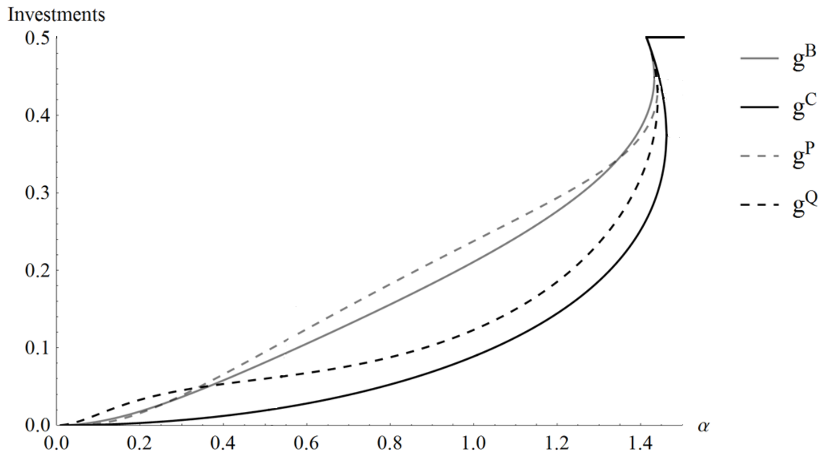


Figure 4: R&D effort, $b = 1$, $\alpha \in [0, 1.5]$



Bertrand competitors or Cournot and PQ competition we find support for Schumpeter's Mark 1 conjecture; more intense competition leads to more innovation. Comparing Bertrand and PQ (joint) levels of investment we find support for both conjectures with Mark II applying for small α and Mark I for larger α. Remarkably, this goes against the common interpretation that the first conjecture applies to small markets and the second conjecture to large markets.

Turning to Figures 6 and 7, we see that the relation between both quantities and prices of the two PQ competitors are robust to the introduction of horizontal product differentiation. So is the relation between Cournot at Bertrand quantities, but not for prices; as market size increases, Bertrand firms eventually are able to reach such high levels of product differentiation that they can optimally set higher prices than Cournot competitors, see also Figure 5. When comparing Cournot (Bertrand) market outcomes with those resulting from PQ competition we see that the only relation that is robust to the introduction of the innovation stage is that between q^C and q^Q (q^B and q^P). All other market variables start off in the same relation as in the model without innovation for small markets, but, as α increases, the relation is reversed.

Finally, turning to profits, represented in Figure 8, we see that there are - not surprisingly in view of our previous remarks regarding prices and quantities - many intersections. Based on the relation between the size of profits for different values of α we can establish the equilibrium strategies for the first stage of the game. This is treated in detail in the following subsection.

3.3 First stage: market variable selection

Going through Figure 8 from left to right (see also Figure 9 for a zoom-in), one can see that for sufficiently small markets, $\alpha < \alpha_{QC}$, the relation between profits, $\pi^C > \pi^Q > \pi^B > \pi^P$, established by Singh and Vives (1984) is robust to the introduction of product innovation. At α_{QC} the profit of the quantity setter in PQ competition becomes higher than the Cournot profit. This has no impact in determining the unique subgame-perfect equilibrium of the game, nor does the crossing at α_{BC} where Bertrand profits become higher than Cournot profits, turning the first stage game into a Prisoner's dilemma with price-setting playing the role of cooperation and quantity-setting standing in for defection. At α_{PC} the profit obtained by the price setter in PQ competition becomes higher than Cournot profit, turning the problem of choosing market variables into a Hawk-Dove game with quantity-setting playing the role of the more aggressive Hawk strategy. Finally, at α_{BQ}, Bertrand profit becomes larger than the profit obtained by a PQ quantity-setter. The game is again again dominance solvable, but

of market competition between firms spurs innovation. His second conjecture, Schumpeter (1942), dubbed Mark II, was that less acute competition gives firms the slack they need in order to divert resources to innovation.

Figure 5: Product Substitutability, $b = 1$, $\alpha = [0, 1.5]$

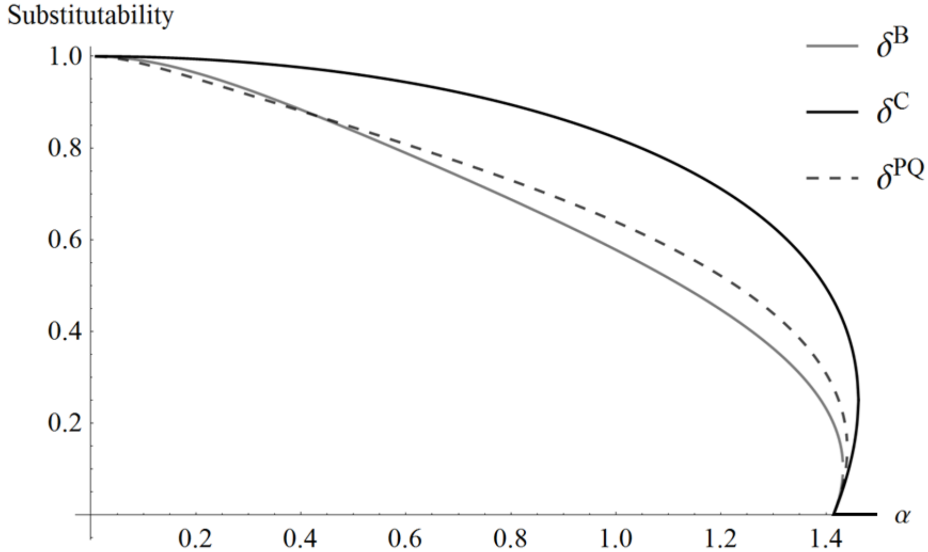


Figure 6: Quantities, $b = 1$, $\alpha \in [0, 1.5]$

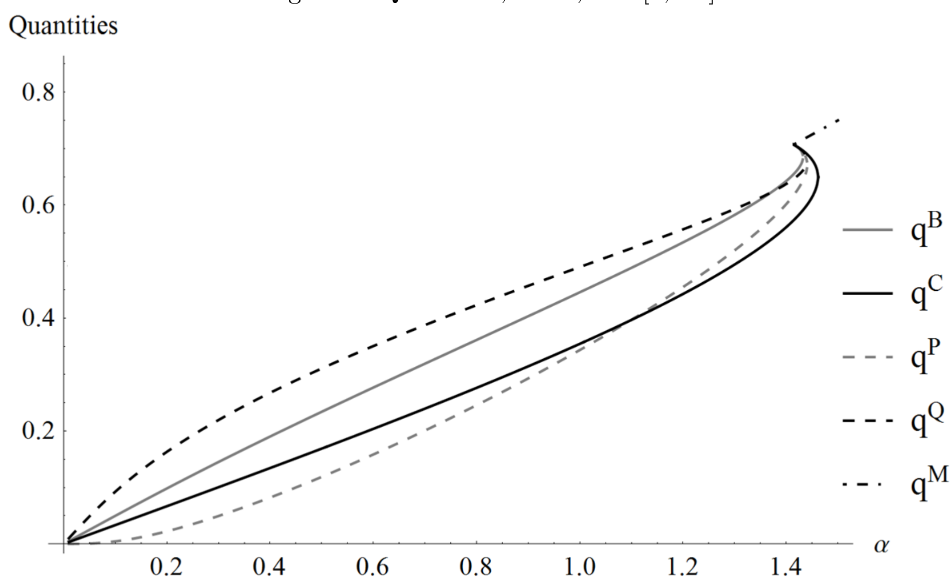
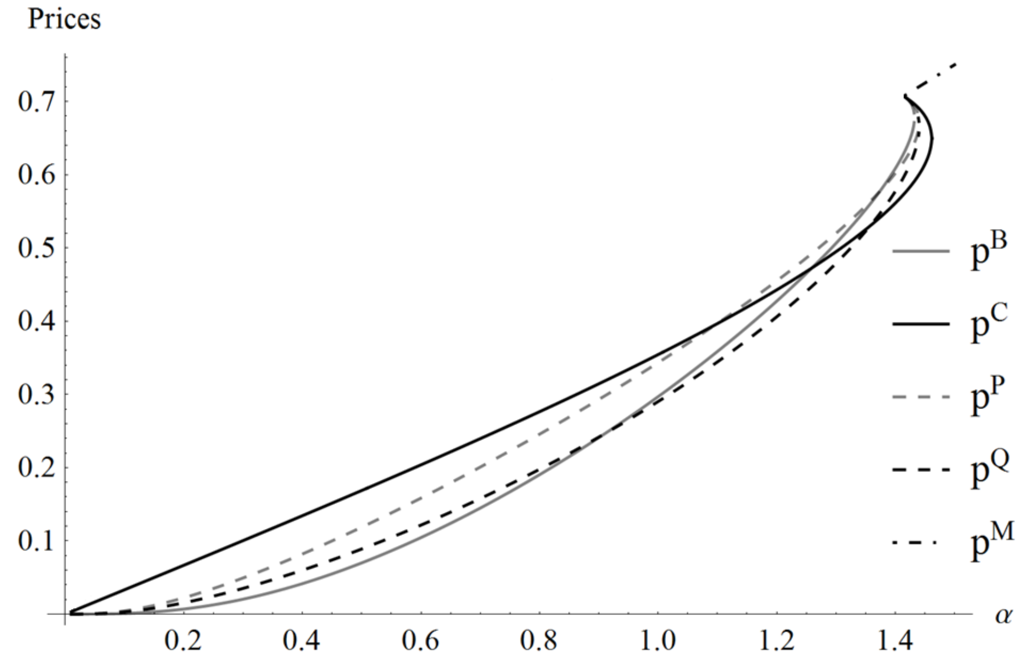


Figure 7: Prices, $b = 1$, $\alpha \in [0, 1.5]$



this time it will play out as Bertrand competition. While the reversals occurring at α_{QC} and α_{BC} may be of less interest[11], For $\alpha \in [\alpha_{PC}, \alpha_{BQ}]$ the game has more than one subgame perfect equilibrium, of which two asymmetric equilibria in pure strategies result in PQ competition[12]. For $\alpha > \alpha_{BQ}$ Bertrand becomes the unique equilibrium type of competition until market size reaches the critical value of α^s. The patterns used in the background of Figure 8 are meant to give the reader a direct graphical appreciation of the size of the parameter space where all the reversals take place. All in all, the parameter space where non-Cournot equilibrium types of duopoly can be observed is 5.09% of the total parameters space that supports duopoly competition.

When market size becomes higher than α^s, there are again multiple equilibria, but this time they spawn from the player's actions during the last two stages of the game, see Figure 9. This happens as the *boundary* solution, $g_i = \frac{1}{2b}$, where the duopoly separates into two monopolies, becomes an equilibrium R&D effort given any type of competition. Remarkably, this takes place at once for all four subgames that start with an R&D stage, under Bertrand, Cournot and PQ competition[13]. Perhaps another remarkable coincidence is that the initial profit ranking found by Singh and Vives (1984) applies to the profits from the *secondary* solutions as long as they exist together as interior solutions, that is, for $\alpha \in [\alpha^s, \alpha^c_B]$. In Figure 9 we can also notice that all *secondary* profits are larger than all the *main* profits[14] but smaller than monopoly profits. These considerations allow us to fully characterize the model's solutions for the remaining region of the parameter space, $\alpha \geq \alpha^s$.

For $\alpha \in [\alpha^s, \alpha^c_B]$ all types of competition can be subgame perfect outcomes. More precisely we have the following types of subgame perfect equilibria, categorized by their outcome:

- Monopoly: There are 4×3^3 subgame perfect equilibria which consist of at least one of the market types (Cournot, Bertrand or the any of the two PQ subgames) deploying maximum R&D efforts

[11] The equilibrium type of competition remains Cournot for all $\alpha < \alpha_{PC}$.

[12] The symmetric equilibrium in mixed strategies will realize into all three types of competition

[13] This happens because all types of competition become monopolies at the maximum bound effort and profits are identical in price-setting and quantity-setting monopolies.

[14] The exception is π^P_s which is surpassed by π^Q_m right before the solutions from PQ competition turn complex.

Figure 8: Profits, $b = 0.05$, $\alpha = [0, 1.5]$

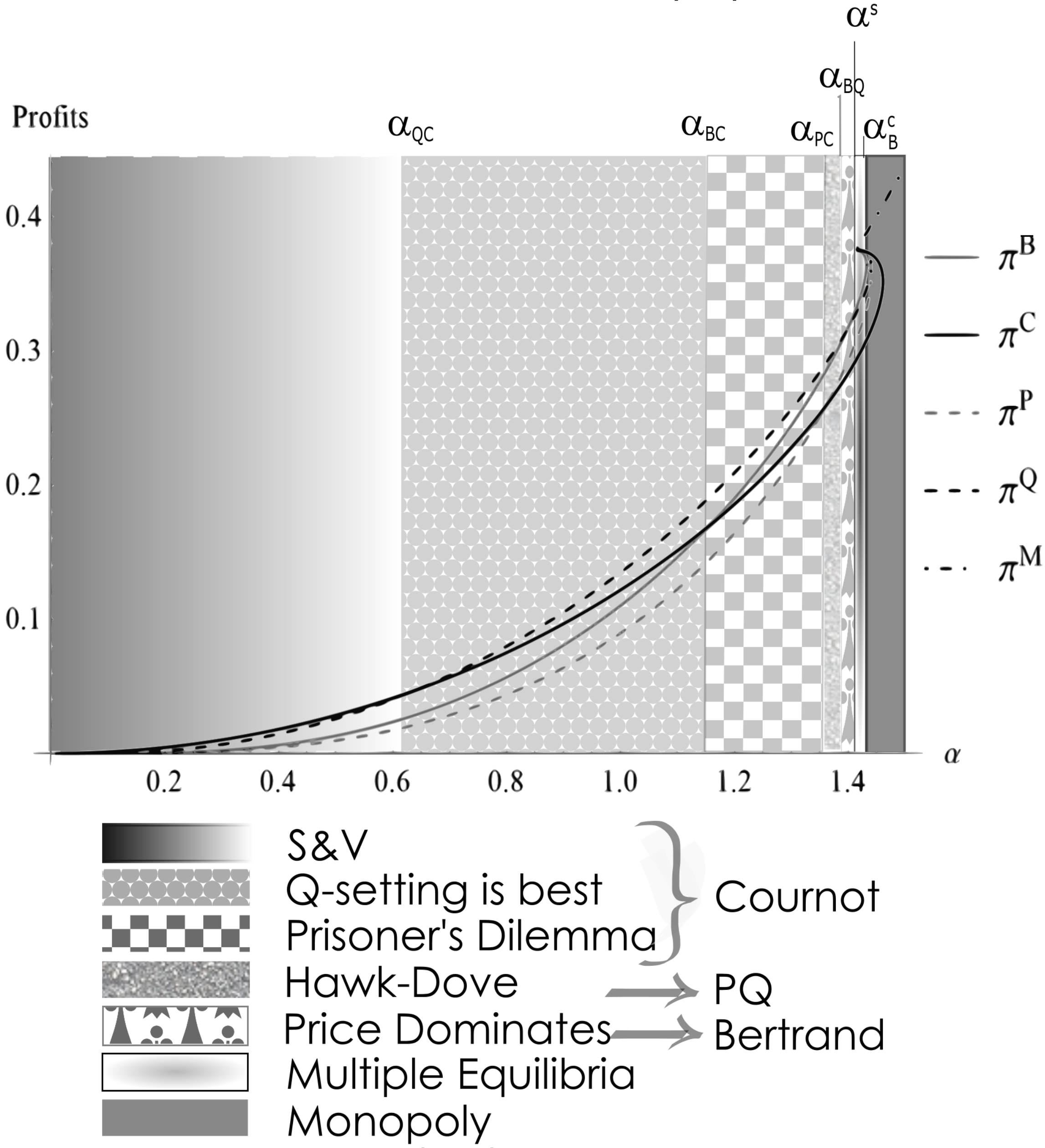
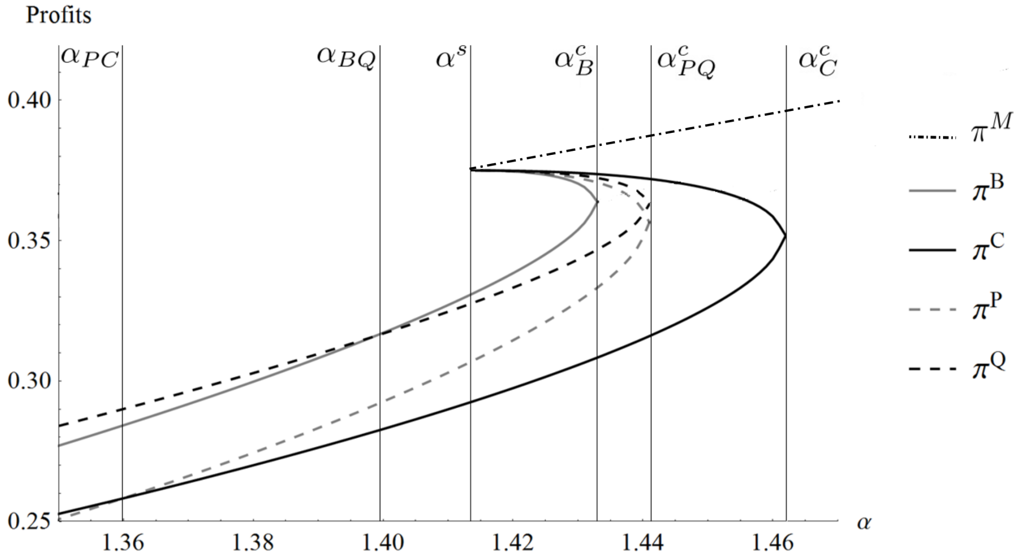


Figure 9: Profits in the multiple equilibria region (*secondary* solution - upper branch, *main* solution - lower branch), $b = 1$, $\alpha = [1.35, 1.47]$



while off the equilibrium path, any of the pairs of efforts below is undertaken.

$$[(g_i, g_j), (g_i, g_j), (g_i, g_j)] \in \left\{ \left[\left(g_i^{C,m}, g_j^{C,m}\right), \left(g_i^{Q,m}, g_j^{P,m}\right), \left(g_i^{P,m}, g_j^{Q,m}\right), \left(g_i^{B,m}, g_j^{B,m}\right) \right] \times \right.$$
$$\times \left[\left(g_i^{C,s}, g_j^{C,s}\right), \left(g_i^{Q,s}, g_j^{P,s}\right), \left(g_i^{P,s}, g_j^{Q,s}\right), \left(g_i^{B,s}, g_j^{B,s}\right) \right] \times$$
$$\left. \times \left[\left(g_i^{C,M}, g_j^{C,M}\right), \left(g_i^{Q,M}, g_j^{P,M}\right), \left(g_i^{P,M}, g_j^{Q,M}\right), \left(g_i^{B,M}, g_j^{B,M}\right) \right] \right\}$$

- Bertrand Competition (with *main* efforts). This specific equilibrium is the one that begins its existence at $\alpha > \alpha_{PC}$. When, on the equilibrium path, firms both choose to compete in prices and then invest $\left(g_i^{B,m}, g_j^{B,m}\right)$, there is just one pattern of subgame-perfect off-equilibrium path play that is consistent with it: in all other subgames the firms will have to choose the effort levels given by the *main* solution.

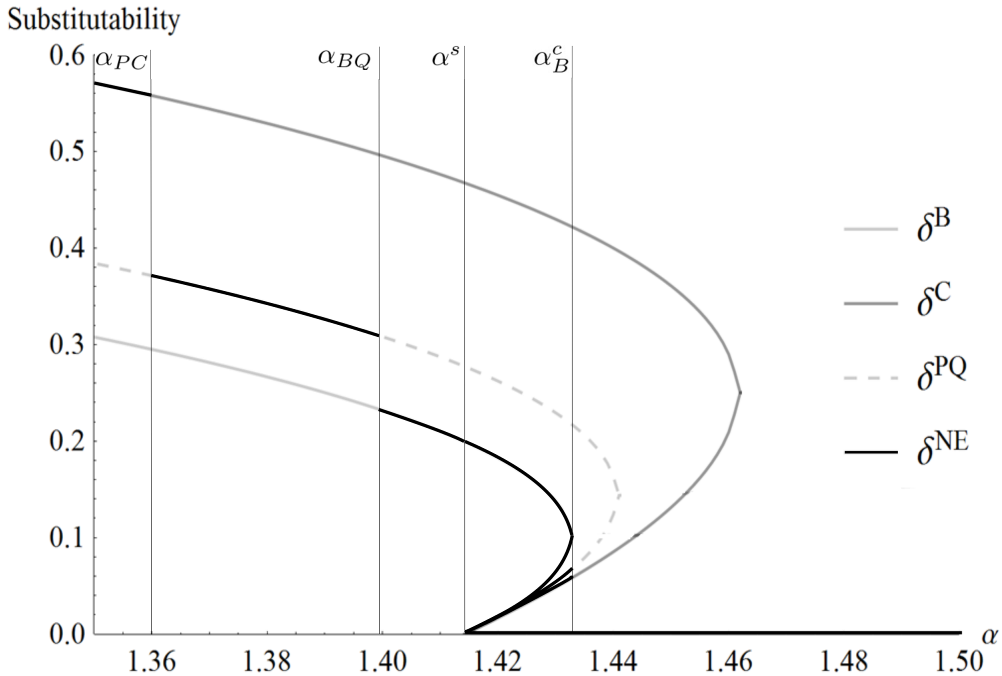
- Bertrand Competition (with *secondary* efforts). This new equilibrium is very similar to the one above, in that it relies on the exact same off-equilibrium path play.

- PQ competition (with *secondary* efforts). There are 2^3 such equilibria where in at least one of the two PQ subgames *secondary* efforts are played on the equilibrium path. Sub-game perfect off-equilibrium path play that is consistent with these equilibria requires that in the other PQ subgame either *main* or *secondary* efforts are played. The same restriction also applies to the Bertrand subgame. For the Cournot subgame equilibrium strategies are still confined to the *main* efforts.

- Cournot Competition (with *secondary* efforts). There are 2^3 equilibria where where firms both compete in quantities and invest according to the *secondary* solution along the equilibrium path, while, in the off-equilibrium path subgames, either *main* or *secondary* efforts are deployed.

When $\alpha > \alpha^c_B$, as interior solutions turn complex, there is a unique (monopoly) equilibrium in the Bertrand subgame which will be also the unique outcome of the game. Since any of the three effort levels are equilibria in the off-equilibrium subgames, there are 3^3 such equilibria for the full game. As α further increases, the number of such equilibria drops to 3 when $\alpha > \alpha^c_{BQ}$. This happens as $2 \times 3PQ$

Figure 10: δ^{NE}, $b = 1$, $\alpha = [1.35, 1.5]$



monopoly equilibria appear. Finally, when $\alpha > \alpha_C^c$, all interior solutions have disappeared and there are just 4 subgame perfect Nash equilibria, one for each of the four subgames that start at the R&D stage, where *upper boundary* R&D efforts are exerted in all subgames and product competition takes place in all of four combinations of the strategic variables.

Notice that employing a further equilibrium refinement on the basis of forward induction we could reduce the number of equilibria between α^s and α_B^c, eliminating the equilibria that support other outcomes than price-setting monopoly.

The analysis above also implies that, for the whole game, the Nash-Equilibrium outcome in terms product substitutability, δ^{NE}, is not a smooth function of market size, α, exhibiting jumps at three critical points, α_{PC}, α_{BQ} and α^s, as shown in Figure 10. This points towards a straightforward empirically testable prediction of the model: as markets increase in size one should observe discrete increases in the amount of differentiation between products, in particular when products are only weak substitutes.

4 Discussion and conclusion

The work presented here shows that introducing horizontal product innovation to the two stage game proposed by Singh and Vives (1984) can systematically change the strategic structure of the first stage of the game where market variables are chosen. The possibility to alter the intensity of competition through costly effort results in multiple reversals in the ranking of the profits obtained in each of the four possible market-subgames. Under more intense types of competition firms have stronger incentives to differentiate their products. When firms sell to larger markets (or are more efficient in their R&D efforts) they can afford larger R&D efforts that become more economically efficient in reducing the intensity of competition compared to the choice of market variables. The results highlight a tradeoff between these two means to reduce the intensity of competition: committing to a decision variable that reduces the intensity of competition (i.e. quantity setting) or altering the product in order to target a niche in market demand where the firm has more market power. Market size increases lead the firms to prefer the latter method of reducing competitiveness to the former. The result is not counter intuitive if we keep in mind that, with exogenous product substitutability, as δ approaches 0 (i.e. the duopolists become two monopolies) the profit differences between Cournot, Bertrand and PQ competition gradually vanish. As market size increases, the between market type difference in the amount of resources that firms can devote to costly specialization diminishes.

As far as the robustness of the results presented here, it is important to mention that the sequentiality of the decision making, observability and commitment to past choices (i.e. of the market variable) play a crucial role. Without these features we can expect the model to behave in a considerably different manner. Already at the basic level of the problem, without any investment, Klemperer and Meyer (1986) show that when quantities or prices are simultaneously chosen without any precommitment to a specific market variable the firms are indifferent between being price-setters or quantity setters[15]. In the model presented here commitment to the strategic variable chosen in stage one is likewise essential. It is the anticipation of tighter competition in the third stage of the game that drives firms to higher levels of product differentiation and eventually to higher profits.

Although it was perhaps not more than a theoretical exercise initially, the choice of a specific market variable was given an economic interpretation by Klemperer and Meyer (1986) who contrast two approaches for supplying consulting services: the consultant can either set an hourly rate or charge a fee per consulting project and subsequently adjusts the amount of time spent per project depending on how many other ongoing projects the consultancy is involved in. The first market strategy resembles price-setting whereas the second is more akin to quantity-setting. In their work it is demand uncertainty in combination with the shape of the marginal cost function that determine the type of market competition in equilibrium, therefore in the absence of asymmetries it would be impossible for firms to end-up in the asymmetric equilibrium where one firm sets prices and the other one sets quantities. However, Tremblay et al. (2013) provide an example of PQ competition from car dealership markets[16]. The model presented here is able to explain the empirically relevant PQ competition without relying on any ex-ante asymmetry.

The results presented here contrast those found in previous work by Symeonidis (2003) and by Qiu (1997) who only qualify Singh and Vives's results regarding welfare, but find no profit reversal. In Symeonidis (2003) product innovation increases market demand only for the firm's own product which will intuitively boost a quantity setter's incentives to innovate while Qiu (1997) considers only cost reducing-innovation which also turns out to be more appealing for Cournot competitors compared to Bertrand competitors. In both cases, innovation has a direct negative effect on the payoffs of a competitor through the (inverse) demand functions. In the model presented here, product innovation is horizontal, with firms investing to reduce product substitutability and thus imposes a strictly positive externality on the competitor. It would be interesting to compare the strength of these to effects when firms can invest in heterogeneous types of product innovation - horizontal and vertical - or in (horizontal) product and process (cost reducing) innovation. Further research may also consider extending the setup to n-firm oligopolies and investigating other demand structures. In particular, one can conjecture that with network goods, as in Pal (2014), the parameter space where non-Cournot markets are equilibria would become larger.

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[15] The work by Tasnádi (2006) shows that indifference between the two market strategies also obtains when firms do ex-ante commit to a market variable but produce a homogeneous good under severe capacity constraints.

[16] They also show that with cost asymmetries, PQ competition can be the outcome of a model with endogenous choice of strategic variables.

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Appendix A

In all three market scenarios considered, Cournot, Bertrand and PQ, making the variable change $\bar{g}_i = bg_i$ in the net profit maximization program (8) leaves it depending on only one parameter, $\bar{\alpha} = \alpha b$.

Profits, net of R&D costs, under Cournot competition are:

$$\pi^C(g_i, g_{-i}; \alpha, b) = \left(\frac{\alpha}{3 - bg_i - bg_{-i}}\right)^2 - \frac{g_i^2}{2}$$

Making the variable change $\bar{g}_i = bg_i$ and factoring out α^2 the program becomes:

$$\pi^C(\bar{g}_i, \bar{g}_{-i}; \bar{\alpha}) = \alpha^2 \left[\left(\frac{1}{3 - \bar{g}_i - \bar{g}_{-i}} \right)^2 - \frac{\bar{g}_i^2}{2\bar{\alpha}^2} \right],$$

where $\bar{\alpha} = \alpha b$. Now the maximand of the profit function will only depend on $\bar{\alpha}$ as a parameter and, since our variable change is invertible, the maximand of the initial and the reformulated program will also be related by: $\bar{g}_i^\star = bg_i^\star$.

For Bertrand and PQ competition the same argument applies since the net profits after the variable change are:

$$\pi^B(\bar{g}_i, \bar{g}_{-i}; \bar{\alpha}) = \alpha^2 \left[\frac{1 - \delta^2}{\left[2 + \delta - \delta^2\right]^2} - \frac{\bar{g}_i^2}{2\bar{\alpha}^2} \right]$$

and respectively:

$$\pi^Q(\bar{g}_i, \bar{g}_{-i}; \bar{\alpha}) = \alpha^2 \left[\left(1 - \delta^2\right) \left(\tfrac{2-\delta}{4-3\delta^2} \right)^2 - \tfrac{\bar{g}_i^2}{2\bar{\alpha}^2} \right]$$
$$\pi^P(\bar{g}_i, \bar{g}_{-i}; \bar{\alpha}) = \alpha^2 \left[\left(\tfrac{2-\delta-\delta^2}{4-3\delta^2} \right)^2 - \tfrac{\bar{g}_i^2}{2\bar{\alpha}^2} \right].$$

with $\delta = 1 - \bar{g}_i - \bar{g}_j$.

Appendix B

Computing the first order conditions we can dispose of the factor α^2. Using the variable change $\bar{g}_i = bg_i$ and substituting $\delta = 1 - \bar{g}_i - \bar{g}_{-i}$ whenever possible, the first order conditions for program (8) under Cournot competition are:

$$2\bar{\alpha}^2 = \bar{g}_i (2 + \delta)^3 .$$

Under Bertrand competition the first order conditions are:

$$2\bar{\alpha}^2 = \bar{g}_i \frac{\left[2 + \delta - \delta^2\right]^3}{1 + \delta^3}$$

For PQ competition where firm i sets quantities and firm j sets prices we have:

$$2\bar{\alpha}^2 = \bar{g}_i \frac{\left(4 - 3\delta^2\right)^3}{(2 - \delta)\left(4 - 4\delta - 5\delta^2 + 6\delta^3\right)}$$

$$2\bar{\alpha}^2 = \bar{g}_j \frac{\left(4 - 3\delta^2\right)^3}{\left(4 - 4\delta + 3\delta^2\right)\left(2 - \delta - \delta^2\right)}.$$