Critical Slowing Down as Early Warning Signals for Financial Crises?

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Abstract

The global impact of the recent financial crisis has once more stressed the urgency of new approaches to designing early warning signals (EWS) for financial crises. In the recent literature on constructing EWS through identifying characteristics of critical slowdown on the basis of time series observations, finance has repeatedly been coined as an important potential application area. On the one hand, this appealing idea is supported by the fact that there is ample empirical and experimental evidence to suggest that nonlinearities play a role in the expectations feedback governing market dynamics. On the other hand, financial markets differ from many natural complex systems, for which evidence of critical slowing down has been reported, in that market dynamics are not necessarily captured well by an ordinary differential equation, the fixed point of which may lose stability through a saddle-node bifurcation, as is the case for the cusp catastrophe. Also, financial time series exhibit persistent near unit root behaviour. In this paper we consider a number of historical financial crises, to investigate whether there is indeed evidence for critical slowing down prior to market collapses. The four events considered are Black Monday 1987, the 1997 Asian Crisis, the 2000 Dot.com bubble burst, and the 2008 Financial Crisis. Our analysis shows evidence for critical slowing down before Black Monday 1987, while the results are mixed and insignificant for the other financial crises.

Keywords: Time series, bifurcation, complex dynamical system, critical slowing down, early warning signal, financial instability

JEL Classification: C10, C53

1 Introduction

The recent financial crisis has intensified theoretical and empirical research on the underlying instabilities of economic systems. Several papers have been concerned with development of an early warning system that could give policymakers and market participants

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warnings on an upcoming financial crisis. However, most of the available contributions are based on "dynamic stochastic general equilibrium" (DSGE) models. These models seek to understand the whole economy with interacting markets under the assumption that the set of prices will result in an overall equilibrium. Unfortunately, when the crisis came, the serious limitations of such financial models become apparent. They were unable to capture the observed magnitude of stock price fluctuations and failed to provide predictions on several extreme events in financial history. A good understanding of dynamic behaviour of financial system is still lacking. The global financial crisis, which began in 2008, is seen as a stress test for DSGE models or even a challenge to their validity. It may be that something is missing from these conventional economic models that prevents them from describing the behaviour of financial markets. In November 2010, the then president of the European central bank, J. Trichet, addressed at the ECB Central Banking Conference stating that "Macro models failed to predict the crisis and seemed incapable of explaining what was happening to the economy in a convincing manner", and he found "In the face of the crisis, we felt abandoned by conventional tools." (J.Trichet, 2010). The lessons of the financial crisis for macroeconomic and financial analysis are profound. They lead us to ask the questions as: What are the determinants of crises? Can crisis be predict? Can crisis be avoided with sufficient early warnings? We need to develop complementary tools to improve the robustness of our overall framework.

During the last decades, there has been a growth of theories recognising that economic systems should be considered as complex systems (Sornette, 2009; Scheffer *et al.*, 2009; Ball, 2012; Farmer *et al.*, 2012). Unlike traditional economic theories under the assumption of general equilibrium, they describe the economic systems as multi-equilibrium processes, as in ecology or weather systems. They consider market crashes as mainly endogenously driven events. Therefore, these crashes could be predicted by studying the underlying mechanisms. Moreover, the advantage of these concepts is that we could be inspired by other disciplines of the science of complexity: ecology, physics, engineering, psychology, biology and so on. Within those fields, sophisticated tools for analysing complex dynamic systems have been developed. Those tools have been proved to be helpful in understanding important complex phenomena in global weather forecasting, ecology, epidemiology, crowd psychology and so on.

Several researchers have recently applied complexity tools, and already demonstrated their value and potential benefits, in economics and finance (Sornette, 2009; Quax *et al.*, 2013). Applying complexity approaches to financial systems can also help deepen our understanding on the dynamic behaviour of financial markets and (parts of) the economy. In this paper, we will apply concepts from complexity theory to real financial data related to historical crises for the first time. We will also investigate which crisis can be linked to critical transitions in an underlying complex system. In particular, we will investigate whether we can develop an early warning system based on the "composite" indicators in different types of historical crisis.

The exploration of indicators of critical transitions in complex systems has been quite fruitful. Important signals that have been suggested in the complexity literature as an early warning indicator are measures of "Critical slowing down". This is based on the slowing down of the dynamics of a complex system approaching a critical point. Several authors developed methods to extracting signals of critical slowing down from time series data. Kleinen *et al.* (2003) observed that how power spectral properties changed as an earth system moving close to a bifurcation point in a hemispheric thermohaline circulation (THC) model. Held and Kleinen (2004) measured the increased memory as an indicator of critical slowing down. The first-order autoregressive coefficient of time series was taken as the propagator of the original time series. This method was applied to North Atlantic hemispheric thermohaline circulation (THC) model, provideing an early warning signal for the climate system. Livina and Lenton (2007) developed another way of detecting critical slowing down by using detrended fluctuations analysis (DFA). This analysis was originally developed by Peng *et al.* (1994) to detect DNA sequences' long-range correlations. They found an early warning signal for an upcoming critical transition in the North Atlantic hemispheric thermohaline circulation (THC) system by investigating model output, as well as Greenland ice core paleotemperature data. The subsequent work by Dakos *et al.* (2008) was the first to study critical slowing down in real time series. The results showed increased autocorrelation in eight climate time series and demonstrated that critical slowing down could serve as an early warning indicator preceding historical critical transitions. Moreover, Lenton *et al.* (2012) contributed to early warning system by offering some general guidelines of choosing the parameters needed for the analysis. They improved the robustness of ACF and DFA techniques and gave more evidences of critical slowing down in both palaeodata and climate model output. Recently, Kefi *et al.* (2013) expanded the theory of critical slowing down to a broader class of situations where a system becomes increasingly sensitive to perturbations even without non-catastrophic transitions. They showed that critical slowing down could even be used in a more general sense as an early warning signal.

The possibility of applying the early warning tools to financial data was suggested by Scheffer *et al.* (2009). Encouraged by the successes of early warning signals in many complex systems, numerous efforts have been made to explore the warnings in financial time series. Due to the demanding of massive datasets, few results have been made. However, there are still some successful examples. For instance, by using information dissipation length (IDL) as indicator, Quax *et al.* (2013) detected the early warning signals prior to Lehman Brothers collapse in both USD and EUR interest rate swaps (IRS). They suggested the IDL may be used as an early warning signal for critical transitions. Tan and Cheong (2014) observed critical slowing down in the U.S. housing market. They detected strong early warning signals associated with a sequence of coupled regime shifts during the period of subprime mortgage loans transition and the supreme crisis. They also found the weaker signals during Asian financial crisis and technology bubble crisis. However, up till now, no evidence of early warning signal has been found in the time series data of stock markets. To fill this gap, this paper attempt to apply complexity theory of "critical slowing down" in real financial data for the first time. Four financial crisis are analysed, which are the Black Monday, the Asian Crisis, the Dot-com Bubble and the 2008 Financial Crisis. Autocorrelation at lag-1 and Variance are considered as the "composite" early warning indicator to examine whether financial systems slow down before the critical point is approached.

We will first explain in Section 2 some theoretical background of nonlinear dynamic systems. Before performing the analysis, we will describe the data and methodology in Section 3 and 4. Subsequently, we analyse how well such early warning indicators perform in financial time series. The results of our analyses are presented and discussed in Section 5. Section 7 provides a summary and conclusions.

2 Theory

The proposed mechanisms behind critical transitions in complex systems mathematically corresponds to *bifurcations*. A bifurcation is a qualitative change in the dynamical systems as a control parameter varies. Thompson *et al.* (1994) reviewed different types of bifurcations in dissipative dynamical systems. Three categories are classified based on their continuous or discontinuous dependence on the control parameters. These categories are safe bifurcation with continuous growth of a new stable attractor, explosive bifurcation with discontinuous increasing attractor which grows to a newly enlarged attractor along with itself, and dangerous bifurcation in which current attractor simply disappears, forcing the system to jump in a fast dynamic transient to a remote and entirely new attractor. Figure 1 illustrate this classification in a straightforward way. The critical transitions in complex systems are often considered to be dangerous bifurcations (Thompson *et al.*, 1994; Sieber and Thompson, 2012).

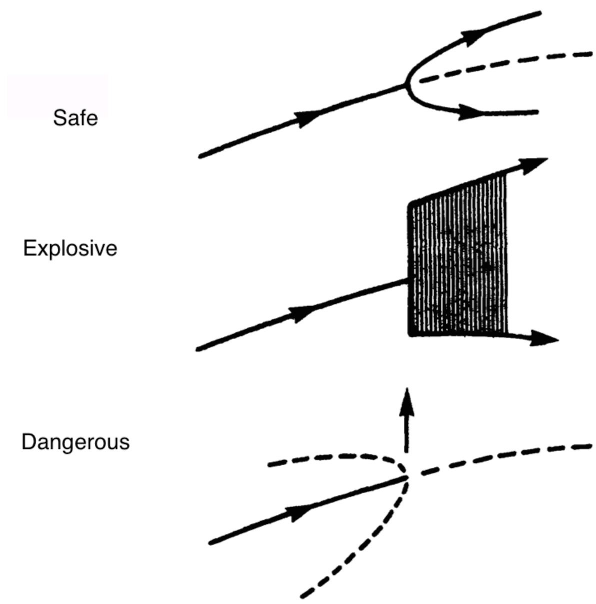


Figure 1: Classification of bifurcations according to their outcome (Source: Thompson *et al.* (1994)).

Figure 2 illustrates an example of saddle-node bifurcations. With the increasing of a single control parameter, there exists a critical point. Even a small perturbation would lead to a large qualitative change when the system is very close to this critical point value. Once this threshold is exceeded, the whole system transits toward a different attractor. Even if the control parameter is reversed, the response will typically remain close to the new attractor. The original attractor will not be re-achieved. This highlights the irreversibility of critical transitions. Scheffer *et al.* (2012) proposed that this bifurcation can also be used to describe the dynamical behaviour of financial systems, systematic crashes in stock markets for instance. It suggests the possibility of applying complexity theory to financial systems.

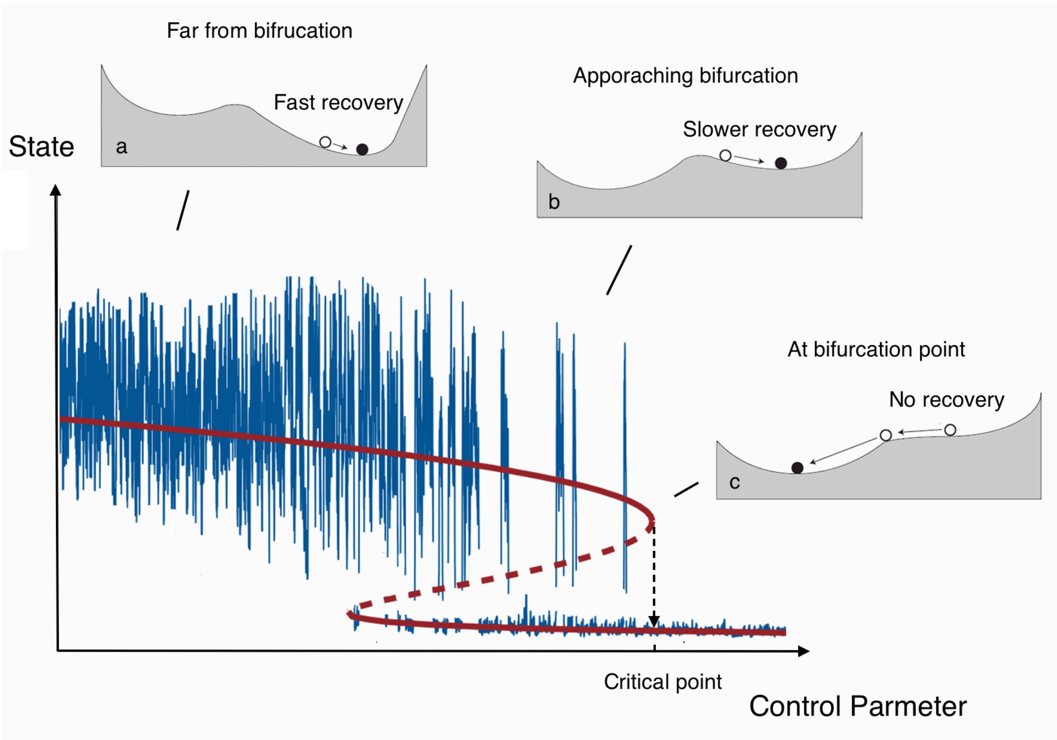


Figure 2: A saddle-node fold with noise. *a*, *b* and *c* describe the critical slowing down as an early warning indicator that the system lost resilience on the way of approaching the critical point. Local minima represent stable attractors while the ball shows the present state of the system. *a*. Far from bifurcation: small variance and fast fluctuations. *b*. Approaching bifurcation: larger but slower fluctuation with increasing variance; *c*. At bifurcation point: irreversible transition to a new local minimum.

To develop tools to forecast critical transitions based on time series, it is necessary to assume that the observed time series is generated by a rather general nonlinear dynamical system. In the early warning literature it is common to assume that the system is one-dimensional and that the bifurcation of interest can be represented in its normal form, is driven by Gaussian white noise, and has drift control parameter ρ:

$$dx = f(x,\rho)dt + g(x,\rho)dW, \quad (1)$$

where x is the state of the system, $f(x,\rho)$ determines the deterministic part of the system, while $g(x,\rho)dW$ determines the stochastic part. dW is a white noise process. The system is assumed to be one-dimensional. The equilibrium of the undisturbed system is stable in all directions on the way of approaching the critical value. By varying the control parameter ρ, the system closes to a threshold. When a real eigenvalue of the Jacobian matrix $DF_{\rho}(\bar{x})$ of the steady state finally crosses +1 (with the other real eigenvalue smaller than 1 in absolute value), a saddle-node bifurcation occurs. This bifurcation is corresponding with a critical transition in a time series. This scenario is descried in detail in Rahmstorf (2001), Lenton *et al.* (2008) and Sieber and Thompson (2012). It offers ways to provide early warnings before the critical transition actually happens. As long as we understand the statistical properties of the system approaching critical transition, we may predict the time of the transition in advance, up to some uncertainty.

Numerous attempts have been made to develop an early warning system in financial systems, such as binomial/multinomial logit/probit models, binary recursive trees, Markov switching models and so on. However, their failure to give warnings ahead of the financial crisis in 2008 makes their predictive ability questionable. At the same time, the exploring for indicators of critical transitions in complex systems has been quite fruitful. The most important signal suggested in literature as an early warning indicator is related to a complex theory of "Critical slowing down". When a dynamical system approaches a critical point, we can expect it to become increasingly slow in recovering from small perturbations. This can be indicated by the linear decay rate (LDR) decreasing to zero. The theory of "Critical slowing down" is used to describe this phenomenon, as is also illustrated in Figure 2. Figure 2*a*, *b* and *c* show the behaviour of a dynamical system approaching a saddle-node bifurcation. The local minima of the potential well represent stable attractors and the ball shows the present state of the system. On the way of approaching the bifurcation point, the local minimum on the right is shallower, recovery speed of the ball is increasingly slowing down in response to the small perturbations. When this minimum finally vanishes, the ball suddenly rolls into the minimum on the left. This implies that the system transits into a different steady state. The mechanisms behind this behaviour can be explained in mathematical terms. Approaching a saddle node bifurcation, the maximum real part of the eigenvalue of the Jacobian matrix tends towards 1. It indicates the slower recovery from perturbations (Kuehn, 2011 and Wagener, 2012). Therefore, as long as we are able to detect the signal of slowing down, it would be possible to predict potential critical transitions with some accuracy.

In what follows, by exploring the critical slowing down prior to some extreme events in financial history, we would be able to assess how well such early warning signals perform in financial time series.

3 Data

For the analyses time series from subsystems of the economy which clearly indicate critical transitions from one state to another are needed. Stock market prices have clear value for helping us forecast these potential transitions. Therefore, time series of daily stock market prices are taken as the dynamical subsystem to be analysed in our work. As long as we find the early warning signals in the stock market price system, we are able to acknowledge the occurrence of financial crisis.

Four financial crisis are analysed: Black Monday (October 19, 1987), the Asian Crisis, the Dot-com Bubble and the 2008 Financial Crisis. Although the direct causes of these crisis are different, they share the common characteristic that the stock prices for these events displayed the same bubble and burst pattern. Cusp catastrophe theory may possibly be used to describe these critical transitions in financial systems.

A particular time series is studied for each crisis based on its characteristic. For instance, the most popular time series data of stock prices in the literature is Standard & Poor 500 (S&P 500) index. Therefore, we employ it to detect the early warning signals prior to the Black Monday and the 2008 financial crisis. The Hongkong Hangseng index is the best candidate to describe the Asian events, so we use Hangseng index to analysis the Asian Crisis. The Dot-com Bubble is an information technology crisis which is boosted by rapidly growth of equity values in internet sector. Thence, the subsystem we choose is NASDAQ Composite. It is related with technology companies and growth companies. Most recently, the 2008 financial crisis is along with credit crisis. In addition with the S&P 500, we analysed the TED spread [1], since it is an indicator of perceived credit risk. Moreover, as the volatility index of the S&P500, the VIX index [2] is also analysed. Detailed data information is illustrated in Table 1.

The daily time series data of stock prices, such as the S&P 500 index, the NASDAQ composite and the Hangseng index, are downloaded from Thomson Reuters Datastream [3] for the period from May 1986 until May 2011. The TED spread is derived by calculating the differences between the three-month LIBOR and the three-month T-bills interest rate. These datasets are also available from the Datastream. The Volatility index VIX is downloaded from the online Chicago Board Options Exchange (CBOE) database[4] (`http://www.cboe.com/micro/vix/historical.aspx`).

[1]The TED spread is the difference between the interest rates on interbank loans (the three-month Eurodollars contract as represented by the London Interbank Offered Rate (LIBOR)) and on short-term U.S. government debt ("T-bills"). It is an indicator of perceived credit risk in the general economy. When the TED spread increases, that is a sign that lenders believe the risk of default on interbank loans is increasing

[2]VIX is a trademarked ticker symbol for the Chicago Board Options Exchange Market Volatility Index, a popular measure of the implied volatility of S&P 500 index options. Often referred to as the fear index or the fear gauge, it represents one measure of the market's expectation of stock market volatility over the next 30 day period.

[3]Financial database containing company data, equities and macro-economic data. Holds time series information for over two million financial instruments, securities and indicators for over 175 countries and 60 markets worldwide, with up to 50 years of historical depth for some series.

[4]The largest U.S. options exchange and creator of listed options, which offers equity, index and ETF options, including proprietary products, such as S&P 500 options (SPX) and options on the CBOE Volatility Index (VIX)

Since we are interested in extracting early warning signals prior to a critical transition, the time series before each transition are carefully selected. The critical transition points are determined by visual inspection according to original records in international news or articles (such as Wikipedia). For the examples that do not have information on timing, we simply choose the points with maximum value in the corresponding period. Since the random growth in stock prices data is in percentage term and not in absolute term, we take logarithms of data. Doing so also linearises the exponential growth in original series and stabilise variance of the analysed residuals.

Table 1: Summary of financial time series

Crisis	Time	Time Series	Sample Size (N)
Black Monday	1987	S&P500 index	200
Asian Crisis	1997	Hangseng index	500
Dot.com	2000	NASDAQ composite	400
2008 Financial Crisis	2008	S&P500 index	398
		TED spread	280
		VIX index	80

4 Methodology

4.1 Detrending

In order to achieve a stationary stochastic process, the first step is to remove the trend pattern in original time series. Under this condition, the residuals can be further analysed by using linear and nonlinear time series techniques. Moving averaging is the most commonly used technique in detrending. In this paper, we introduce the weighting scheme and use gaussian kernel smoothing. This allows data near the given time point to receive larger weights. Moreover, this method could ensure the positive autocorrelations in the residuals and facilitate our analysis.

In the analysis of many other complex systems, additional interpolation is needed to have evenly spaced time series. This may produce spurious results (Dakos *et al.*, 2008). Because stock market system can be already considered as a time discrete dynamical system, which is with fixed time-step $\Delta t = 1$ trading days, we skip the interpolation and therefore avoid its possible adverse effects. We detrend time series using a gaussian kernel,

$$G(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}, \tag{2}$$

and the moving average is

$$MA_j = \frac{\sum_{i=j-N}^{j-1} G(i) z_{j-i}}{\sum_{i=j-N}^{j-1} G(i)}, \tag{3}$$

where z is the logarithm of original time series with fixed time-step $\Delta t = 1$. The bandwidth is σ. It is also the standard deviation of the Gaussian kernel distribution. In kernel density estimation, the choice of bandwidth is very important. A large bandwidth would leave out possible details of the trend, underestimate the magnitude of peaks and troughs, and produce an oversmoothed estimate. Therefore when a large bandwidth is used, the estimator may have large biases. Taking account of this, people may want to choose a small bandwidth, as small as the data allows. However, there is always a trade off. When too small bandwidth is applied, it may result in only a few local data points available to reduce the variance of the estimator. It will reduce the reliability of the estimation results. In this paper, we choose bandwidth in such a way that we filter out the slower trends visible in the data but still keep the details of the records. Dakos *et al.* (2008) have already had reasonable results by setting the bandwidths around 10 to 25. In this paper, we use bandwidth $\sigma = 10$ and test them with $\sigma = 20$ [5]. By subtracting moving average from the logarithm of the original time series. The residuals time series is given by

$$y_j = z_j - MA_j, \tag{4}$$

which fluctuates around 0.

4.2 Leading Indicators

An increasing in autocorrelation and variance is expected from critical slowing down. It can be estimated for a moving window using time series techniques. Three early warning indicators are considered: AR(1), ACF(1) and Variance.

AR(1) indicator When approaching a critical transition, consecutive observations in the state of the system become increasingly similar to each other. Autocorrelation at lag-1 is considered as the leading indicator to measure critical slowing down. Its quantities can be estimated from an autoregressive model of lag-1:

$$f(t_{j+1}) = e^{-\kappa \Delta t} f(t_j) + \theta_\eta \eta_j, \tag{5}$$

which is also called AR(1) model. $f(t_j)$ is residual y_j as time series. $\Delta t = t_{j+1} - t_j$ and η_j is a zero mean innovation. κ indicates the magnitude of the recovery rate. $\lambda = e^{-\kappa \Delta t}$ is AR(1) coefficient, which is the autoregressive coefficient at lag1. In a saddle-node bifurcation scenario, κ vanishes on the way to the bifurcation point. As κ goes to 0, λ goes to 1.

The random disturbances are assumed to be white noise with zero mean and variance equal to 1. Therefore, the first order autocorrelation coefficient λ is approximated as constant in a local time window of length w. We estimate λ by an ordinary least-square (OLS) fitting method of

$$y_{k+1} = \lambda y_k + u_k, \tag{6}$$

[5] reference from Dakos *et al.* (2008)

with u_k white noise, over the set of indices $k = j - w + 1, ..., j$. The local window slides from left to right and traces a series of AR(1) coefficients vary with respect to index. This new series can be interpreted as the increased slowing down. Therefore, it indicates that the system is driven gradually closer to a bifurcation.

Apart from bandwidth, window size is also a very important parameter. Smaller window size allows us to track short term changes in autocorrelation. However, too small window size with very few observations can make the estimation of autocorrelation less reliable. Dakos *et al.* (2008) proved that half the size of analysed time series give appropriate estimations. In this paper, we follow their rules and also choose half the size of analysed time series as the sliding window size.

ACF(1) indicator An alternative and more straightforward way to estimate autocorrelation at lag-1 is by using the first value of the autocorrelation function (ACF)

$$\rho_1 = \frac{E[(y_t - mu)(y_{t+1} - mu)]}{\sigma_y^2}, \tag{7}$$

where μ is the mean of y_t in the window considered, and σ_y^2 the variance. Like AR(1) indicator, moving window produces a proxy series of ACF(1). It also serves as an indicator to detect critical slowing down prior to a critical transition.

Variance indicator An increased slowing down also induces an increased amplitude on the way of approaching threshold. This amplitude is correspond with variance and is measured by standard deviation:

$$St.Dev. = \frac{1}{N-1} \sum_{t=1}^{N} (y_t - \mu)^2. \tag{8}$$

This proxy series of variance produced by moving window also serves as one of the early warning indicators precede a critical transition.

For each indicators, we test the trends by estimating the noparametric Kendall rank correlation τ. It is a statistic tool used to measure the association between two measured quantities. Its quantity is in the range of $-1 \leq \tau \leq 1$. If Kendall's τ is close to 1, the agreement between two rankings is perfect. If Kendall's τ is close to -1, The two rankings are disagreement. A high Kendall's τ suggests a strong trend. In this paper, a strong upward trend which is measured by positive Kendall's τ is desired.

5 Results

The early warning signals are evaluated over two approaches. Firstly, we observe the early warning signals before real critical transitions. By using six time series, we examine four well-known extreme financial events in history - the Black Monday, the Asian Crisis, the Dot-com Bubble and the 2008 Financial Crisis. Secondly, we examine the likelihood of spurious early warnings. Random walk are tested as a surrogate time series. The possibility of obtaining our early warning signals by chance is checked by using bootstrapped time series. In the end, we perform an extensive analysis to examine the robustness of results with respect to the choice of parameters.

5.1 Financial Time Series

First of all, we examine whether there is evidence of "Critical slowing down" in time series data of stock prices. Four financial crisis are investigated. However, the existence of underlying bifurcation is uncertain.

The Black Monday In a single day, Dow Jones Industrial Average (DJIA) lost almost 22%. By the end of that month, most of the major exchanges had dropped by more than 20%. Stock markets around the whole world crashed, beginning in Hong Kong, spreading to Europe, and hitting the United States later after other markets had declined by a significant margin. This event marked the beginning of a global stock market decline, making "Black Monday" one of the most tragic days in recent financial history.

Figure 3 shows the analysis of early warning indicators around half a year precede "Black Monday" by using Standard & Poor 500 index (S&P500) time series. The original time series in Figure 3(a) is the logarithm of daily S&P500 index. It starts from 200 days before the crash and ends to 100 days after it. Stock markets raced upward during the first half of 1987, but experienced a great depression in the last few months. The vertical dashed line identifies the critical transition in the time series. Since we are interested in searching for the early warning signals before the critical transition, analysis stops at the dashed line before the termination occurs. To facilitate explanation, we set the x-axis of the critical transition as 0 to distinguish the days before and after it. The smoothed central line shows the smoothed time series used for filtering. The dashed arrow shows the width of the moving window. Half size of the time series is selected as moving window which is suggested by Dakos *et al.* (2008). Figure 3(b) is the remaining residuals used to estimate the early warning indicators.

Figure 3(c), (d) and (e) give the examples of early warning indicators of AR(1), ACF(1) and Variance. They show that the great crash in the "Black Monday" is preceded by the overall upward trends in these indicators. All of the positive trends are confirmed by the positive Kendall rank correlation coefficient τ. Moreover, the p-value of each Kendall's tau is close to 0 which demonstrates the significances of the increasing trends. Therefore, the examples of early earning indicators showed an increase in the period preceding the critical transition, suggesting that the system of S&P500 time series indeed slows down before the critical transition.

The Asian Crisis Using the same techniques, we examine the Hangseng time series. Figure 4 shows the analysis of early warning indicators around one and a half year before the Asian Crisis. Figure 4(a) is the logarithm of daily Hangseng index from November 1995 to July 1998. This time series increases in the beginning but collapses around mid-1997, which illustrates the Asian financial crisis in July 1997. The Asian Crisis is a series of currency devaluations along with stock markets declining. The currency market first failed in Thailand

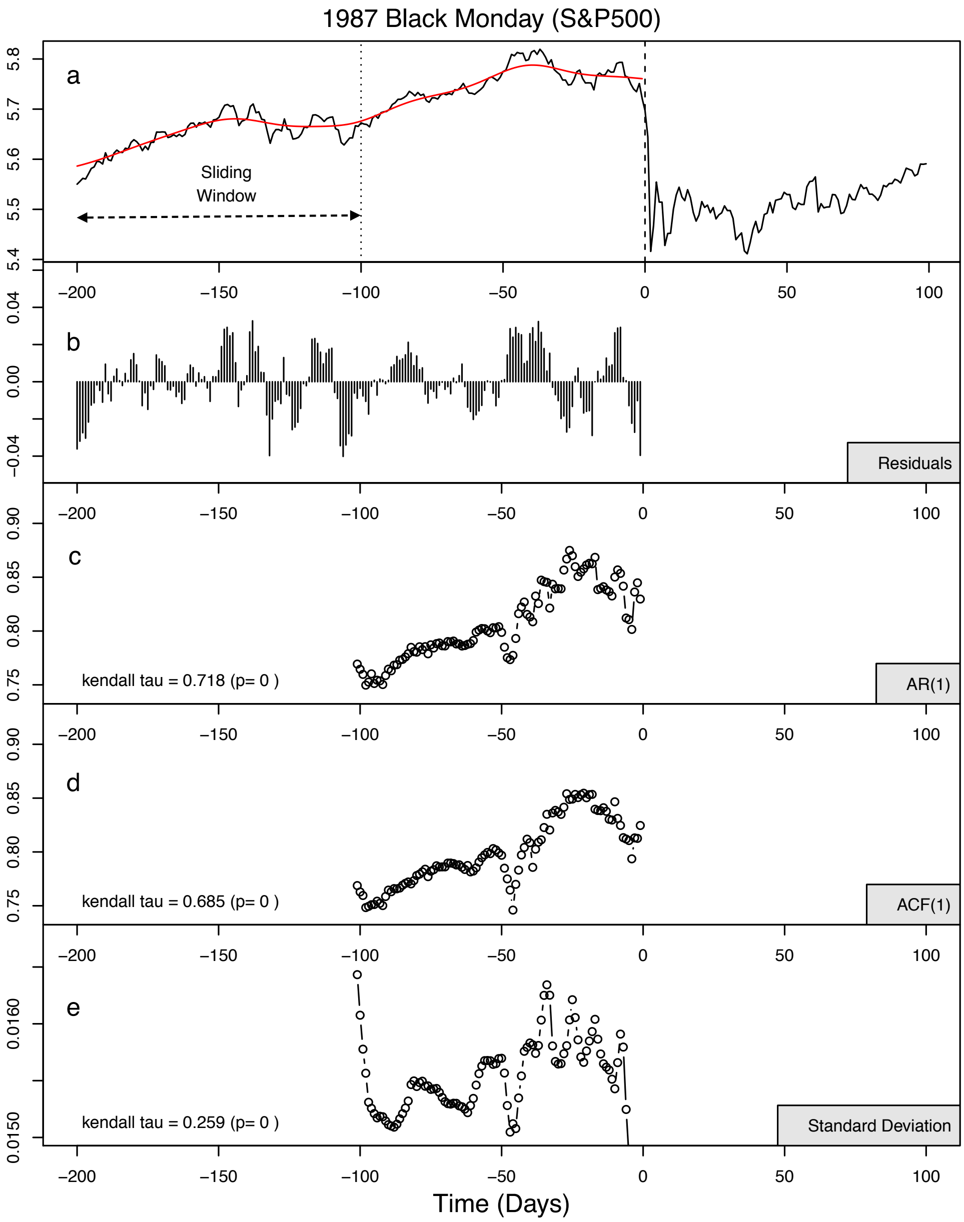


Figure 3: Detecting the early warning indicators for "Black Monday" using time series S&P500. (a) Logarithm of the daily S&P500 time series. (b) Analysis of the residuals time series. (c) Early warning indicator from AR(1) function. (d) Early warning indicator from ACF(1) function. (e) Early warning indicator from Standard deviation. The vertical dashed line in (a) identifies the critical transition. Analysis stops there before the termination occurs. The dashed arrow shows the width of the moving window used to compute the indicators shown in $(c)(d)$ and (e). The red line is the smoothed time series used for filtering.

because its government no longer pegged their local currency, the Thai Baht, to the U.S. dollar. The currency crisis rapidly caused stock market declines spreading throughout South Asia. Thailand, South Korea and Indonesia were the countries most affected by the crisis. As a result of the crisis, the stock markets in Japan and most of Southeast Asia fluctuated dramatically.

Figure 4 has a similar format as Figure 3. The smoothed central line in Figure $4(a)$ shows the moving average used for filtering. The dashed arrow shows the width of the moving window, which is the half size of the analyzed time series. Figure $4(b)$ is the remaining residuals used to estimate the early warning indicators.

The examples of upward trends in indicators of AR(1), ACF(1) and Variance shown in Figure $4(c)$, (d) and (e) indicate critical slowing down before critical transitions. All of the trends are significant as measured by the Kendall's τ. The increased slowing down before July 1997 in Hangseng index gives evidence on early warnings before the Asian Crisis.

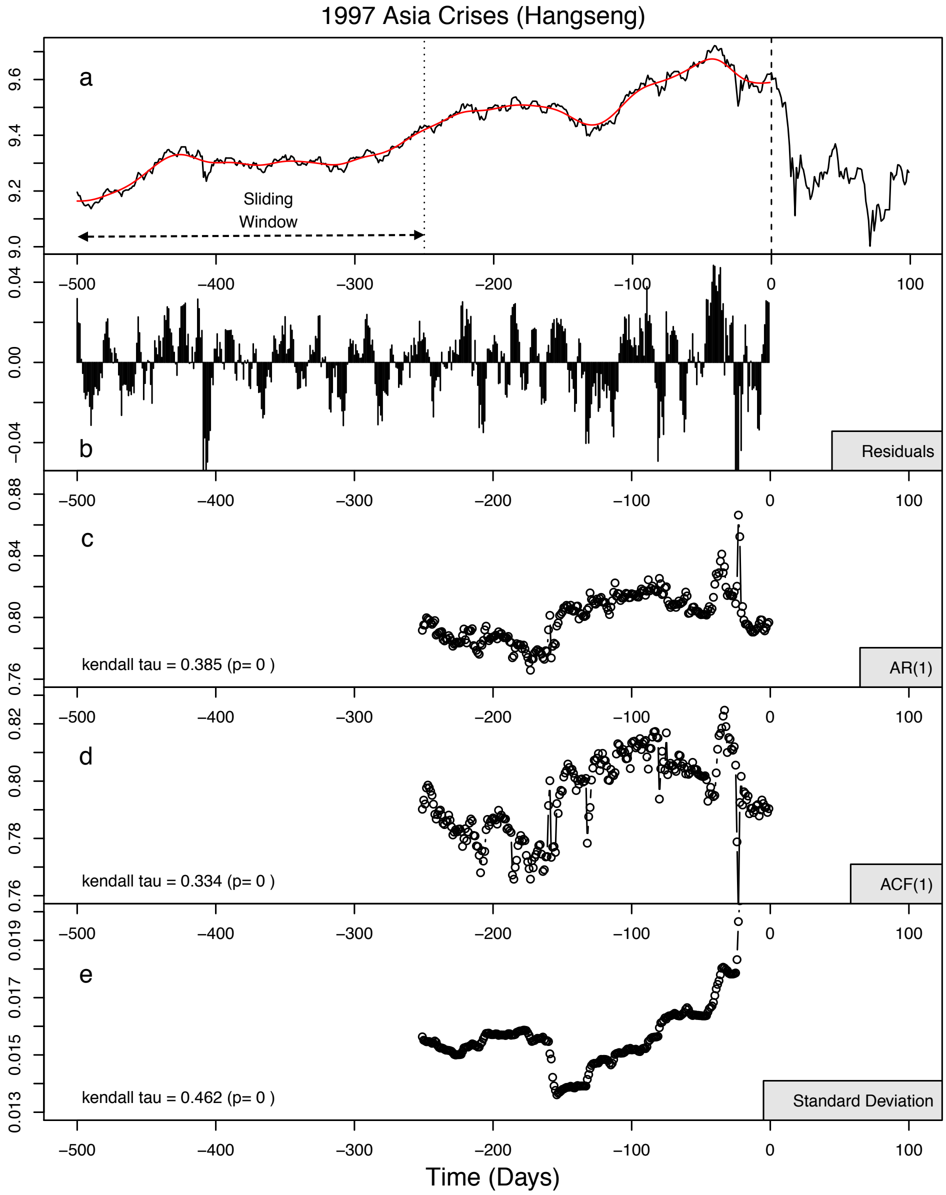


Figure 4: Detecting the early warning indicators for the Asian Crisis using time series Hangseng index. (a) Logarithm of the daily Hangseng time series. (b) Analysis of the residuals time series. (c) Early warning indicator from AR(1) function. (d) Early warning indicator from ACF(1) function. (e) Early warning indicator from Standard deviation. The vertical dashed line in (a) identifies the critical transition. Analysis stops there before the termination occurs. The dashed arrow shows the width of the moving window used to compute the indicators shown in $(c)(d)$ and (e). The red line is the smoothed time series used for filtering.

The Dot-com Bubble Figure 5 presents the analysis of early warning indicators around one year before the Dot-com bubble collapse. Boosted by the rising of commercial growth of internet, NASDAQ composite index occurred speculative bubble, as shown in Figure 5(a). It peaked around year 2000, the latter part followed a typical boom and bust cycle. When the bubble "bursts", the stock prices of dot-com companies fall dramatically. Some companies went out of business completely, such as Pets.com. Some others survived but their stocks declined more than 80%, such as Cisco and Amazon.com.

Like the Black Monday and the Asian Crisis, the analysis of early warning indicators give the evidence on increased slowing down in NASDAQ time series before Dot-com Bubble. The bubble collapse is preceded by an overall upward trend in the examples of early warning indicators. All the results are robust based on the Kendall's τ analysis. It suggests a possibility that critical slowing down could serve as an early warning signal for Dot-com Bubble.

The 2008 Financial Crisis The financial crisis of 2008 is known as the greatest financial crisis since the Great Depression of the 1930s. It was triggered by the bursting of United States housing bubble which peaked approximately 2005 - 2006. Banks began to give out more loans than ever before to potential home owners. The rising subprime mortgage increased thereafter. When housing bubble finally bursted in the latter half of 2007, the secondary mortgage market collapsed. Over 100 mortgage lenders went bankrupt during 2007 and 2008. Several major financial institutions failed, including Lehman Brothers, Merril Lynch, Washington Mutual, Citigroup and so on. The world wide economies experienced a great depression and stock markets around the world went down.

Figure 6(a) and (b) show the analysis of early warning indicators around one year ago before the 2008 financial crisis by using two time series, S&P500 and TED spread. Both of the analysis show mixed results. In the analysis of TED spread time series, the examples of AR(1) and ACF(1) indicators produce strong upward trends with Kendall's tau close to 0.7. It indicates the increased slowing down precede critical transitions in TED spread time series on September 2008. Just around this time Lehman Brothers went bankrupt. However, for the same period, the examples of variance indicator show downward trends. In Figure 6(b), the analysis of S&P500 time series gives a reversal of the results. AR(1) and ACF(1) indicators show downward trend while Variance indicator show upward trend. The mixed results suggest that none of the analysis gives promising early warnings. It also highlights the importance of applying "composite" indicators. None of the indicators alone would be able to give us accurate predictions.

In order to obtain more evidences, we also analyse the volatility index (VIX) in Figure 7. What interests us is that VIX index is an estimated time series itself. It is a popular estimation of the implied volatility of S&P 500 in the next 30 days. Figure 7(a) presents the logarithm of the daily S&P500 time series while Figure 7(b) shows the logarithm of the volatility of the S&P500 index. Because we are interested on volatility time series, the x-axis of the critical transition in volatility index in Figure 7(b) is set as 0. As shown in Figure 7(a) and (b), this critical transition in VIX is 15 days precede the real critical transitions in the daily S&P500

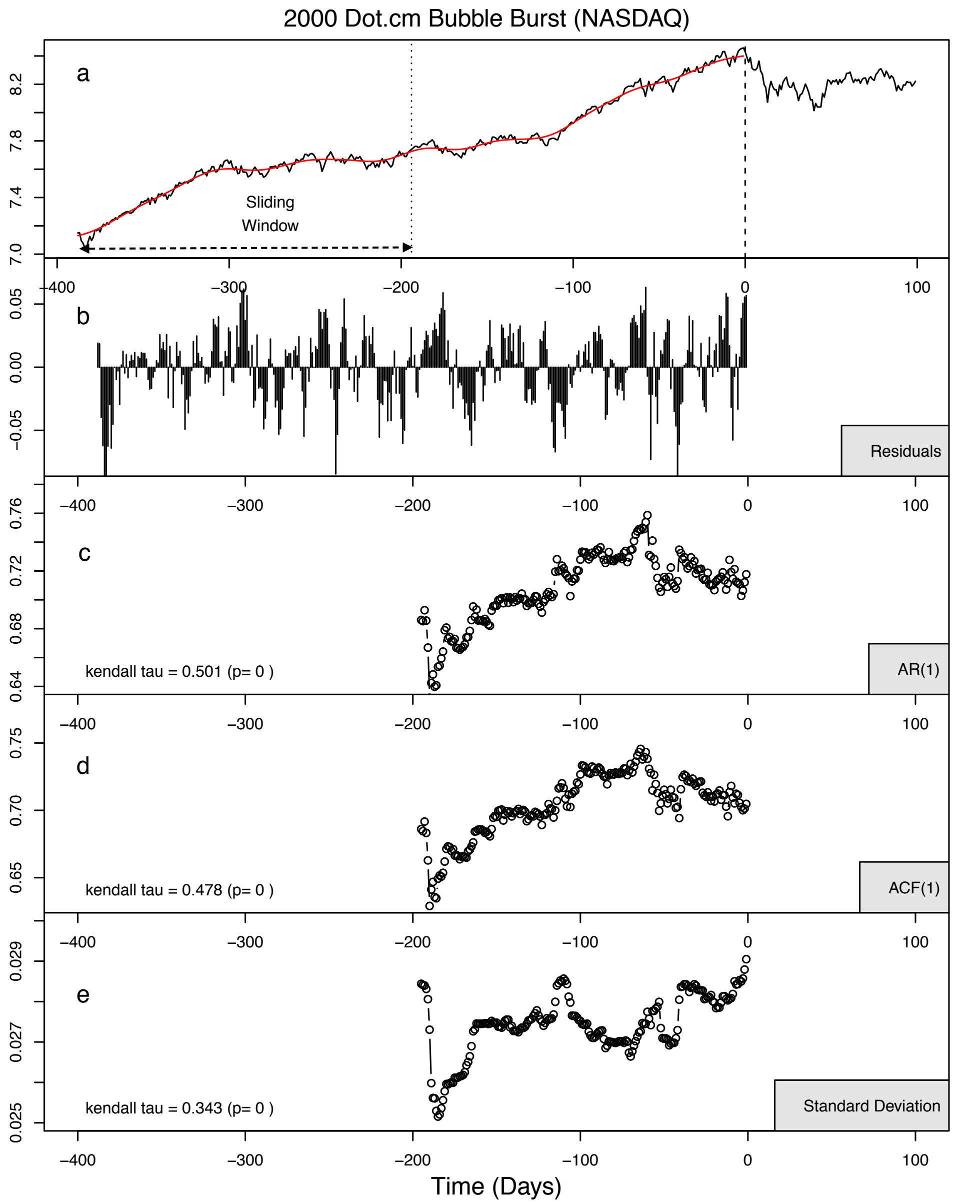
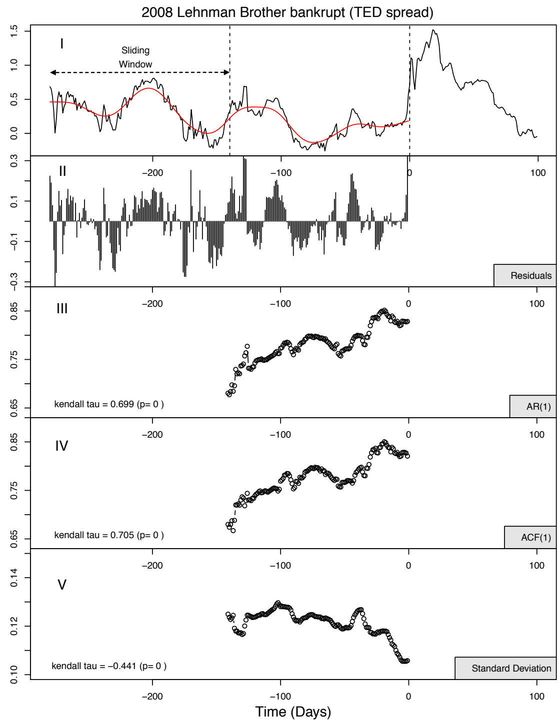
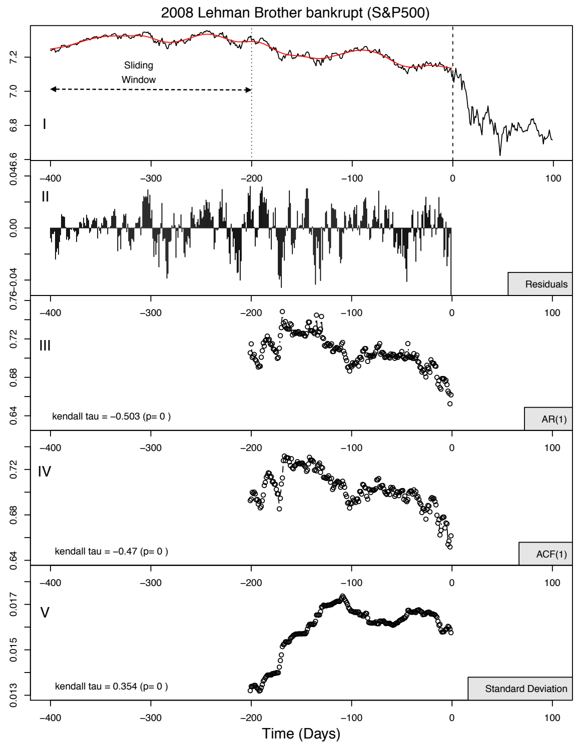


Figure 5: Detecting the early warning indicators for the Dot-com Bubbles using NASDAQ composite index. (a) Logarithm of the daily NASDAQ time series. (b) Analysis of the residuals time series. (c) Early warning indicator from AR(1) function. (d) Early warning indicator from ACF(1) function. (e) Early warning indicator from Standard deviation. The vertical dashed line in (a) identifies the critical transition. Analysis stops there before the termination occurs. The dashed arrow shows the width of the moving window used to compute the indicators shown in $(c)(d)$ and (e). The smoothed line is the smoothed time series used for filtering.



(a) TED spread



(b) S&P 500

Figure 6: Detecting the early warning indicators for the 2008 Financial crisis using TED spread and S&P500 index. (a) Analysis using TED spread. (b) Analysis using S&P500 index. For each of the analysis, (I) Logarithm of the daily original time series. (II) Analysis of the residuals time series. (III) Early warning indicator from AR(1) function. (IV) Early warning indicator from ACF(1) function. (V) Early warning indicator from Standard deviation. The vertical dashed line in (I) identifies the critical transition. Analysis stops there before the termination occurs. The dashed arrow shows the width of the moving window used to compute the indicators shown in $(III)(IV)$ and (V). The red line is the smoothed time series used for filtering.

time series. In Figure $7(b)$, the time series kept rising after the critical transition for almost a month and decreasing slightly thereafter. However, it did not shift back to the equilibrium before critical transition. The VIX index is also called fear index or fear gauge. It measures the expectations of markets on future stock market volatility. Therefore, our analysis implies that people expected increased volatility of stock market prices already before the financial crisis actually happen. This fear of the increased volatility of stock market prices quickly accumulates and lasts for a long time. The whole process coincides with the depression of economy. It also shows that the early warnings on the critical transitions in volatility index can be considered as the early warnings on the crash in S&P500 index.

The same early warning methodology is applied to VIX time series. The analysis shows significant upward trends in the examples of AR(1) and ACF(1) indicators precede the critical transition in VIX time series. It demonstrates the increased slowing down and potential early warnings before critical transition in volatility index. However, the example of variance indicator is just horizontal with a slightly downward trend. Moreover, p-value of the estimated Kendall's tau indicates insignificance of the trend. Therefore, no early warning is found by variance indicator.

All the above results and the parameters used in the analysis are described in Table 2. The symbols "(+)" indicate early warning signals are detected, while the symbols "(-)" indicate the transitions are not preceded by indicators. As shown in the table 2, the window size we choose is half of the sample size in each example as suggested by Dakos *et al.* (2008). The choice of bandwidth is 10 under the condition that we do not overly smooth the data but still give the stationary time series. We also checked each example under the bandwidth as 20. Although the kendall's τ coefficients are slightly different, they give similar trends as in the above examples.This simple test suggests the robustness of the results respect to the choice of the bandwidths.

Table 2: Studies of early warning indicators for critical transitions in different time series. N is sample size. w and σ is window size and bandwidth used to do the analysis. τ is estimated kendall's tau coefficient. symbol (+) indicates early warning signals are detected. (-) indicates the transitions are not preceded by indicators.

Extreme Event	Time Series	N	w	σ	τ		
					AR(1)	ACF(1)	Variance
Black Monay	S&P500	200	100	10	0.718***(+)	0.685***(+)	0.259***(+)
Asian Crisis	Hangseng	500	250	10	0.385***(+)	0.334***(+)	0.462***(+)
Dot-com	NASDAQ	398	194	10	0.501***(+)	0.478***(+)	0.343*** (+)
2008 Crisis	S&P500	400	200	10	-0.503***(-)	-0.470***(-)	0.354***(+)
	TEDspread	280	140	10	0.699***(+)	0.705***(+)	-0.441***(-)
	VIX	80	40	10	0.463***(+)	0.446***(+)	-0.149 (-)

$\star\star\star$ significant at 1% level, $\star\star$ significant at 5% level, $\star$ significant at 10% level.

The summarise of the results in Table 2 suggests the possible early warnings indicated by the examples of AR(1), ACF(1) and variance precede the crashes of the stock market prices

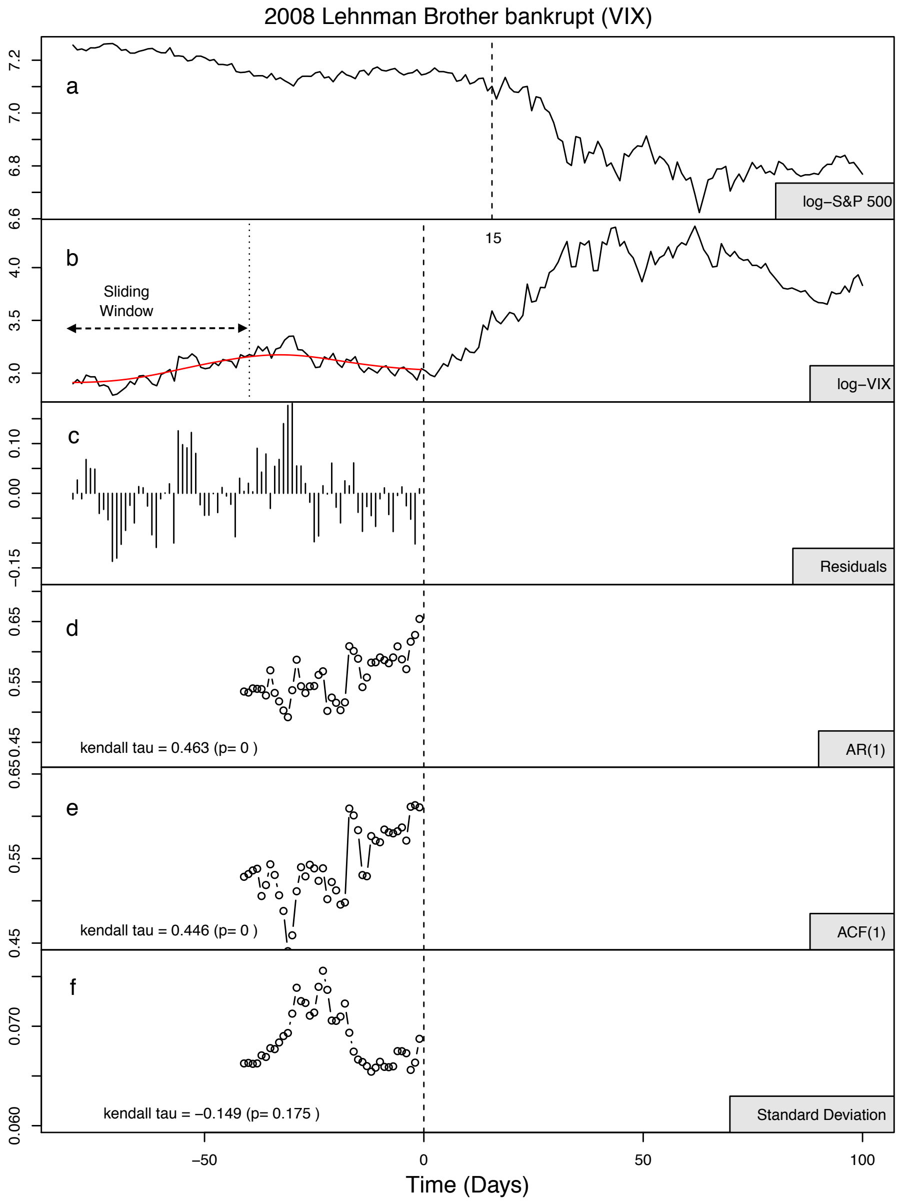


Figure 7: Detecting the early warning indicators for the 2008 Financial crisis using volatility index (VIX). (a) Logarithm of the daily S&P500 index. (b) Logarithm of the volatility of the S&P500 index. (c) Analysis of the residuals time series. (d) Early warning indicator from AR(1) function. (e) Early warning indicator from ACF(1) function. (e) Early warning indicator from Standard deviation. The vertical dashed line in (a) and (b) identifies the critical transitions in the time series of the S&P500 index and the volatility index. Analysis stops at the critical transitions in volatility index before the termination occurs. The dashed arrow shows the width of the moving window used to compute the indicators shown in $(d)(e)$ and (f). The red line is the smoothed time series used for filtering.

in the Black Monday, the Asian Crisis and the Dot-com Bubble. However, the signals for the 2008 Financial Crisis are mixed. They lead us to the question whether there is increased slowing down prior to the 2008 Financial Crisis. The AR(1) and ACF(1) methods in TED spread and VIX time series suggest so but they give the opposite results in S&P500 time series. Are these signals just spurious early warnings? Or were there no bifurcations underlying the 2008 financial crisis? To solve these questions, more work is still to be done.

5.2 Bootstrapped Time series

In order to test the likelihood of having the trend statistic estimation of kendall's τ by chance, we apply the same early warning methodology on surrogate time series. We also calculate the probability of the trend statistics in surrogate time series higher than the original records. The surrogate time series are generated by bootstrapping the financial time series of the S&P500 index, the Hangseng index, the NASDAQ composite, TED spread and the VIX index in three different ways.

Firstly, we bootstrap the residuals after detrending following the test in Dakos *et al.* (2008). By resampling the order, we generate surrogate time series with similar means and variances.

Secondly, instead of residuals, the log-returns of the original time series are bootstrapped. Similar with the first method, we bootstrap the time series by randomly picking data with replacement. Moreover, we take the cumulative sum before detrending. Because we are using differences instead of residuals, these surrogate processes are more stationary than the processes in the first method.

Thirdly, the surrogate time series are generated by fitting log-returns to GARCH(1,1) model [6]:

$$\begin{aligned} y_t &= \sigma_t \epsilon_t, \\ \sigma^2 &= \omega + \alpha y_{t-1} + \beta \sigma_{t-1}^2, \; t = 1, ..., T, \end{aligned} \tag{9}$$

where ϵ_t is a white noise process with $\epsilon_t \sim N(0,1)$, σ_t is the volatility. It is a stochastic process which is assumed to be independent from ϵ_t and ω. ω, α, β are parameters that satisfy $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$ to ensure the positivity of the conditional variance. The process y_t is stationary respect to $E[log(\beta + \alpha\epsilon_t^2)] < 1$.

For the time series data of the S&P500 index, the Hangseng index, the NASDAQ composite, the TED spread and the VIX index, 1000 surrogate time series are estimated under each bootstrap method. The trend statistics of Kendall's τ coefficients and p-values are represented by histograms. Figure 8 shows one of the examples of the analysis. It presents the analysis of the Kendall's τ coefficients of the examples of AR(1) indicator in 1000 surrogate S&P500 time series, under the bootstrap method of fitting the GARCH(1,1) model. Figure

[6] Generalized autoregressive conditional heteroskedasticity, originally introduced by Engle (1982), Bollerslev (1986) represent the dynamic evolution of conditional variances.

8(a) presents the probability distributions of the p-values while Figure 8(b) shows the histogram of the Kendall's τ coefficients. The dashed line represents the trend statistic of the original τ^* in the Black Monday example.

The likelihood of obtaining trend statistic estimates by chance is estimated by the subsets that the surrogate trend statistics are higher than the trend statistics of the original time series, $p(\tau \geq \tau^*)$. This subset is indicated by the arrow in Figure 8(b). It refers to the probability that surrogate kendall's τ lie on the right hand of the original kendall's τ. The p-values as $p(\tau \geq \tau^*)$ would evaluate the significances of the trend statistic estimations of the early warnings in financial examples. The results for the examples of AR(1) indicators are shown in Table 3.

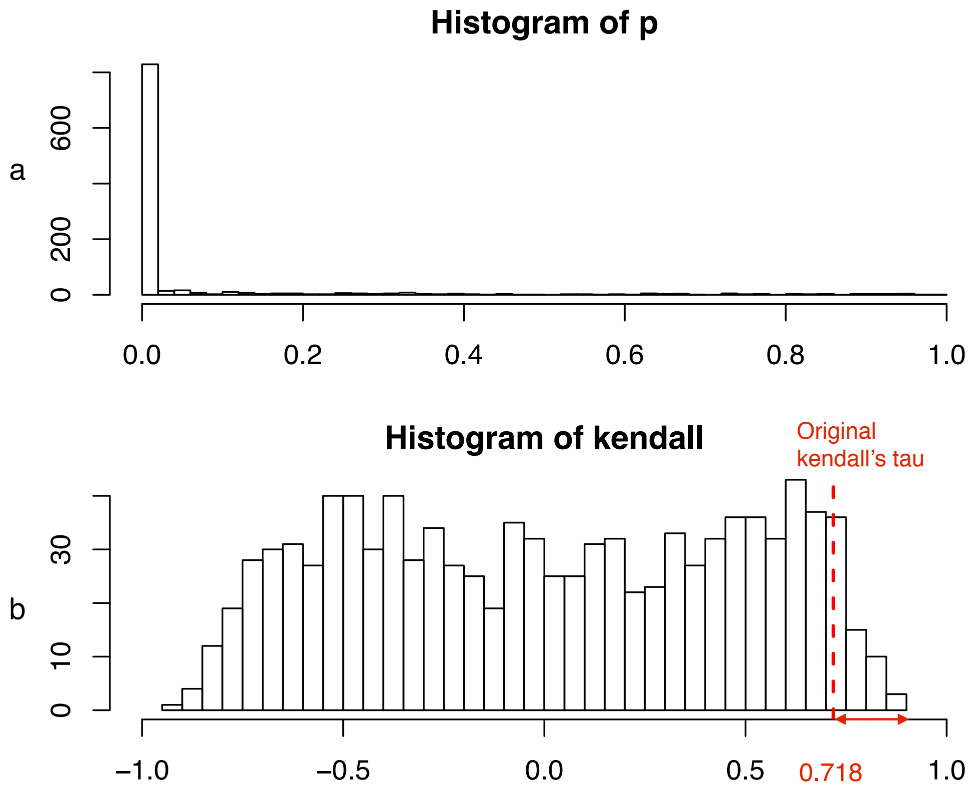


Figure 8: Analysis of the Kendall's τ coefficients of the examples of AR(1) indicator in 1000 surrogate S&P500 time series, under the bootstrap method three which is to fit GARCH(1,1) model. (a) presents the bootstrap density of the p-values; (b) shows the histogram of the Kendall's τ coefficients. The Red dashed line represents the trend statistic of the original τ^* in the Black Monday example. The Red arrow indicates the subset which the surrogate trend statistic is larger than the trend statistic of the original residual records. From this subset, only values equal to or higher than the original record are used to estimate the likelihood of acquiring trend statistic estimates this far in the tail by chance.

Table 3 shows the likelihood of obtaining trend statistic estimates by chance, which is estimated by the probability of the surrogate estimated trend statistic kendall's τ larger than the trend statistic of the original residual records. Only the results for the examples of AR(1) indicators are shown here. The probabilities vary from case to case, but the differences between the three bootstrap methods are small. This suggests the reliability of the bootstrap results. As shown in Table 3, the results are mixed. The probability of having the observed trends by chance are less than 10% in the examples of the Black Monday (S&P500) and the 2008 Financial Crisis (TED spread) in all three bootstrap methods. It implies that the early warning signals we found in both examples are significant, with less than 10% significance level. In particular, the performance of the example of the Black Monday (S&P500) is fairly good. Its results are significant at 5%. It highlights the reliability of the early warning signals precede Black Monday crash in S&P500 time series. In other cases, the probability of obtaining the trends by chance are around 20% - 30%. The significance levels of the early warnings we found in the Asian Crisis, the Dot-com Bubble and the 2008 Financial Crisis(VIX) are only around 30%.

Table 3: The likelihood of obtaining trend statistic estimates by chance, estimated by the probabilities of the surrogate estimated trend statistic (kendall's τ) larger than the trend statistic of the original residual records. Only the results for the examples of AR(1) indicators are shown here. Symbol (+) indicates early warning signals are detected. (-) indicates the transitions are not preceded by indicators. Surrogate set $N = 1000$.

Extreme Events	Time Series	Bootstrap $P(\tau > \tau^*)$ [7]			τ^*
		I. Permute Residuals	II. Permute Return	III. GARCH (1,1)	
Black Monday	S&P500	0.055*	0.045**	0.060*	0.718(+)
Asian Crisis	Hangseng	0.300	0.287	0.309	0.385(+)
Dot-com	NASDAQ	0.354	0.335	0.350	0.501 (+)
2008 Crisis	S&P500	0.205	0.200	0.183	-0.503(-)
	TEDspread	0.072*	0.061*	0.074*	0.699(+)
	VIX	0.311	0.30	0.288	0.463(+)

$\star\star$ 5% level, $\star$ 10% level

5.3 Random financial time series

Spurious early warnings can occur in the fluctuations in a single regime without transiting to a different one. In order to test this possibility, we analyse the early warnings on random segments in financial time series. Figure 9 shows an example of the test of the early warning indicators for random segments in Hangseng time series. It follows the same format of figures as in the analysis of financial time series. The analysis is based on a randomly picked segment from a long period of Hangseng time series from 1988 to 2009. However, the sample size is kept the same as it in the example of the Asian Crisis. As shown in Figure 9, there is

[7] $\tau < -\tau^*$ for 2008 Financial Crisis case using S&P500

no evidence of early warnings in this example with respect to the early warnings of AR(1), ACF(1) and Variance indicators.

Many other random segments of the financial time series are tested in the same way. However, the trends of the early warning indicators are diverse. Some of them show no early warnings while the others do. The early warning indicators of 1000 random segments are analysed. Figure 10(a), (b) and (c) show the histograms of the examples of AR(1), ACF(1) and Variance indicators individually. The dashed lines are Kendall's τ of the indicators of the real critical transitions in the Asian Crisis. The arrows indicate the subsets which the segments trend statistics are higher than the trend statistics of the true critical transitions. The fractions of these subsets are indicated by percentages in the figure. In all three indicator examples, the fractions of the subsets are around 20% - 30%. Following the ideas on the analysis of bootstrapped time series in Section 5.2, Figure 10 implies that the examples of early warning indicators of AR(1), ACF(1) and variance give the early warnings in Hangseng index before critical transitions but only with 20% - 30% significance levels.

5.4 Random Walk

To compare with financial time series, we do the analysis in random walk process. A simple random walk is presented as:

$$y_t = y_{t-1} + \epsilon_t, \tag{10}$$

where ϵ is white noise.

We generate 1000 random walk process realisations and analysis the early warning indicators in them. Figure 11 shows the histograms of Kendall's τ and p-values of AR(1) and Variance indicators. Because there are no critical transitions and bifurcations happening in random walk, the early warning indicators are independent with time. This is indicated by $\tau \to 0$. Furthermore, due to gaussian detrending in the methodology, $-1 < \tau < 1$ is expected as shown in the figure. Moreover, if it is the methodology which creates spurious trends in Kendall's τ, the distribution would left skewed and clustered at 1. However, Figure 11 shows all the Kendall's τ are distributed evenly between -1 and 1, which rejects the hypotheses above. According to p-value distributions in Figure 11(b) and (d), most of them are significant. These results suggest that this early warning methodology does not give rise to false early warnings.

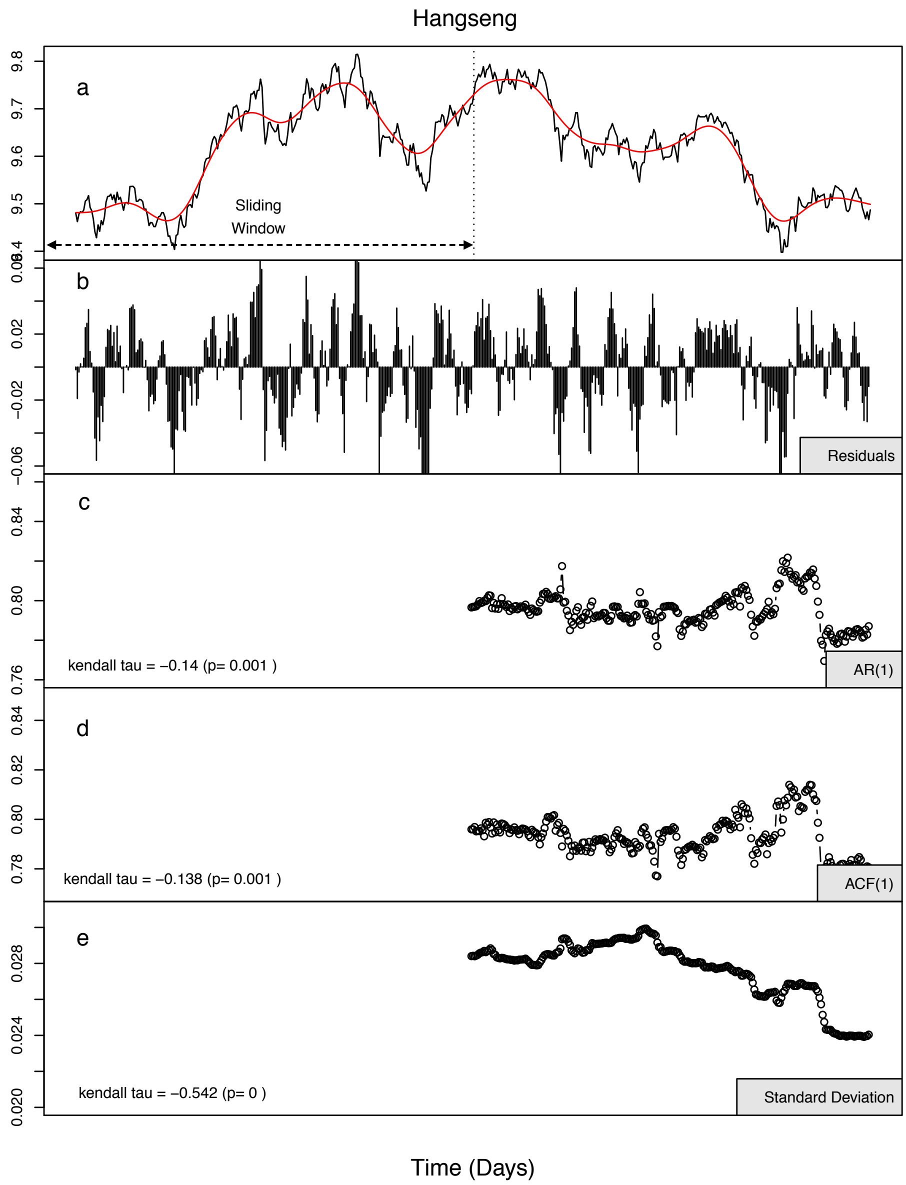


Figure 9: Testing the early warning indicators for random segments in Hangseng time series. (a) Logarithm of the random segments in Hangseng time series. (b) Analysis of the residuals time series. (c) Early warning indicator from AR(1) function. (d) Early warning indicator from ACF(1) function. (e) Early warning indicator from Standard deviation. The dashed arrow shows the width of the moving window used to compute the indicators shown in $(c)(d)$ and (e). The red line is the smoothed time series used for filtering.

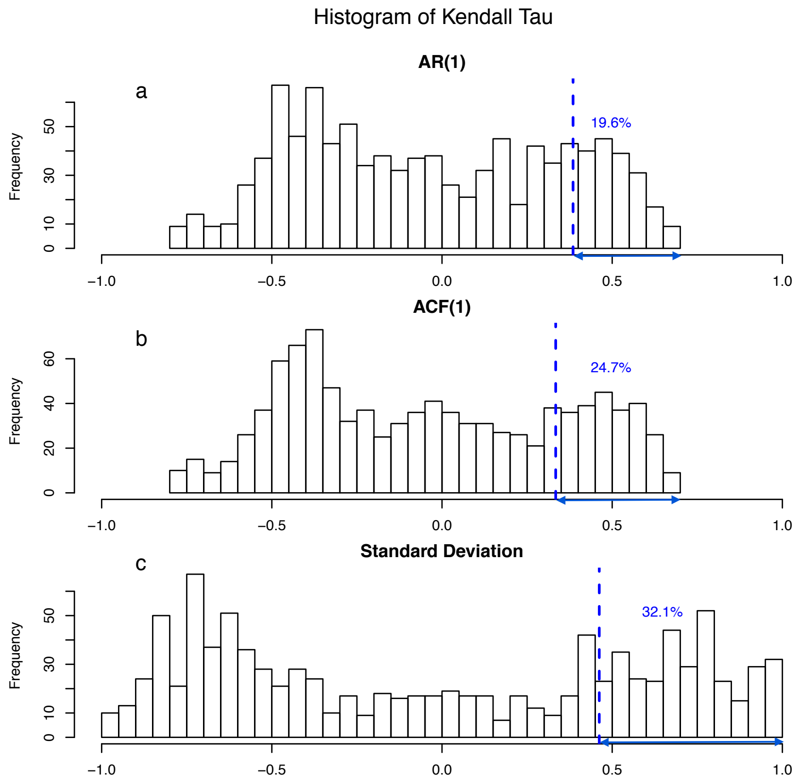


Figure 10: Histograms of Kendall's tau by Randomly segments in Hangseng time series from 1988 to 2009. The number of time series is N=1000. a, b and c show the histograms of the examples of AR(1), ACF(1) and Variance indicators individually. Blue dashed lines are the kendall's tau of the indicators of the real critical transitions in Asian Crisis. Blue arrow indicates the subsets which the segments trend statistic is higher than the trend statistic of the true critical transition. Blue percentage numbers indicate the fractions higher than original time series.

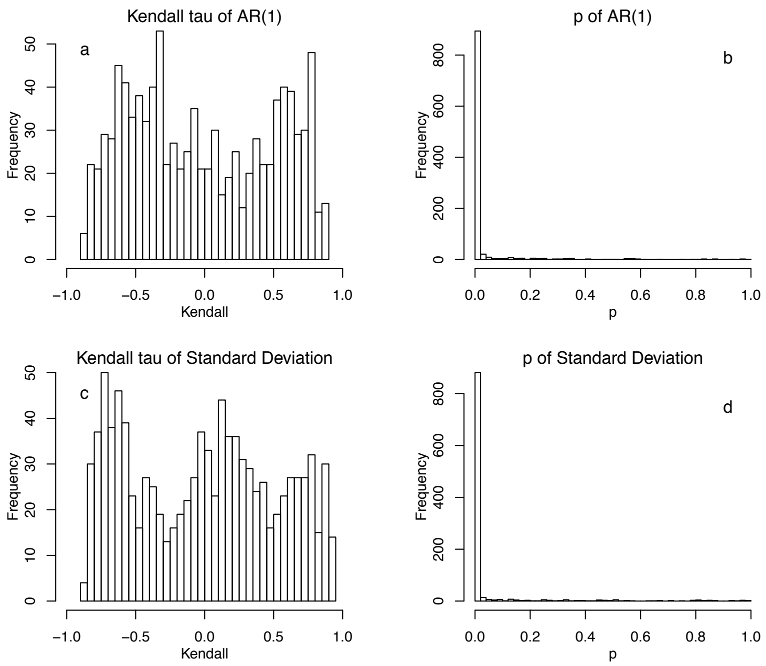


Figure 11: Histograms of Kendall's tau and p-value in the examples of AR(1) and Variance indicators in random walk process. (a) and (c) show the histograms of the Kendll's tau in the examples of AR(1) and Variance indicators while (b) and (d) show the distributions of p-values.

6 Robustness of parameters

The early warning analysis in this paper is sensitively influenced by two key parameters: bandwidth size and sliding window size. Bandwidth size is a very important parameter when filtering out long term trends in original time series. There is a trade off when making the choice. A too narrow bandwidth would not only remove the long run trends but also the short run fluctuations which we intend to study; a too wide bandwidth would not remove enough long run trends. There would be still some slow trends left which may lead to spurious trends of the indicators. A similar trade off also happens to the window size. A smaller window size is good to track short run changes, but a too small window size with too few sample points would make the estimations less reliable.

In order to check the robustness of the parameters in our analysis, we perform an additional analysis by using rolling window and rolling bandwidth. The contour plots in Figure 12 and 13(a) show the influence of parameters on the observed trends of AR(1) indicators in the examples of the Black Monday and the Asian crisis. The black dot indicates the combination of window size and bandwidth size that shows the strongest positive trends. The white dot indicates the parameters used in our early warning analysis. The kendall's tau distributions in Figure 12 and 13(b) confirm the strong positive trends of kendall's tau in the contour plots. This robustness analysis indicates that the results are quite robust respect to the choices of parameters. It also shows that even more significant trends could be obtained by moving parameters to black dots, which in turn confirms the robustness of parameters in this paper.

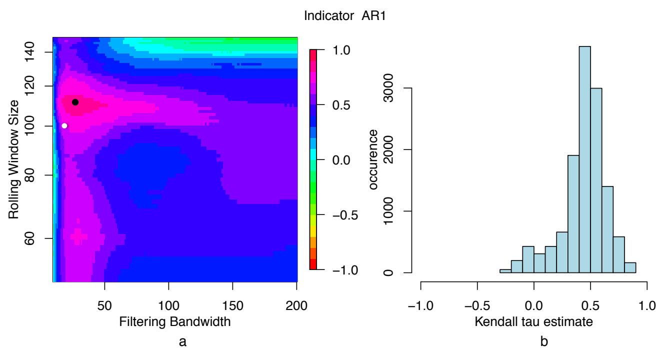


Figure 12: Analysis of the robustness of parameters in the example of Black Monday: window size and bandwidth. (a) Contour plots of the rolling window size and bandwidth size. Black dot indicates the assemble of window size and bandwidth size that shows the strongest positive trends. White dot is the parameters used in the early warning analysis (b) Histogram of the kendall's tau in (a)

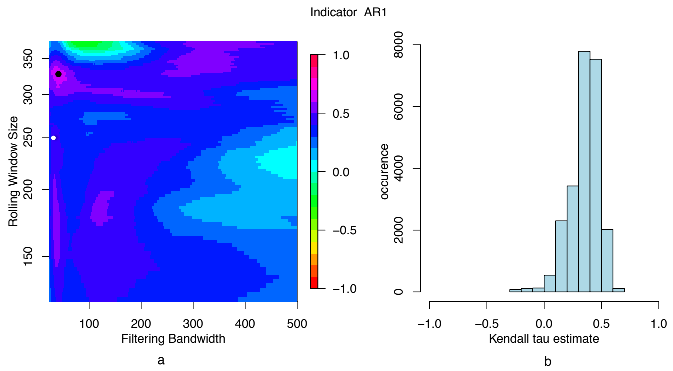


Figure 13: Analysis of the robustness of parameters in the Asian Crisis: Window size and bandwidth. (a) Contour plots of the rolling window size and bandwidth size. Black dot indicates the assemble of window size and bandwidth size that shows the strongest positive trends. White dot is the parameters used in our early warning analysis (b) Histogram of the kendall's tau in (a)

7 Summary and Conclusions

The theory of "Critical slowing down", measured as the increased first order autocorrelation coefficients towards +1, is considered to be an early warning signal for an upcoming critical transition. This theory is applied to the analysis of time series by Held and Kleinen (2004) and Livina and Lenton (2007). They used the increased slowing down before critical transitions to identify the bifurcations in climate systems. The method has been firstly applied to the North Atlantic thermohaline circulation and Greenland ice core paleotemperature using climate model output. Dakos *et al.* (2008) developed this methodology further and tested it in real climate data for the first time. Their analysis provides robust empirical evidence. The first order autocorrelation coefficient indeed increases as the system approaches a critical transition.

Our paper investigates whether there is evidence of "critical slowing down" in financial systems for the first time. Four financial crisis are analysed by using six time series. The results suggest the increases in AR(1), ACF(1) and variance indicators precede the crashes of the stock market prices in the Black Monday, Asian Crisis and Dot-com Bubble. However, the signals for the 2008 Financial Crisis are mixed. In order to estimate the likelihood of spurious early warnings, the same methodology is applied to various surrogate time series. The analysis of the bootstrapped time series confirms the reliability of the early warning signals preceding the Black Monday crash in S&P500 time series, but suggests the possibility of spurious early warnings in the Asian Crisis, the Dot-com Bubble and the 2008 Financial Crisis(VIX) are around 30%. Analysis of random financial time series also shows the failure to detect the critical transitions in financial time series are around 20% - 30%. The random walk analysis suggests that the methodology does not give rise to false early warnings. An additional analysis on the robustness of parameters is performed in the end. It shows that the results are fairly robust with respect to the choices of the parameters.

This paper fills the gap between the theory of "critical slowing down" and its application to financial data. It explores the forecasting ability of the critical slowing down indicators in financial time series. The results detect fairly good early warnings before the crashes in Black Monday. The early warnings before the Asian Crisis, the Dot-com Bubble and the 2008 financial crisis are only detected at 20% - 30% significance level. It suggests that critical slowing down can serve as one of the complementary early warning indicators of financial crisis. However, in order to increase the accuracy of the predictions, more sophisticated methods are still desired.

There are a number of possible reasons why the results are mixed. Firstly, so far the tools of detecting critical slowing down are usually linear based. With the increase of the complexity of the nonlinear dynamics, the predictions of bifurcations may fail. Sieber and Thompson (2012) tried to extend the techniques to nonlinear features. However, they did not find discernible trends in the nonlinear proxies. The methodology used in this paper is also based on a first order linear approximation of the dynamics. In this case, due to the complexity of financial dynamic systems, false early warnings can be expected. Secondly, the transitions in complicated financial systems may happen far from local bifurcations and do not have to experience the cusp catastrophe transitions. For instance, early escape due to a noise induced transition (Thompson and Sieber, 2010). In particular, the emergence of new technology and financial instruments nowadays makes the financial markets even more complex. This could explain the failure to detect the 2008 financial crisis using advance economic and financial approaches. Thirdly, the catastrophe theory approach is based on one dimensional systems with only one control variable, while the situation in financial systems are far more complicated. Perhaps a multivariate approach is required to capture the sophisticated dynamic behaviours in financial systems. Fourthly, asset pricing models are usually based on the assumption that the fundamental price follows geometric random walk process. In particular, the fluctuations around fundamental price are endogenous for heterogenous agents models (HAMs) (Boswijk *et al.*, 2007). Therefore they always have the marginally stable eigenvalue 1 already.

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