Monetary Policy and Speculative Stock Markets

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Abstract

This paper studies the relationship between asset prices and macroeconomic aggregates when financial markets are speculative, and whether monetary policy can mitigate potential spillovers from financial markets. I augment a model with financial constraints on working capital with asset markets, where excess volatility of these markets is endogenized using a behavioral model for financial speculation. The presence of credit constraints links asset returns to optimal leverage and the price level. This link can induce a dynamic feedback loop that can amplify excess volatility in asset prices. I estimate this model to match key moments of empirical European data. The endogenous process of financial market speculation and the feedback from asset prices to the price level are key features to replicate these moments well, and to provide an explanation for the relationship between asset prices and macroeconomic aggregates. Nonlinear analysis suggests that central banks can offset the impact of speculation on either output or inflation by carefully targeting asset prices, but not on both, and can furthermore dampen excess volatility of stock prices. However, the scope of such policy to stabilize economic activity is limited narrowly due to its undesirable response to real economic shocks.

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1 Introduction

"I consider asset price bubbles and monetary policy to be one of the most challenging issues facing a modern central bank at the beginning of the 21st century."
— Jean-Claude Trichet, 2005

Observers of the great recession have argued that when facing a liquidity trap, expansionary monetary policy will, instead of fostering steady growth, rather fuel financial markets. These in turn might further destabilize the economy and comprise hazard. Some economists (Borio and Lowe, 2002, Cecchetti, 2000) have suggested to let monetary policy target asset prices to prevent aforementioned spiral of bubbles, instability and unconventional monetary policy. When suspecting such a feedback between financial markets and real aggregates, two postulates are implied. The first postulate is the existence of a mutual link between asset prices and real activity, i.e. that causality might run in both directions. Recent economic literature stresses the relevance of firms' financial structure. This suggests to also study the macroeconomic impact of equity, and equity prices. Yet there is limited insight on how and when stock prices impact on real activity. The second postulate is that asset prices do not only reflect a discounted fundamental value but embed distortions themselves, possibly biased by speculation. Such destabilizing speculative process implies that traders are not fully rational and that the existence of potential positive profits through speculation makes financial markets more prone to instability than real markets.

This work tries to answer the question whether causality does not only run from real activity to stock markets, but also in the opposite direction. Starting with the microfoundations of such linkage, I use a parsimonious macroeconomic model to replicate statistical key-moments of the data. As it turns out, both the linkage as well as a speculative process are necessary to sufficiently match the data. Having thus motivated a positive role for policy, I ask whether a Taylor-type interest rate rule that also targets asset prices is able to mitigate the impact of speculation on real activity and to reduce excess volatility on stock markets.

The first contribution of this work is hence to identify the potential channels of how stock prices could feedback into macroeconomic activity. The first channel transmits through the *external finance premium* introduced by Bernanke et al. (1999) (BGG). When firms' financial structure is relevant and borrowing constraints depend on the quality of equity, an increase in equity prices lowers costs for external funding. Secondly, since stock market prices reflect future expected returns, potential holders will only purchase shares of those firms that reflect the highest expected return per invested unit. If certain shares are valued below the market price, holders are incentivised to liquidate the firm and reinvest the return in a firm with higher return to equity. If firms are ex ante identical, all firms must provide the same return on equity.[1] Further below I show that both concepts result in the same mathematical model.

I formulate a DSGE model of a monetary production economy and extend it by a sector of financial intermediaries. Firms lever their profits by borrowing from the financial intermediaries and pledge their equity as collateral, while borrowing conditions and finance costs depend on the quantity of collateral offered. The external finance premium thus depends on firms net

[1] A further potential channel is a wealth effect that works through aggregate demand: increasing stock prices raise the nominal value of assets held by households, and amplifies consumer demand. Unfortunately such effect is ruled out in a representative agent framework where seller and buyer are identical and changes in asset prices level out to zero in aggregate. Since that, an increase in asset prices falls short to increase households' real spending opportunities.

worth. I assume that firms issue equity shares, can choose their net worth by deciding over dividends payed to shareholders. It is furthermore assumed that firms can also raise capital from shareholders. If firms seek to maximize the dividends per share, I show that under reasonable general assumptions asset prices are linked to the profit rates and are competitive among firms. This connects asset prices and return on equity, whereas optimality requires return on equity to equal the external finance premium. Return on equity in turn depends on price-setting which, in aggregate, determines the consumer price level. Thus, if firms maximize expected returns on equity, a link between asset prices and real activity is probable.

The second methodological contribution concerns the interaction of speculative and rational agents. Financial markets work fundamentally different than commodity markets. While commodities are a means to an end, notably consumption or production, people hold assets because of their expected return in the future. Hence, their value is directly expressible in monetary terms. Focussing on the resale value of a financial asset reveals that beliefs about prices are to a certain extent self-fulfilling. Furthermore, incentive schemes differ notably between financial assets and real goods. To illustrate, let us imagine a firm that has anticipated the current price correctly in the previous period. Then profits are maximal since the optimal production volume is chosen with respect to the firm's cost function. But if prices were overestimated, the firm will be unable to sell the produced stock profitably and incurs a loss. Asset markets work differently. If a positive price change has been overestimated by a trader, he will still realize a higher profit than a second trader that expected the price change correctly. Unlike commodity markets, traders in asset markets can benefit from overoptimistic forecasts at least in the short run. This can lead to herding behaviour: instead of focussing on the underlying fundamental, it can be *behaviorally rational* (Hommes, 2013) for traders to follow the majority in their beliefs. Such a mechanism is not well captured by rational expectations. To incorporate these anomalies and to allow for realistic asset price dynamics I make a distinction between expectations on *real economic aggregates* like output and inflation and expectations of financial market prices. While expectations on the real side of the economy are modeled to be perfectly rational, financial traders form boundedly rational expectations that embed speculative dynamics. This enable me to explicitly study the feedback between stock prices and real aggregates to derive implications for monetary policy. Restricting the bounded rationality to a small part of the model addresses the prominent critique of *the wilderness of bounded rationality* (Sims, 1980) while retaining a strong forward-looking component in the framework. It enable me furthermore to make use of nonlinear dynamic theory to identify both, qualitative and quantitative changes in dynamics depending on the central banks policy rule.

My third and central contribution is to study the role of monetary policy to stabilize the economy via asset price targeting using the model outlined above. The fundamental idea behind this concept is that the interest rate impacts on asset prices through the discount factor, i.e. a higher interest rate leads to a devaluation of assets. A different branch of the literature most prominently represented by Galí (2013) uses the concept of rational asset price bubbles to analyze the role of such policy. Apart from an ongoing dispute on whether such concept is plausible or not, the work of Assenmacher and Gerlach (2008) show using a VAR model that asset prices react almost instantaneously to the interest rate in support of the classic theory in which the interest rate matters through their role as the discount factor. Bernanke and Gertler (2000) use a model similar to mine where stock prices are represented as the price for capital and bubbles are exogenous. They provide the benchmark result that asset price targeting is rather harmful in terms of welfare.

Relation to other literature

Similar to my model, Winkler (2014) uses BGG-type frictions to combine asset prices and real activity, and empathizes the role of learning in an otherwise rational model. This enables to reproduce excess volatility of asset prices as well as a relatively high standard deviation of stock prices. As in my model, under rational expectations a monetary policy that targets asset prices induces a welfare-loss, while it is argued that under learning, carefully targeting asset prices might lead to a welfare improvement Miao et al. (2012) and Miao et al. (2016) build a Bayesian model with rational stock price bubbles which affect the economy through endogenous borrowing constraints. Similar to my model, the feedback between asset prices and asset price expectations plays a key role on the formation of a stock price bubble. Compared to this literature, the emphasis of my analysis lies on two effects: first, under which conditions a monetary policy that targets asset prices could stabilize financial markets and as such mitigate the intensity of spillovers? And secondly, whether and how it is possible to unlink economic aggregates from asset prices such that excess volatility would not matter for economic welfare. I show that this question is closely interrelated to this feedback loop, because asset price targeting can effectively mitigate the direct feedback of expectations on current prices, which is important independently of whether a bubble is due to bounded rationality or other mechanism.

	π	y	s
SD	.018	.014	.184
π	1	-0.358	-0.475
y	–	1	0.579
s	–	–	1

Table 1: Cross-correlations and standard deviations of inflation, output and real stock prices, Core-Europe from 1977 to 2014

Episodes with booms and busts are recurrent phenomena. In analysing the housing market and equity prices in industrialized economies during the postwar period, the IMF (2003) found that busts in both markets arise frequently (on average every 13–20 years) with entailed drops in prices averaging around 30% and 45% respectively. These busts are associated with losses in output that reflect declines in consumption and investment. Table 1 summarizes key statistics of European data and embedding a set of stylized facts:

i) While standard deviations of inflation and output are roughly on the same level, the standard deviation of asset prices is roughly 10 times in magnitude.

ii) Inflation is (weakly) countercyclical, i.e. negatively correlated with output.

iii) Stock market prices and output are positively correlated.

iv) Stock market prices and inflation are negatively correlated and as such more than inflation and output.

These stylized facts are well in line with empirical work, for instance by Campbell (1999) on stock prices and consumption. Sargent (2008) and Barro (1990) argue that stock prices are

[1]Monthly data from the OECD, stock prices are the MSCI-Europe index. Time series are HP-filtered with $\lambda = 10e5$. Prices in 2005, stock prices and output are divided by CPI.

a good indicator for investment and hence future output, which implies a lead-lag structure of asset prices and output. Winkler (2014) conducts a vector auto-regression (VAR) on asset price shocks. He finds that the response of total factor productivity (TFP) is insignificant or even negative, while the asset price shock has significant effects on investment. Thus, he concludes that the classical view that stock price changes reflect new information about productivity changes might be controversial. Abbate et al. (2016) report similar findings by using a time-varying FAVAR. To my best knowledge, the relationship between inflation and stock prices has not been subject to detailed studies yet.

The model used here adopts the general New Keynesian literature (Woodford, 2003, Galí, 2008). Financial frictions are implemented inspired by BGG, but with the simplification that labor is the only input factor. This allows to focus on the dynamics of agents' interaction and interconnection of asset prices and macroeconomic activity. Boundedly rational expectations gained popularity for being able to endogenously reproduce asset price bubbles by specifying a behaviorally intuitive and simple mechanism of expectation formation. Empirical regularities of stock market prices such as return volatility (Shiller, 1981), return predictability (Fama and French, 1988) and fat tails of asset price distributions are hard to explain unless one relaxes the hypothesis of rational expectations. There is a growing literature on behaviourally rational agents that have been introduced to macroeconomic modelling, specifically with respect to expectations on output and inflation. For an extensive overview see e.g. Evans et al. (2001). Mankiw et al. (2003), Branch (2004), Pfajfar and Santoro (2010) and Pfajfar and Santoro (2008) provide empirical evidence in support of *heterogeneous* expectations using survey data on inflation expectations. Hommes et al. (2005), Pfajfar and Zakelj (2012), Assenza et al. (2013) and Hommes (2011) find evidence of simple, heterogeneous forecasting mechanisms in laboratory experiments with human subjects. Taken this evidence as a starting point, heterogeneous expectations have found their way into macroeconomic modelling, see for instance Anufriev et al. (2008), Brazier et al. (2008), Tuinstra and Wagener (2007), Branch and McGough (2009), Branch and McGough (2010), De Grauwe (2011), Evans and Honkapohja (2003) and De Grauwe and Macchiarelli (2013). From the perspective of this literature, recessions are not due to shocks to fundamentals but rather to massive coordination failure.

As to theoretical insights on the link between asset markets and real macroeconomic activity, Martin and Ventura (2011) rely on Gertler et al. (2010) to create a linkage between credit volume, firms' value and real activity and implement the idea of *rational bubbles* (Martin and Ventura, 2010). Tallarini (2000), Rudebusch and Swanson (2012) and others explore the dynamics implied by Epstein-Zin preferences (Epstein and Zin, 1989), while a different branch of the literature followed the idea of habit-formation specifications (e.g. Abel (1990), Ljungqvist and Uhlig (2000, 2015)). Also see Kliem and Uhlig (2016) for a brief overview of the literature and the estimation of such type of model. Generally these methods report mixed success with fitting both, macroeconomic dynamics and asset price volatility. One of the few attempts to connect an agent-based stock market to the goods market via national income is conducted by Westerhoff (2012), where the author shows that this link has rather destabilizing effects. Greenwood and Shleifer (2014) provide a summary on survey data that documents the failure of rational expectations. According to this data, the rational expectations hypothesis can, al least concerning asset markets, almost always be rejected.

This paper is structured as follows. In Section 2 I formally present the macroeconomic part of the model and provide microfoundations for the mutual linkage between asset prices and macroeconomic aggregates. This model is used to study the equilibrium dynamics under rational expectations and to estimate and identify relevant parameter ranges in Section 3. I then endogenize fluctuations in asset prices in Section 4, where I also present simulation results and policy analysis. Section 5 concludes.

2 Model

The economy is populated by a continuum of identical households, a heterogeneity of firms, a financial intermediary and a monetary authority.

2.1 Households

Households are indexed by i. They face a standard problem of maximizing the expected present value of utility by deciding over consumption of a composite good C_t and time devoted to the labour market H_t. For every unit of labour supplied they receive the real wage W_t. Furthermore they can deposit monetary savings D_t at the financial intermediary. The maximization problem for individual agents is then

$$\max_{\{C_{i,t}\},\{H_{i,t}\},\{D_{i,t}\}} E_t \sum_{s=t}^{\infty} \beta^{s-t} \left(\frac{\zeta_{i,t} C_{i,s}^{1-\sigma}}{1-\sigma} - \xi \frac{H_{i,s}^{1+\gamma}}{1+\gamma} \right)$$

s.t. the budget constraint in real terms states

$$C_{i,t} + D_{i,t} \leq W_t H_{i,t} + R_t \frac{P_{t-1}}{P_t} D_{i,t-1} + \mu \int_0^{\bar{\omega}_t} \omega H_t / X_t dF(\omega) \ \forall t = 1, 2, ...$$

where $\mu \int_0^{\bar{\omega}_t} \omega H_t \ X_t dF(\omega)$ are the audition costs for defaulting wholesalers, which will be explained further below. Audition costs are, via the financial intermediary, distributed equally among households and do not enter optimality conditions. Each household is subject to an idiosyncratic preference shock $\zeta_{i,t}$. The composite consumption good consists of differentiated products from the retail sector and is sold in a monopolistically competitive market. The composite good and the aggregate price index for the consumption good are defined by the CES aggregators

$$C_t = \left(\int_0^1 C_{l,t}^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} \qquad \text{and} \qquad P_t = \left(\int_0^1 P_{l,t}^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}}.$$

Optimization yields the usual Euler equation and a clearing condition for the labor market

$$\zeta_t C_t^{-\sigma} = E_t \left\{ \beta R_{t+1} \frac{P_t}{P_{t+1}} C_{t+1}^{-\sigma} \right\} \tag{2.1}$$

$$\xi H_t^{\gamma} = \frac{\zeta_t W_t}{C_t^{\sigma}} \tag{2.2}$$

where ζ_t denotes the i.i.d. aggregate demand shock that is due to the idiosyncratic preference shocks. Since individual shocks are not observable for other agents, at time t the aggregate shock is not observable either. Given optimality, the budget constraint needs to hold as an equality and agents obey the transversallity condition

$$\lim_{s \to \infty} \beta^{s-t} E_t C_s^{-\sigma} D_s = 0.$$

Household deposits are given to the sector of financial intermediaries.

2.2 Firms

To maintain analytical tractability, firms are divided into a wholesale and retail sector. Wholesalers borrow money from the financial intermediary to finance production and their shares are traded at the stock exchange. Their (homogeneous) good is sold to the retail sector where diversification takes place and the then heterogeneous goods are sold to the households with monopolistic profits.

2.2.1 Wholesale Sector

Let labor be the only production factor and index wholesalers by j, then the CRS production function is

$$Y_{j,t} = \omega_{j,t} H_{j,t},$$

where $\omega_{j,t}$ is a firm-specific idiosyncratic productivity shock similar to the households' preference shock. A more careful definition can be found in the appendix. The negative correlation between stock prices and inflation indicates that if stock prices are relevant for the macro economy, it should be motivated through the firms' financial structure. Therefore I specify two additional assumptions:

i) Unlike creditors, shareholders can liquidate the firm at any time without costs.

ii) Since that, wholesalers maximize the expected future stream of dividends on equity.

To simplify the optimization problem I allow for negative dividends to be payed, which implies that firms can obtain financing resources from their shareholders as well. Shareholders will comply if expected future profits are appropriately high. This is a reasonable assumption since shareholders are willing to increase firms' equity to seize the opportunity of higher future profits. A similar approach is chosen by Martin and Ventura (2010) for aggregated investment. Wholesalers are price takers. Let X_t be the gross markup of retail goods over wholesale goods. Then equivalently X_t^{-1} is the relative price of wholesale goods. This implies that R_t^H, the gross return on employing one unit of labor, is given by

$$R_{t+1}^H = (X_t W_t)^{-1}, \tag{2.3}$$

where the reciprocal definition of X_t ensures $R_t^H > 1$, a necessary condition for positive external finance. Let us denote firm j's equity by $N_{j,t}$. The expected return on equity implied by the stock price, $E_t R_{t+1}^S$, is then $\frac{R_{t+1} S_{j,t}}{N_{j,t}}$, given no arbitrage. Goods are produced and sold in the current period, but returns are realized at the beginning of the next period. Then firms decide upon their equity and distribute the rest as dividends Θ_{t+1}. Finally, the firm's shares are traded at the stock exchange. As shown further below, given no-arbitrage the price $S_{j,t}$ of one share of the firm needs to satisfy $S_t = E_t \frac{\Theta_{t+1} + S_{t+1}}{R_{t+1}}$ or equivalently

$$S_t = E_t \sum_{s=t}^{\infty} \prod_{l=t}^{s} R_{l+1}^{-1} \Theta_{s+1} \tag{2.4}$$

where the normalization of the number of shares to unity is implied. Dividends in period t are composed of $\Theta_t = H_{t-1}/X_{t-1} - N_t$, and N_t is the amount necessary to finance production costs

$N_t = W_t H_t$.

Simple example without external finance: The link between stock prices and return per unit of labor R^H_{t+1} can be explained more intuitively in a world without external finance. Hence, let me briefly abstract from external finance. Recall that every period firms choose how much of their returns to retain and how much to distribute. The Lagrangian is:

$$\max_{\{H_t\},\{N_t\},\{\lambda_t\}} E_t \sum_{s=t}^{\infty} \prod_{l=t}^{s} R^{-1}_{l+1} \left[H_{s-1}/X_{s-1} - N_s\right] - \lambda_s \left(W_t H_s - N_s\right).$$

The first-order condition is $H_t/X_t = N_t R_{t+1}$, which combined with the definition of expected future dividends gives $E_t \Theta_{s+1} = R_{t+1} N_t - E_t N_{t+1}$. Inserting this result into Equation (2.4) implies that stock prices reflects the value of equity perfectly:

$$S_t = \frac{R_{t+1} N_t - E_t N_{t+1} + \frac{E_t\{R_{t+2} N_{t+1}\} + \dots}{R_{t+2}}}{R_{t+1}} = N_t.$$

It follows that the optimal labor demand $H_t = R_{t+1} S_t X_t$ is determined by the prices prevailing in the financial market in combination with wholesale prices and the economies interest rate. Once we drop the assumption that expectations on financial market prices are perfectly rational, this can lead to coordination failure.[2]

Full model: Let me now return to the wholesalers' problem with external finance. The volume of external finance demanded is firms' working capital $W_t H_{j,t}$ minus equity, hence

$$B_{j,t} = W_t H_{j,t} - N_{j,t}. \tag{2.5}$$

Regarding the external finance premium I follow the lines of the BGG financial accelerator mechanism closely, where the borrowing process follows a costly state verification (CSV) approach. I follow a similar mechanism in the appendix to establish that the interest rate on loans from the intermediary, denoted by R^B_{t+1}, is a risk-premium on the prevailing interest rate which depends on the individual firm's leverage,

$$R^B_{j,t+1} = z\left(\frac{N_{j,t}}{W_t H_{j,t}}\right) R_{t+1}$$

with $\frac{\partial z}{\partial N_t} < 0$. Intuitively, when the leverage ratio decreases, the premium on external finance falls because the amount of collateral increases and the loan becomes less risky. I show in the appendix that optimality requires the return on assets to be equal to the rate paid on external funds, $R^S_{t+1} = R^B_{t+1}$. Otherwise wholesalers would have an incentive to increase or decrease the

[2] Another perspective is that the expected return implied by asset prices is

$$R^s_{t+1} = \frac{S_t}{N_t} R_{t+1}$$

which in combination with Equation (2.3) gives us the same pricing equation for the simplified problem without external finance. Note that without external finance $N_t = H_t/X_t$, i.e. equity and working capital are the same. Under rational expectations this implies $\frac{S_t}{N_t} = 1$.

borrowing volume. Similarly to the example above, an increase in S_t will also have an increasing effect on equity N_t. Hence, once a functional form of $z(\cdot)$ is known, we can use

$$\frac{S_{j,t}}{N_{j,t}} = z\left(\frac{N_{j,t}}{W_t H_{j,t}}\right)$$

to eliminate N_t. Since $R^S_{t+1} = R^B_{t+1}$, these returns also need to equal R^H_{t+1}. Plugging the result into Equation (2.3), substituting for W_t and H_t and log-linearising the result gives us an aggregate representation of the price X_t for wholesale goods:

$$x_t = -\eta y_t - i_t + \nu s_t.$$

A competitive market for wholesale goods implies equal prices. Likewise, all firms have to offer the same R^S_{t+1}. This implies that the stock market evaluation of shares determines the amount of equity and profits. In general equilibrium, to comply to market forces and implicit expectations on future dividends, relative prices have to rise. This mechanism can be summarized by *the pressure to perform* combined with the fact that managers can not distinct whether aggregate stock prices are overvalued or not.

Note that this result contains two effects: an increase in stock prices puts pressure on firms to increase prices. In an economy with constant marginal costs, firms' only chance to increase profits per labor unit, is to raise the price level. But an increase in stock prices also decreases the leverage ratio, which lowers the cost for external finance and lets prices decrease. In this model, the second effect prevails as intuition suggests.

2.2.2 Retailers

Retailers buy the homogeneous good $Y_{j,t}$ from entrepreneurs and differentiate to sell it in a monopolistic competitive consumer market. This implements the Calvo pricing mechanism as it is standard in the literature. For details on the solution given the markup X_t, see Bernanke et al. (1999). Letting resellers be denoted by l, it can be shown that setting the optimal price P^*_t, given the corresponding demand $Y^*_{l,t}$, satisfies

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,k} \left(\frac{P^*_t}{P_{t+k}}\right)^{-\epsilon} Y^*_{l,t+k} \left[\frac{P^*_t}{P_{t+k}} - \left(\frac{\epsilon}{\epsilon-1}\right) X_t^{-1}\right]\right\} = 0. \tag{2.6}$$

Taking into account that each period the fraction θ of retailers is not allowed to change prices in period t, the aggregated price level follows

$$P_t = [\theta P_{t-1}^{1-\epsilon} + (1-\theta)(P^*_t)^{1-\epsilon}]^{\frac{1}{1-\epsilon}}$$

where P^*_t needs to satisfy Equation (2.6). Log-linearizing the combination of both equations yields the Phillips Curve (3.1) depending on the log-linearised markup x_t.

2.3 Financial Intermediation

There is a continuum of financial intermediaries indexed by k. Each of them takes the deposits $D_{k,t}$ received from households as given and invests a fraction in the stock market by holding a $J_{k,t}$-share of stocks at the real stock price S_t and issues the rest as credit volume $B_{k,t}$ to the wholesalers. I assume that investment in the financial market is done by traders that are each associated with a financial intermediary. Furthermore the intermediary has access to central bank money for which he will have to pay the real central bank rate R_{t+1}. Next periods' real dividends net of seized collateral are expected to be $E_{k,j}\Theta_{t+1}$. Market clearing requires:

$$R^D_{t+1}D_{k,t} = \hat{E}_{k,t}[P_{t+1}\Theta_{t+1} + P_{t+1}S_{t+1}]J_{k,t} + z^{-1}R^B_{t+1}B_{k,t}$$

subject to the constraint $D_{k,t} \geq P_tS_tJ_{k,t} + B_{k,t}$. From the fact that the opportunity costs of finance are given by the central bank interest rate, optimality requires $R^D_{t+1} = z^{-1}R^B_{t+1} = \frac{\hat{E}_t\{P_{t+1}\Theta_{t+1}+P_{t+1}S_{t+1}\}}{P_tS_t} = R_{t+1}$. In case of homogeneous and rational expectations the pricing equation can be aggregated straightforwardly:

$$R_{t+1}P_tS_t = E_t\{P_{t+1}\Theta_{t+1} + P_{t+1}S_{t+1}\}. \tag{2.7}$$

Let a capital letter without time subscript denote the respective steady-state value. In equilibrium, Θ is a function of the markup X and total output Y. $\frac{\partial\Theta}{\partial Y}$ depends on the labor-income-to-profit ratio and is here set to unity. Note that, when log-linearising equation (2.7)) the coefficient of $(1-\beta)$ of E_ty_{t+1} is very small. The log-linear version of the asset pricing equation is thus almost independent of expectations on next periods' output and markup. This yields the following expression for the percentage deviation of stock prices from their steady state value, where r_{t+1} denotes the real interest rate $r_{t+1} \equiv i_{t+1} - E_t\pi_{t+1}$:

$$s_t = (1-\beta)E_ty_{t+1} + \beta E_ts_{t+1} - r_{t+1}.$$

2.4 Central Bank and Government

The central bank follows a standard contemporaneous Taylor Rule. Additionally I allow to adjust the nominal interest rate i_{t+1} in response to the real stock market prices s_t:

$$i_{t+1} = \phi_\pi\pi_t + \phi_s s_t.$$

A policy that increases the nominal interest rate when stock market prices increase I will call *asset price targeting* (APT). Asset price targeting is my only additional policy measure that is explicitly implemented in the model. One problem the monetary authority faces when responding to movements in stock prices is that it is not ex-ante identifiable whether a deviation in asset prices represents a shift in fundamentals or not. In order to establish a practicable mechanism, the central bank hence must *always* react to price movements in terms of deviation from steady state, independently of whether these are identified as bubbles or a correct anticipation of real future movements. I furthermore abstract from governmental expenditures here and assume that the government issues no debt.

3 General Equilibrium and Estimation

In this section it is assumed that all expectations are formed homogeneously and rationally. The linearized economy is characterised by the following set of equations:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + v_t^\pi, \tag{3.1}$$
$$y_t = E_t y_{t+1} - \sigma^{-1} r_{t+1} + v_t^y, \tag{3.2}$$
$$x_t = \eta y_t + i_{t+1} - \nu s_t, \tag{3.3}$$
$$s_t = (1-\beta) E_t y_{t+1} + \beta E_t s_{t+1} - r_{t+1} \tag{3.4}$$
$$i_{t+1} = \phi_\pi \pi_t + \phi_s s_t, . \tag{3.5}$$

Shock terms and the real interest rate are given by

$$v_t^\pi = \rho_\pi v_{t-1}^\pi + \varepsilon_t^\pi, \qquad \varepsilon_t^\pi \sim N(0, 0.1)$$
$$v_t^y = \rho_y v_{t-1}^y + \varepsilon_t^y, \qquad \varepsilon_t^y \sim N(0, 0.1)$$
$$r_{t+1} = i_{t+1} - E_t \pi_{t+1}.$$

Equation (3.1) is the New-Keynesian Phillips curve which links inflation π_t to the markup x_t. Equation (3.2) is referred to as the dynamic IS-curve. Equation (3.4) states the no-arbitrage condition for the stock market. If stock prices do not feed back to the real economy, this equation would not contain any relevant information for macro dynamics. The major difference to the standard New Keynesian model thus lies in Equation (3.3) which implements the financial accelerator and outside-option effect discussed in Section 1. It imposes a relationship between output, markup, nominal interest rate and stock market prices. Stock market prices can be thought to act on the market return on investment and impact the firms' pricing decision, optimal leverage and, in turn, the external finance premium. This also establishes the linkage between the textbook model and the BGG-type credit frictions where ν is the price elasticity of the markup with respect to stock market prices.[3] This also nests the Woodford-Type model as a special case where $\nu = 0$. For stock prices to be relevant for the macroeconomy it is hence necessary that $\nu \neq 0$, since otherwise this equation collapses to the markup x_t simply being a fraction of wages w_t and the interest rate. Equation (3.5) is the Taylor rule with inflation and asset price targeting.

$v^y \sim N(0, \sigma_y)$ represents the aggregate of individual preference shocks ζ_t and translates to a demand shock. Since individual preferences are not publicly observable, the realization of the shock is not ex-post observable. $v^\pi \sim N(0, \sigma_\pi)$ is an aggregate productivity shock that results from idiosyncratic productivity shocks to wholesalers. Similar as to the demand shock, v^π is not observable in the aggregate since it affects producers individually. Once I deviate from the assumption of rationality, the non-observability of both shocks is an important ingredient of my model. Both shocks follow an AR(1) structure with ρ_π and ρ_y respectively.

Equations (3.1) to (3.4) can be represented as a 3-dimensional system of the endogenous

[3] Note that (3.3) can also be interpreted as to incorporate the idea that creditors use stock prices as a proxy for firms' collateral. Given $\bar{\nu}$ both mechanisms lead to the same model with only marginally different parameterization.

variables π_t, y_t and s_t:

$$\underbrace{\begin{bmatrix} 1-\phi_\pi\kappa & -\kappa\eta & \kappa\nu \\ \phi_\pi\sigma^{-1} & 1 & \phi_s\sigma^{-1} \\ \phi_\pi & 0 & 1+\phi_s \end{bmatrix}}_{\mathbf{M}} \underbrace{\begin{bmatrix} \pi_t \\ y_t \\ s_t \end{bmatrix}}_{\mathbf{x}_t} = \underbrace{\begin{bmatrix} \beta & 0 & 0 \\ \sigma^{-1} & 1 & 0 \\ 1 & 1-\beta & \beta \end{bmatrix}}_{\mathbf{P}} \underbrace{\begin{bmatrix} E_t\pi_{t+1} \\ E_t y_{t+1} \\ E_t s_{t+1} \end{bmatrix}}_{E_t\mathbf{x}_{t+1}} + \underbrace{\begin{bmatrix} v_t^\pi \\ v_t^y \\ 0 \end{bmatrix}}_{\boldsymbol{v}_t}. \tag{3.6}$$

The matrix $\mathbf{N} = \mathbf{M}^{-1}\mathbf{P}$ and particularly the eigenvalues of $\mathbf{N}$ take a non-trivial form if expressed as a mapping from the parameter space.

Estimation and Identification

Some of the deep parameters are fixed to values that are commonly found in the literature. I let $\beta = 0.99$, representing the short-term perspective of a quarterly model and, in accordance, set the shocks' autocorrelation to $\rho_\pi = 0.9$ and $\rho_y = 0.7$ respectively. Other values are consistent with the calibration of BGG, as I pick $\gamma = 0.3$ and $\omega = 0.66$. Then $\eta = \frac{\sigma+\gamma+\tilde{\nu}}{1-\tilde{\nu}} \approx 1.3$ and $\kappa = (1-\omega)(1-\beta\omega)/\omega \approx 0.086$. $\bar{\nu}$, the elasticity of the external finance premium with respect to net worth determines the elasticity of marginal costs to changes in stock prices, is defined by $\nu = \frac{\tilde{\nu}}{1-\tilde{\nu}}$. The central banks policy in the baseline setup is described by $\phi_\pi = 1.3$ and ϕ_s, the response in interest rate with respect to stock prices, is set to zero implying that the central bank does not target stock prices when setting the policy rate.

To identify parameter values for the benchmark model with rational expectations I make use of a grid-based minimization technique that is explained in detail in the appendix. The underlying intuition of this technique is to find the parameters of the global minimum of a distance measure between the simulated moments and those presented in Table 1.

I furthermore also make use of Bayesian estimation, considering a cost-push shock as well as an exogenous shock on asset prices. These results are redirected to the appendix for the following reasons. First, once I extend the model by a nonlinear speculative process the use of Bayesian estimation is not feasible since finding the likelihood function of a model potentially embedding deterministic components is highly nontrivial. Hence, if I want to create a benchmark calibration it is advisable to use the same calibration technique when identifying the rational expectations model. Secondly, Bayesian estimation targets more statistical moments than the ones considered here, namely the autocorrelation parameters of endogenous variables. Since the model presented here tends to be too simplistic to explain the data in richer detail, I rather target specific important moments than the whole spectrum and profit from the simplicity when identifying economic mechanisms. Thirdly, since several of the model parameters are close to the boundaries of determinacy, some priors have to be chosen to be relatively tight. Furthermore, for reasonable values of risk aversion σ stock prices and output display a relatively similar response to changes in interest rates. Since the data correlation between stock prices and output is relatively strong, it is difficult to identify the direction of causality, i.e. whether a change in output is induced by asset prices that shift down marginal costs or directly by the interest rate setting. This leads to an overall poor identification of parameter values when using Bayesian estimation, which, regardlessly, confirms the parameterization above.

Table 2 shows the parameter values estimated in the grid-based minimization procedure. Standard deviations necessary to match the data are lower than in the Bayesian estimation since less moments are targeted. A further important result is that ν is significantly positive, a fact which I will establish more profoundly further below. Table 3 reveals two key problems

ν	σ_π	σ_y	σ_x	ρ_x
0.23	0.001	0.030	0.005	.90

Table 2: Parameter estimates of RE model

	π	y	s
SD	0.077	0.015	0.152
π	1	-0.484	-0.999
y	–	1	0.517
s	–	–	1

Table 3: Standard deviations & Covariances of RE model

of the rational expectations based approach. First, as is well known, it is impossible to match standard deviations properly. Asset prices are mainly driven by fluctuations in the interest rate, which in turn depend on deviations in inflation from the central banks target. Hence, unrealistically strong fluctuations in inflation are necessary to replicate the standard deviation of asset prices. Secondly, this leads to a very high correlation between inflation and stock prices, which is also not supported by the data. The only reason why stock prices and output are correlated is the existence of a feedback between stock prices and output, i.e. a positive ν.

In the next section I will show how to significantly improve the data fit by introducing speculative dynamics.

4 Endogenous Fluctuations in Asset Prices

In this section I provide intuition why the excess volatility in stock prices, that is necessary to match the stylized facts from Table 1, can be well explained by deviations in expectations about stock prices. However, because asset returns are composed of a payoff and the reselling value, there is reason to assume that expectations in asset prices are more likely to be prone to deviate from the rational expectations hypothesis, which introduces the mechanism that creates excess volatility in stock prices that is necessary to reproduce the data.

Let me now drop the assumption that stock market expectations are fully rational and introduce speculative behavior at the financial market. This is well in line with the research on bounded rationality given different feedback systems, which is summarized by Boehl and Hommes (2017). They find that in markets with negative or weakly positive feedback boundedly rational agents are quickly driven out of the market. High or even exploding positive feedback alone however does not suffice to ensure the long-term survival of boundedly rational agents, but it is also necessary that the payoff function takes a form that is non-proportional to the inverse of the forecasting error. In the model presented here, these conditions are only satisfied in the stock market. Hence agents behave rationally on the commodity and labor market, but not necessarily on the stock market.

The fact that all markets other than the stock market are dominated by rational agent also preserves the forward looking nature of the model that comes along with the rational expectations structure. Let us denote the model consistent rational expectations on inflation and output by $E_t\pi_{t+1}$ and $E_t y_{t+1}$ using the rational expectations operator. Let speculative expectations of stock market prices be denoted by $\hat{E}_t s_{t+1}$.[4] This demands a mechanism of how rational agents deal with the existence of agents that form different, non-rational beliefs. Let me assume that the distribution of agent types is unobservable. Than, rational agents are not

[4] $\hat{E}_t$ here is rather a function than a mathematical operator.

aware of the presence of non-rational agents.[5] Since aggregate shocks are unobservable, rational agents perceive fluctuations induced by speculation as part of this exogenous noise. Let me define $\mathbf{\tilde{v}_t}$ to be the *perceived* exogenous shocks, which, as I will show, actually depend on the real exogenous shocks and the degree of financial market speculation.

Letting $E_t\left[\mathbf{x_{t+1}}|\mathbf{\tilde{v}_t}\right]$ denote the rational expectations solution of (3.6) in terms of these perceived shocks, then I have to find a solution for

$$\mathbf{M}\mathbf{x}_t = \mathbf{P}\begin{bmatrix} E_t\left[\pi_{t+1}|\mathbf{\tilde{v}_t}\right] \\ E_t\left[y_{t+1}|\mathbf{\tilde{v}_t}\right] \\ \hat{E}_t s_{t+1} \end{bmatrix} + \mathbf{v_t}. \tag{4.1}$$

$\mathbf{v_t}$ here denotes the actual stochastic shocks v_t^{π} and v_t^{y}. Let me assume rational agents are New-Keynesians and do not think that asset prices play a role, which is not only consistent with the vast majority of the literature but also an hypothesis that cannot be rejected by Bayesian estimation.

The perceived law of motion for rational agents is

$$\begin{bmatrix} \mathbf{P} & \mathbf{0_{3\times 2}} \\ \mathbf{0_{2\times 3}} & \mathbf{I_{2\times 2}} \end{bmatrix} E_t \begin{bmatrix} \mathbf{x_{t+1}} \\ \mathbf{\tilde{v}}_{t+1} \end{bmatrix} = \begin{bmatrix} \mathbf{M} & \mathbf{0_{3\times 2}} \\ \mathbf{0_{2\times 3}} & \boldsymbol{\rho} \end{bmatrix} \begin{bmatrix} \mathbf{x_t} \\ \mathbf{\tilde{v}}_t \end{bmatrix}, \tag{4.2}$$

where $\boldsymbol{\rho}$ is a diagonal matrix containing the autocorrelation parameters and $\mathbf{x_t}$ the vector of endogenous variables at t. This is a different way to write the system in (3.6) but with perceived exogenous shocks instead of the real exogenous shocks. In the appendix I derive this system and use eigenvector-eigenvalue decomposition to find the (linear) rational expectation solution to (4.2). Let this linear solution be denoted by the matrix $\mathbf{\Omega}$. It needs to hold by definition that

$$\begin{bmatrix} \pi_t \\ y_t \end{bmatrix} = \mathbf{\Omega}\begin{bmatrix} \tilde{v}_t^{\pi} \\ \tilde{v}_t^{y} \end{bmatrix} \qquad \text{and} \qquad \begin{bmatrix} E_t[\pi_{t+1}|\mathbf{\tilde{v}_t}] \\ E_t[y_{t+1}|\mathbf{\tilde{v}_t}] \end{bmatrix} = \mathbf{\Omega}\boldsymbol{\rho}\begin{bmatrix} \tilde{v}_t^{\pi} \\ \tilde{v}_t^{y} \end{bmatrix}. \tag{4.3}$$

It follows directly that we can express the conditional expectations on inflation and output without explicitly solving for the perceived shocks $\mathbf{\tilde{v}_t}$:

$$\begin{bmatrix} E_t[\pi_{t+1}|\mathbf{\tilde{v}_t}] \\ E_t[y_{t+1}|\mathbf{\tilde{v}_t}] \end{bmatrix} = \mathbf{\Omega}\boldsymbol{\rho}\boldsymbol{\tilde{v}_t} = \mathbf{\Omega}\boldsymbol{\rho}\mathbf{\Omega^{-1}}\begin{bmatrix} \pi_t \\ y_t \end{bmatrix}$$

I am plugging this result for the expectations on output and inflation back into (4.1) and rearrange in terms of the actual exogenous states and speculative expectations on the stock market to obtain the actual law of motion. The latter can then be expressed as a mapping $\mathbf{\Psi} : (\boldsymbol{\rho}, \mathbf{\Phi}, \boldsymbol{\phi}) \to \mathbb{R}_{3\times 3}$, where $\mathbf{\Phi}$ is the set of model parameters $(\beta, \sigma, \nu, \eta, \kappa)$ and $\boldsymbol{\phi}$ the two policy parameters:

$$\begin{bmatrix} \pi_t \\ y_t \\ s_t \end{bmatrix} = \mathbf{\Psi}\begin{bmatrix} v_t^{\pi} \\ v_t^{y} \\ \hat{E}_t s_{t+1} \end{bmatrix}.$$

[5] Boehl and Hommes (2017) show that in a system where fully rational and boundedly rational agents coexist, the type of dynamics is similar but *even more unstable* than in systems where rational agents are unaware of the presence of boundedly rational agents.

This represents a solution for the rational expectations equilibrium in terms of the real shock terms with one degree of freedom, which is used for boundedly rational beliefs $\hat{E}_t s_{t+1}$. Note that this actual law of motion, by definition, is not known to any of the agents.

Recall that the Blanchard and Kahn (1980) condition for *determinancy* of the rational expectation solution requires that the number of forward looking variables coincides with the number of eigenvalues of $\mathbf{N} = \mathbf{P}^{-1}\mathbf{M}$ that are lying outside the unit circle. Since the eigenvalues of $\boldsymbol{\rho}$ will always lie inside the unit circle, all three eigenvalues of $\mathbf{N}$ need to be lying *outside* the unit circle. I use numerical calculus to determine the modulus of the eigenvalues of $\mathbf{N}$. Using parameter values from Table 4, these numerical results can be found in the appendix. The system is *determinate* for values of ϕ_π larger than one (which corresponds to standard findings in the literature). To ensure determinacy, ϕ_s is bounded by $[-0.116, 0.465]$ with the interval decreasing in ν. Note that I explicitly do not rule out negative values of ϕ_s to remain agnostic concerning an optimal policy. Naturally, these intervals are the boundaries for the following policy experiments, i.e. I am not considering rational sunspots in the expectations on inflation or output.

Which policy implications can we deduct from this model without further specification of a mechanism for expectation formation on the financial market? In the absence of real shocks the law-of-motion in (4.3) can be reduced to

$$\mathbf{x}_t = \boldsymbol{\Psi}_{\mathbf{:,3}}\hat{E}_t s_{t+1} \qquad \text{and in particular} \qquad s_t = \boldsymbol{\Psi}_{\mathbf{3,3}}\hat{E}_t s_{t+1}.^6 \tag{4.4}$$

Let us consider the calibration from Table 2 and for now disregard any exogenous shocks in order to focus on the impact of a deviation in stock price expectations.[7]

Given this calibration $\boldsymbol{\Psi}_{3,3} > 1$, i.e. the coefficient of the law-of-motion for s_t with respect to expectations on next periods asset prices is larger than one. Learning-to-forecast experiments[8] have shown that systems with positive feedback, especially when close to unit roots, can exhibit large swings and bubbles. From a behavioral perspective it seems quite demanding that economic agents are able to successfully calculate *other* agents' rational expectations solution to find one's own, especially when speculation, short term profits and animal spirits are involved. Hence, encountering such a system is likely to indicate further issues with dynamic instability. Even though the rational expectation solution is determinate, it is rather unlikely that traders will be able to coordinate on the equilibrium and, presumably, will instead be led by the exploding positive feedback loop implied by Equation (4.4). A system of this form, with a coefficient larger than one, is classically called a *exploding feedback loop* once expectations have a strong

[6] I implement the model in Python. I would like to empathize the excellence of contemporary free and open source software, also and especially in comparison with proprietary software. I also want to encourage the reproducibility of research by providing the source code of my work which is available upon request. In the rest of this paper I use *Python-like* notation when referring to certain parts of matrices. So if

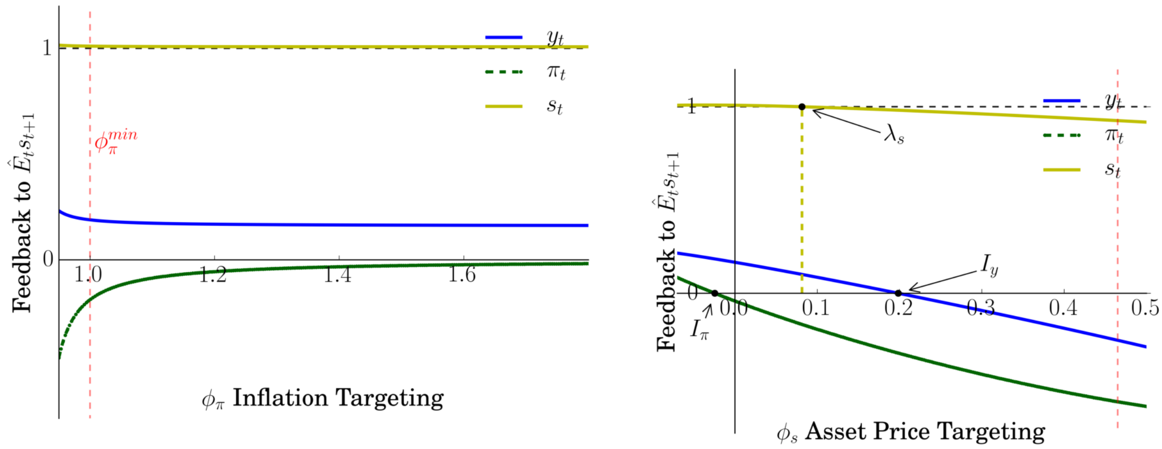
$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix},$$

I use $\mathbf{A}_{2:3,1:2}$ to denote the lower-left square matrix (row 2 to 3, column 1 to 2) $\begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$ or $\mathbf{A}_{3,:}$ to denote the vector in the third row of $\mathbf{A}$ given by $\begin{bmatrix} a_{31} & a_{32} & a_{33} \end{bmatrix}$.

[7] $\tilde{\nu} = 0.23$ lies well in the 90% interval of the estimation and, compared to the value of 0.5 used in BGG, can be considered a conservative calibration.

[8] For a review on laboratory experiments on expectation formation, see Hommes (2011).

backward-looking component. Hence, when stabilizing such system it should *ceterus paribus* be the policy makers' aim to minimize $\mathbf{\Psi_{3,3}}$. Likewise, the second best solution would be to minimize $\mathbf{\Psi_{1,3}}$ and $\mathbf{\Psi_{2,3}}$ and thereby minimizing the impact of stock prices on real activity. Following this line of argument, $\mathbf{\Phi_{3,3}}$ represents a key measure for the probability of excess volatility on the stock market.



(a) $\mathbf{\Psi_{:,3}}$ as a function of ϕ_π, no asset price targeting ($\phi_s = 0$).

(b) $\mathbf{\Psi_{:,3}}$ as a function of ϕ_s, $\phi_s = 1.3$.

Figure 1: Direct responses of output, inflation and asset prices to a 1% change in asset price expectations as a function of central bank policy parameters. Plots for different values of central bank parameters. Responses in deviation from steady state.

Figure 1 shows the values in $\mathbf{\Psi_{:,3}}$ as a function of policy parameters. These can be interpreted as the general equilibrium response of endogenous variables to a one-percent increase in stock price expectations. An increase in ϕ_π (left panel) has quite moderate impact on the system dynamics, with almost flat curves in the relevant interval. Most noteworthy, the response of s_t with respect to $\hat{E}_t s_{t+1}$ is almost constant in inflation targeting ϕ_π, and always larger than one. However, the system changes more drastically with changes in ϕ_s, with values of $\mathbf{\Psi_{3,3}} < 1$ for $\phi_s > \lambda_s \approx 0.09$. This means that if the central bank reacts moderately to stock prices, the positive feedback loop can be mitigated. However, lab experiments suggest that near-unit root dynamic processes are still prone to instability. The diagram reveals two other interesting points. At I_y the impact of speculation on output is exactly offset and for higher values of ϕ_s a positive shock on asset prices actually leads to a decrease in output. This is due to a stronger increase in the interest rate that dampens consumer demand. A similar point although for a negative value of ϕ_s, is I_π. Here the inflation rate is completely unaffected by the immediate impact of a deviation in stock price expectations. The intuition behind these diagrams will be discussed further in detail below. The diagram also reveals another fact, i.e. boundedly rational deviations in asset price expectations can contribute to high ratio of standard deviations of stock prices and output, σ_s/σ_y.

The above results imply that concerning financial stability, the central bank faces a trade-off. They can target to lower the impact of $\hat{E}_t s_{t+1}$ on either π_t or y_t, but accept the extreme dynamic feedback induced by a high $\mathbf{\Psi_{3,3}}$ and its implications on the stability of the nonlinear expectations system. Or policy makers can choose to potentially stabilize the system but increase the impact of stock market expectations on real variables notably. After the analysis of the expectation-feedback system I will take a closer look at this trade-off. For this purpose it is necessary to analyse the explicit dynamics under speculation and to provide numerical results for the central bank policy. For that purpose I implement a concrete mechanism on how financial market traders form expectations $\hat{E}_t s_{t+1}$.

Heterogeneous beliefs

I assume traders to follow the *Heterogeneous Agent Switching Model* (Brock and Hommes, 1998) and let them endogenously choose between simple forecasting heuristics. This model has widely been used in the literature of nonlinear dynamics and expectation formation, and provides a set of empirically relevant properties. Most notably it incorporates the fact that agents can run short-term profits by following simple prediction strategies that were successful in the recent past, and these traders are able to outperform others that believe that the price will return to the rational expectations equilibrium. It also embeds a nonlinear law of motion for asset prices that, most notably, can reproduce a positive correlation between returns and expected returns, and fat tails of the distribution of asset prices. For an intuition of the model dynamics and empirical validation through laboratory experiments see Hommes (2006). Boehl and Hommes (2017) show that in financial markets as modeled here, the efficient market hypothesis does not hold and, given the existence of positive speculative profits, agents that do not act perfectly rational are not necessarily driven out of the market.

Traders are heterogeneous in their forecasting rules. Let there be $H > 1$ predictors of future prices and let each predictor $h = 1, 2, \dots, H$ be of the form $\hat{E}_{t,h} s_{t+1} = g_h s_{t-1} + b_h$. I aggregate over each individual optimality condition (Equation 2.7) to derive the economy wide price for shares S_t.[9] If I let $n_{t,h}$ denotes the fraction of traders using predictor h at time t, then:

$$R_{t+1}\hat{E}_t \left\{ \frac{P_t}{P_{t+1}} \right\} S_t = E_t \Theta_{t+1} + \sum_h n_{t,h} \hat{E}_{t,h} S_{t+1}.$$

Let us also assume that traders take the real interest rate r_{t+1} as given. Log-linearization again yields

$$s_t = \beta \hat{E}_t s_{t+1} - r_{t+1} \qquad \text{with} \qquad \hat{E}_t s_{t+1} = \sum_h n_h \hat{E}_{h,t} s_{t+1}. \tag{4.5}$$

Not surprisingly, the first part here is identical to Equation (2.3) but the second part incorporates the speculative expectations $\hat{E}_t s_{t+1}$. Note that r_{t+1} in turn depends on y_t and π_t, so this equation yet takes the general equilibrium effect of changes in stock market prices into account. Fractions $n_{h,t}$ are updated according to a *performance measure* $U_{h,t}$ of predictor h in period t. As such I chose *realized past profits*:

$$U_{h,t} = (\beta s_t - s_{t-1})(\beta \hat{E}_{t-1,h} s_t - s_{t-1}). \tag{4.6}$$

The choice of the performance measure is an essential ingredient of the model. It determines the properties of the nonlinear part of the dynamic system. Realized profits from trading

[9] I assume that trader k's demand for shares is a linear function of expected profits with some τ:

$$J_{k,t} = \tau \hat{E}_{k,t}\{\Pi_{t+1} + S_{t+1} - R_t \frac{P_t}{P_{t+1}} S_t\}$$

with $\int_0^1 j_t^k dk = 0$ when expressed as log-deviations from steady state.

ν	σ_π	σ_y	γ	α
.11	.002	.008	1.29	1.2

Table 4: Parameter estimates

qualifies in several ways for our purpose. As I have outlined above, the fundamental difference between macroeconomic real markets and stock markets is that participants can make profits from speculation. Instead of being rewarded for an accurate estimate of the price, it is sufficient to *get the sign right*, hence to decide whether to go short or long. Likewise, a trader A that has a high forecast of next periods' prices will invest more money in the asset than some trader B with a relatively lower forecast. If it then turns out that B was correct in terms of point estimates, A will still realize higher profits since he invested more. Precisely this feature is captured by Equation (4.6).[10] Realized past profits (Equation 4.6) cannot be log-linearized around the steady state because steady state profits will always be 0: each agent will demand the same amount. Therefore I use the nonlinear form and adjust for the fact that s_t is a measure for log-deviation from steady state. The approximation error is considerably small. Furthermore I use the constant discount factor β instead of the nominal interest rate to simplify the analysis.[11]

The probability that predictor h is chosen is given by the *multinomial discrete choice model*

$$n_{h,t} = \frac{e^{U_{h,t-1}}}{Z_{t-1}} \qquad \text{and} \qquad Z_{t-1} = \sum_{h=1}^{H} e^{U_{h,t-1}}. \tag{4.7}$$

Next I specify the different predictors. Consider a simple 3-type model where one type of agents are fundamentalists and the other two share a trend-following parameter γ and are either negatively or positively biased by α:

$$\begin{aligned} \hat{E}_{t,1} s_{t+1} &= 0, \\ \hat{E}_{t,2} s_{t+1} &= \gamma s_{t-1} + \alpha, \\ \hat{E}_{t,2} s_{t+1} &= \gamma s_{t-1} - \alpha. \end{aligned} \tag{4.8}$$

Now that expectation formation mechanisms for both types of agents are given, the model is fully specified. It consists of a linear part, associated with the economy and the formation of rational expectations and represented by Equation (4.3), and a nonlinear mechanism for boundedly rational expectation formation given by $\hat{E}_t s_{t+1}$ and the performance measure $U_{h,t}$ (Equations 4.5 and 4.6), the fractions $n_{h,t}$ and the normalization factor (Equation 4.7), and the predictors (Equation 4.8).

The remaining task is to assign values to the set of parameters for the boundedly rational expectation formation mechanism, $\{\gamma, \alpha\}$. Since it is not possible to use standard optimization techniques to find the parameter setup with the best fit, I use a grid method in combination

[10] τ, the demand sensitivity with respect to risk aversion, runs together with the sensitivity of choice, which normally is an integral part of the model. Since the intensity of choice does not have a measurable empirical counterpart, the product of both parameters is normalized to one here. This is to limit the degrees of freedom of the estimation, furthermore the other behavioral parameters can compensate this limitation fairly well.

[11] Including the nominal interest rate $R_t \frac{P_t}{P_{t+1}}$ in the performance measure does not change the dynamics in a fundamental way, but leads to a slight asymmetry of bifurcations. Then it can not be guaranteed anymore that the mean of the time series of *perceived* shocks equals zero. This, however, is a necessary requirement when solving for rational expectations.

	π	y	s
SD	0.019	0.024	0.18
π	1	-0.309	-0.486
y	-0.309	1	0.627
s	-0.486	0.627	1

Table 5: Cross-correlation matrix and standard deviations for simulation, 10.000 iterations

with brute-force computation to find the set of behavioral parameters together with the standard deviations of shocks and the elasticity to stock prices. Details on the optimization algorithm can be found in the appendix. The relevant parameters for cross correlation and standard deviations are thus only the real macroeconomic parameters ν, σ_y, σ_π and the excess volatility of stock prices, which depends on the degree of rationality and trend extrapolation which *emerges* from the speculative process. The behavioral parameters can also be used to fine tune the dynamic process to also match turning points and amplitude. Optimal parameter values are summarized in Table 4. The simulated statistical moments can be found in Table 5.

I will provide economic intuition on why the extended model matches the data well after explaining the intuition behind this model in the following subsection.

Deterministic Simulations

To provide intuition behind the speculative process and the associated macroeconomic dynamics I use bifurcation theory. This also enables me to study the dynamic properties of the system and to identify the relevant types of dynamics that occur given different values for the set of behavioral parameters $\{\gamma, \alpha\}$. For this purpose I switch off shocks and run 11.000 iterations for each combination in the parameter space of which I omit a transition phase of 10.000. All points of endogenous parameters visited in the remaining 1000 simulations are then plotted for each point in the parameter space. The relevant dynamics can be summarized by a bifurcation diagram. The diagram for α is shown in Figure 2 and hence depicts the long-run dynamics as a function of the parameter.[12]

Since this section serves illustrative purposes I chose a value of γ which is slightly lower than the value identified by the estimation procedure.[13] Generally, an increase in the behavioral bias α implies two effects. First the *quantitative* aspects of the dynamics change, i.e. the standard deviation increases. Secondly, the *type* of dynamics changes, each which is indicated by the vertical grey lines. From Figure 2 we learn that for low values of $\alpha < 1.22$ the fundamental steady state is stable and unique, implying that the speculative forces are not strong enough to have an impact on stock prices without exogenous shocks. A stable steady state implies that there is always a fraction of fundamentalists in the market that outweighs the beliefs of biased agents. Exogenous shocks could lead to a temporal increase in the fraction of belief-biased agents, but their belief is still not strong enough to prevent the price from returning to

[12] As a technical note, it is clear that the first-order Taylor approximation around the steady state, that is embedded in the baseline DSGE model, will not hold once stock market prices deviate too much from their steady state value. I accept this inconvenience for the sake of simplicity and argue that introducing further nonlinearities would further complicate the interpretation of results while the added value of such undertaking would be unclear.

[13] For higher values of γ the type of dynamics are the same, but the interval of α in which they occur is very small. The dynamics then change from stable steady state to explosive almost immediately when increasing α.

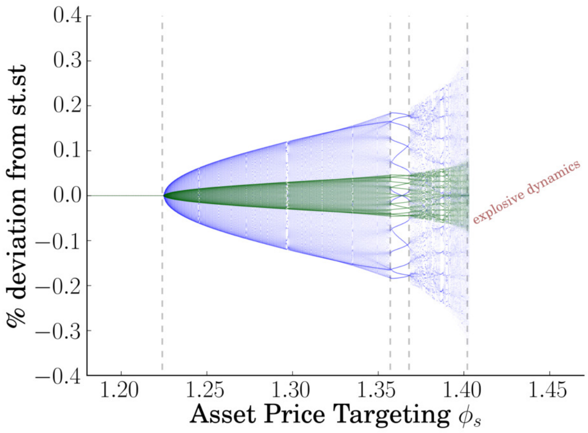
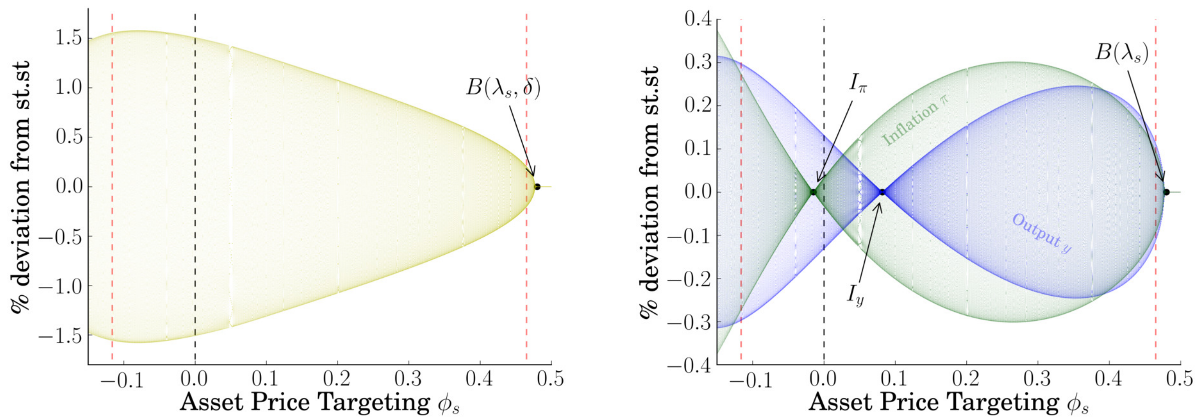


Figure 2: Long-run deterministic dynamics of y_t (blue/light) and π_t (green/dark) with respect to α. $\gamma = 1.29$, all other parameters as in Table 4.

its steady state. When α increases, cycles arise, where the amplitude of these cycles increases with the parameter value. A small deviation from the steady state is now, by increasing the fraction of belief-biased agents, strong enough to ignite deterministic dynamics. Since beliefs are self-fulfilling to a large extent, the fraction of belief-biased agents is increasing each period, which in turn lets the price rise even further. When the price approaches the value predicted by positively (negatively) biased traders, they reduce their long (short) position. This then reduces their profit and other strategies become more attractive. Once alternative beliefs are more and more enforced, the fraction of positively (negatively) biased traders is too small to maintain the high price level. These financial cycles now have impact on output and inflation, which adapt the cyclic movement of stock prices as shown in the graphic. After $\alpha \approx 1.35$ these cycles become unstable from around $\alpha \approx 1.37$ the simulations suggest that the system is close to a homoclinic orbit: the zero steady state is globally stable but locally unstable (Hommes, 2013). Long periods of stability can then be interrupted by bursts in asset prices that are hard to predict, and, through the credit-collateral channel, which can be followed by severe recessions. Increasing α even further, cycles collapse and dynamics become explosive. For values of α higher than 1.4 there is no sufficiently large fraction of fundamentalists that stabilizes the system. Since agents continue to assume price increases in the upcoming period, this process is self-fulfilling and prices explode to infinity in the long run.

The data fitting algorithm chooses 1.2 as value for α, which lies in the region where the deterministic steady state is stable. This result implies that the data is not purely driven by endogenous dynamics, but that the speculative dynamics are sensitive to the real stochastic shocks and, through speculation, can induce excess volatility. Triggered by a series of exogenous shocks, agents might observe the change in stock prices and extrapolate it, follow their beliefs but then, triggered by another exogenous shock, such bubbles might burst unexpectedly while preserving the mean-reverting property of a rational expectations model.

This also explains why the extended model matches the empirical moments well, in particular compared to the rational expectations model. First, introducing speculative dynamics implies excess volatility in the financial market. A series of positive shocks increases the fraction of optimistic traders. Optimism then prevails in the market for the following periods, even in the absence of significant underlying shocks. This lets stock prices go up further and ensures that the standard deviation of stock prices is matched. Optimistic periods are then ended either by a significant negative shock or just by the property of mean reversion (i.e. biased agents reduce their position because their beliefs became self-fulfilling). This endogeneity also ensures that the correlation between stock prices and inflation is not overestimated as in the rational expectations model. Since the impact of an increase in stock prices decreases inflation trough the marginal cost channel, the central bank lowers the interest rate which in turn stimulates demand. This ensures that the correlation between output and stock prices is positive and noticeable, which would not be the case in a model without a feedback from stock prices to real activity. Hence, the property of excess volatility in combination with a mutual linkage is crucial to replicate key-moments of the data.



(a) Bifurcation diagram of asset prices s_t with respect to ϕ_s.

(b) Bifurcation diagram of y_t (blue/light) and π_t (green/dark) with respect to ϕ_s.

Figure 3: Deterministic dynamics for ϕ_s. $\alpha = 1.31$ and $\gamma = 1$, all other parameters as in Table 4.

Policy and Stochastic Simulations

We want to have an understanding of the speculative dynamics when monetary policy responds to asset prices. For that purpose let me set α to a higher value than implied by the estimation and likewise decrease γ, which helps us to understand what would happen in a world that would mainly be driven by speculative dynamics.[14] Still I abstract from stochastic shocks meaning that all dynamics are endogenous. As shown in the previous Subsection an increase in ϕ_s is able to mitigate the impact of stock prices at least on output and decreases the positive feedback within the speculative process. Figure 3 show the long-run system's dynamics as a function of the policy parameter ϕ_s. This diagram reveals that an *increase* in ϕ_s has the same dynamic implications as a *decrease* in α. The left figure implies invariant cycles for all $\phi_s < B(\lambda_s)$. Since the dynamic process for s_t depends crucially on $\mathbf{\Psi_{3,3}}$, the amplitude decreases with the magnitude of policy ϕ_s and the graph exhibits a supercritical Hopf-Bifurcation in $B(\lambda_s)$, after

[14]Similarly to the long run dynamics depending on α, the calibration implied by the data suggests that dynamics jump from stable steady state to explosive with only a marginal change in ϕ_s.

which the dynamics settle to the steady state. The decrease in amplitude can be explained by the central banks response to stock prices. If $\phi_s = 0$ an increase in asset prices decreases marginal costs and induces a drop in inflation, which is counteracted by an decrease of the nominal interest rate. Falling interest rates however increases stock prices even further. If the central bank now increases the interest rate when stock prices go up, this counteracts the first effect of a decrease in interest rates and dampens the positive feedback of stock price expectation to stock prices, which is captured by $\mathbf{\Psi_{3,3}}$. The weaker this feedback, the lesser are speculative dynamics self-fulfilling. In my example calibration depicted here, this leads to the fact that endogenous, deterministic cycles can be completely switched off at $B(\lambda_s)$. Under this parametrization a very high central bank policy with respect to stock prices would manage to shift the system on a stationary steady state path.

The dynamics for y_t and π_t are a linear mapping of the dynamics of s_t through $\mathbf{\Psi}$. Both mappings for y and π contain an inflection point I_y and I_π respectively. These points correspond to these marked in Figure 1b. Since the results depicted here are deterministic and only result from speculative dynamics, this means that at these points the direct impact of speculation and bounded rationality can be completely offset. As explained before, the initial response of the central bank is to decrease the interest rate. If however $\phi_s = I_y$, this is completely mitigated by the *increase* of interest rates in response to stock prices, i.e. the net change in interest rates is zero. Since in this simple model output dynamics work trough intertemporal substitution, output will not deviate from its steady state level. Likewise, if the central bank slightly decreases the interest rate when stock prices increase, this raises demand and, at point I_π, can perfectly set off the negative effect of stock prices on marginal costs. Hence at I_π the net change in marginal costs is zero and, through the Phillips curve, inflation remains at its target level.

The parameters in Table 4 provide a good data fit, have a meaningful behavioral interpretation and reproduce dynamics that match the stylized facts Let me now use the parameters as the starting point to run stochastic simulations on the policy parameter ϕ_s.

In Figure 4 I show standard deviations of simulations where I add real economic shocks v^π and v^y. Remember that under this parameterization there are no deterministic dynamics, hence the process is a combination of stochastic shocks and endogenous responses of financial market speculation to these shocks. The implications from the deterministic case however still hold: an increase of ϕ_s to a value higher than I_y induces unnecessary fluctuations in both inflation and output. In addition to that, the policy response to asset prices also reacts to movements in stock prices that are not induced by speculation but by the stochastic process. For a productivity shock this means that when inflation increases, the Taylor rule implies an increase in interest rate. This in turn will decrease stock prices. But if the central bank also targets stock prices, in equilibrium it will again lower the interest rate, shifting it away from the optimal response to the productivity shock which then overall induces an unnecessary strong response in inflation and output. The response to a demand shock works similarly, implying that asset price targeting always increases volatility in inflation and output. This effect runs in the opposite direction of the stabilizing effect of asset price targeting on speculative dynamics and the respective spillovers to inflation and output. This means that the optimal sensitivity of monetary policy to asset prices is bounded by I_π and I_y. Furthermore, as predicted by the deterministic setup, a policy that *lowers* the interest rate in response to stock price increases also intensifies the feedback loop. Given the parameterization here, this leads to explosive dynamics in the stock market that are transmitted to the real economy. Monetary policy faces a trade-off that can be summarized by *"fragility versus volatility"*: first, increasing ϕ_s decreases the volatility of s_t while keeping the volatility of output at a similar level, but increases the volatility of π_t. Although raising ϕ_s mitigates the impact of speculation on real variables and strengthens the overall stability of the system, under the parametrization identified by the estimation procedure this stabilizing effect is of second order.

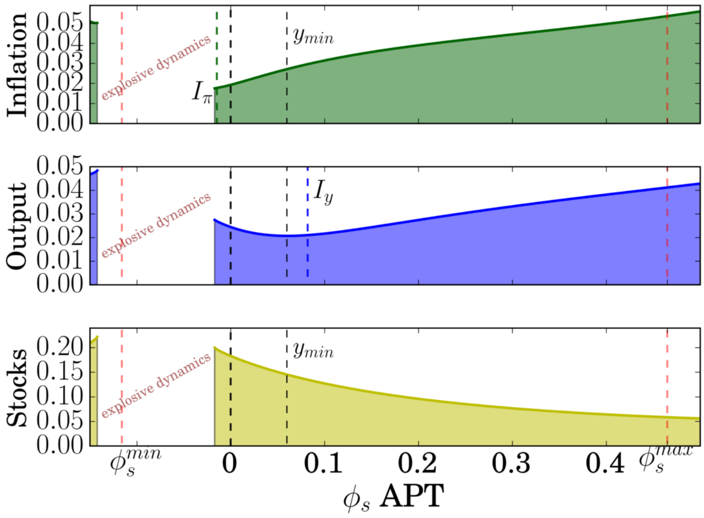


Figure 4: Standard deviations of stochastic simulations for varying values of ϕ_s.

The set-up of my model does not give reason to include asset prices themselves into welfare considerations. Relevant for the wellbeing of households is only their impact on output and inflation. Every welfare or central bank loss function that embeds a relative ranking of alternative central bank policies can be expressed as a convex combination of these both variances. From the point y_{min} on, both the variance of output and inflation are quasi-monotonous in ϕ_s. I therefore conclude that only a very moderate asset price targeting has the potential to increase dynamic stability, decrease volatility and to mitigate the coordination failure induced by speculation in the financial market. Given the model presented here and the simulations depicted in Figure 4 it is unlikely that even such a moderate policy will have a positive effect, neither on welfare nor, taking into account the associated potential welfare losses, on financial stability. A concrete advice for policy however depends on how much a central bank weights fluctuations of output when conducting monetary policy.

This setup can also be used to analyze the policy implications given by the literature on rational bubbles, as proposed most prominently by Galí (2013). Since such rational bubble would presumably grow proportional to the interest rate, it is suggested to actually lower the policy rate when facing asset price bubbles. If such policy is non-discretionary, in my model it would lead to a considerable increase in output and stock price volatility and, even for small values of the sensitivity of such policy, would lead to explosive dynamics in the stock market.

5 Conclusion

In this paper, I show that a causal linkage from stock prices to real activity as well as speculation in the stock market are necessary to replicate key-moments of the empirical data on inflation, output and stock prices. In particular the covariances between stock prices and inflation and stock prices and output, as well as the relatively high standard deviation of stock prices are well explained by the excess volatility that is induced by financial market speculation.

Methodically, by arguing that financial markets and commodity markets are fundamentally different, I introduce speculative behavior in the asset markets. Given a small number of consistency assumptions, I show that it is possible to find the rational expectation solution on all other markets even tough behavior in the asset market is boundedly rational. Simulation results using this model suggest that, enabled by positive speculation profits, any kind of speculation or herding, induced by bounded rationality, destabilizes the economy. Depending on the parametrisation, financial market interactions can lead to large and persistent booms and recessions. I thus find that instability is an inherent threat to economies with speculative financial markets.

I provide theoretical evidence that the central bank's interest rate setting amplifies the expectation feedback in the financial market, and that this can lead to an overall unstable dynamic process. My estimation procedure identifies the link from asset prices to real aggregates to be small but of macroeconomic significance, which implies a potential role for macroprudential policy to mitigate the negative externalities running though this mechanism.

Using this model as a counterfactual, I show that if asset prices impact on real variables, a monetary policy rule that also targets asset prices can mitigate the excess volatility of stock prices but at the cost of intensifying real shocks. The model however suggests that such policy is bounded very narrowly by the unwanted side effects ("collateral damage") of asset price targeting. This result does not only hold for a policy that raises the interest rate when facing stock price bubbles but also for a policy that lowers the interest rate. The latter can easily amplify the speculative process and destabilize the economy furthermore.

Apart from the theoretical results provided in this paper, policy institutions should be careful with regard to asset price targeting since such policy implies adding another structural link between asset prices and macroeconomic aggregates. Such additional link embeds the risk of other unforeseeable complications, independently of how tight the natural link from asset prices to real activity is. This is particularly true because stock prices impact solely through a signaling effect. Then an artificial inflation of stock prices in recessions might be counterproductive. This however leaves room for other macroprudential policies that potentially limits speculation itself or reduces profits from speculation in financial markets (i.e. policies such as short-selling constraints or leverage requirements) and indicates that such policy would contribute to overall economic stability. This work also suggests that neither stock prices nor indices on stock prices are a good indicator to base decisions on (lending, evaluation of competitors,...). Furthermore practitioners should be aware that, regarding stock prices and real activity, causality might run in both directions.

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Appendix

A Entrepreneurs' optimization problem

I follow Bernanke et al. (1999) closely, but instead of assuming risk in the productivity of capital I assume that there is idiosyncratic risk in labor productivity, i.e. firm js ex post gross return on one unit of labor is ω_j which is i.i.d. across time and firms with a continuous and once-differentiable c.d.f $F(\omega)$ over a non-negative support and with an expected value of 1. I assume that the hazard rate $h(\omega) = \frac{dF(\omega)}{1-F(\omega)}$ is restricted to $h(\omega) = \frac{\partial(\omega h(\omega))}{\partial \omega} > 0$. The optimal loan contract between financial intermediary is then defined by a gross non-default loan rate, $Z_{j,t+1}$, and a threshold value $\bar{\omega}_{j,t}$ of the idiosyncratic shock $\omega_{j,t}$. For values of the idiosyncratic shock greater or equal than this value, the entrepreneur will be able to repay the loan, otherwise he will default. $\bar{\omega}_{j,t}$ is then defined by

$$\bar{\omega}_{j,t} R_t^H H_{j,t} = Z_{j,t+1} B_{j,t}.$$

Dropping firms' subscripts, as in Bernanke et al. (1999) the optimal contract loan contract must then satisfy

$$\left\{ [1 - F(\bar{\omega}_t)]\, \bar{\omega}_t + (1-\mu) \int_0^{\bar{\omega}_t} \omega dF(\omega) \right\} H_t/X_t = R_t(W_t H_t - N_t),$$

and the expected return to the wholesaler is (dropping time-subscript of ω_t for better readability)

$$E\left\{ \int_{\bar{\omega}}^{\infty} \omega dF(\omega) - (1 - F(\bar{\omega}))\bar{\omega} \right\} H_t/X_t.$$

Given constant returns to scale, the cutoff $\bar{\omega}$ determines the division of expected gross profits H_t/X_t between borrower and lender. Let me define

$$\Gamma(\bar{\omega}) = \int_0^{\bar{\omega}} \omega f(\omega) d\omega - \bar{\omega} \int_{\bar{\omega}}^{\infty} f(\omega) d\omega,$$

the expected gross share of profits going to the lender with $\Gamma'(\bar{\omega}) = 1 - F(\bar{\omega})$ and $\Gamma''(\bar{\omega}) = -f(\bar{\omega})$. This implies strict concavity in the cutoff value. I define similarly the expected monitoring costs:

$$\mu G(\bar{\omega}) = \mu \int_0^{\bar{\omega}} f(\omega) d\omega,$$

with $\mu G'(\bar{\omega}) = \mu \omega f(\omega)$. For a proof that the following result is a non-rationing outcome, please refer to BGG. The resellers problem of choosing the optimal equity can be solved by maximizing discounted profits over equity, or maximizing return on investment and including investment as part of the optimization problem:[15]

$$\begin{aligned} \max_{\{H_t\},\{\bar{\omega}_t\},\{N_t\},\{\lambda_t\}} \quad & E_t \sum_{s=t}^{\infty} N_t^{-1} \prod_{l=t}^{s} R_l^{-1} \left[(1-\Gamma(\bar{\omega}_s)) \, H_s/X_s - N_{s+1} \right] \\ & -\lambda_s \left([\Gamma(\bar{\omega}_t) - \mu G(\bar{\omega})] \, H_t/X_t - R(W_t H_t - N_t) \right) \end{aligned} \tag{A.1}$$

The first-order conditions for this problem can be written as:

$$\begin{aligned} H &: (1-\Gamma(\bar{\omega}_t)) \, (X_t N_t R_t)^{-1} - \lambda_t \left([\Gamma(\bar{\omega}_t) - \mu G(\bar{\omega})] \, / X_t - R_t W_t \right) && = 0 \\ \bar{\omega} &: \Gamma'(\bar{\omega}_s)(N_t R_t)^{-1} - \lambda_t \left[\Gamma'(\bar{\omega}_t) - \mu G'(\bar{\omega}) \right] && = 0 \\ N &: -\frac{S_t}{R_t N_t^2} - R_t \lambda_t && = 0 \\ \lambda &: [\Gamma(\bar{\omega}_t) - \mu G(\bar{\omega})] \, H_t/X_t - R(W_t H_t - N_t) && = 0 \end{aligned}$$

Combining the first three conditions makes clear that there exists a connection between the optimal choice of labor, prices and stock prices. Using the optimality condition for the cutoff value $\bar{\omega}_t$ and rearranging yields

$$\frac{\Gamma'(\bar{\omega}_t)}{\Gamma'(\bar{\omega}_t) - \mu G'(\bar{\omega}_t)} = \frac{S_t}{R_t N_t}$$

where I can write the LHS as a function $\rho(\bar{\omega})$. Bernanke et al. (1999) show that under reasonable assumptions $\rho(\bar{\omega})$ is a mapping from $\bar{\omega}$ to $\mathbb{R}^+$. I can use the inverse function to establish that the premium payed on external funds depends on the return payed on internal funds. As noted in the main body, this is intuitive since the marginal costs of external and internal finance need to be equal. Likewise I can define the risk premium on external funds to be a function of the leverage ratio (if $N_t = W_t H_t$, the premium is obviously one) and establish the relationship in the main body.

B Bayesian estimation

To avoid stochastic indeterminancy a third shock needs to be added. Here I consider an additional cost-push shock on marginal costs x_t

$$v_t^x = \rho_x v_{t-1}^x + \varepsilon_t^x, \qquad \varepsilon_t^x \sim N(0, \sigma_x)$$

added to Equation (3.3), which then reads

$$x_t = \eta y_t + i_{t+1} - \nu s_t + v_x.$$

Likewise an exogenous shock on stock prices v_s also follows an AR(1) structure and hits the economy in the zero profit condition.

[15] In fact the linkage between stock and wholesale prices would be clearer if I would allow wholesalers to have some monopolistic power and hence scope to adjust prices in response to pressure from the financial market. I do not model this here in detail since it seems unnecessary given the fact that the monopolistic competition is already implemented in the retail sector and the artificial division between both sectors is only due to reasons of analytical tractability.

	Prior distribution			Posterior distribution $\sigma_x = 0$			Posterior distribution $\sigma_s = 0$		
	Type	Mean	St. error	Mean	5%	95%	Mean	5%	95%
σ_π	Uniform	[0,1]	–	0.0009	0.0008	0.0010	0.0122	0.0113	0.0132
σ_y	Uniform	[0,1]	–	0.0038	0.0034	0.0041	0.0162	0.0153	0.0170
σ_s	Uniform	[0,1]	–	0.0006	0.0005	0.0006	–	–	–
σ_x	Uniform	[0,1]	–	–	–	–	0.1913	0.1745	0.2058
ρ_s	Beta	0.85	0.1	0.9984	0.9984	0.9984	–	–	–
ρ_x	Beta	0.85	0.1	–	–	–	0.5678	0.5343	0.5966
σ	Normal	1	0.05	0.9598	0.8710	1.0371	1.3133	1.3075	1.3181
γ	Normal	0.3	0.05	0.3030	0.2193	0.3903	0.3000	0.2238	0.3873
ω	Beta	0.66	0.05	0.6570	0.5795	0.7328	0.6593	0.5760	0.7518
$\tilde{\nu}$	Normal	0.0	0.3	-0.0187	-0.5341	0.5017	0.0034	-0.4916	0.4707
ϕ_π	Normal	1.300	0.01	1.2951	1.2774	1.3120	1.3616	1.3591	1.3636
	Log data density:				4413.36			2110.33	

Table 6: Parameter estimates

Priors and the result of the estimation can be found in Table 6. Most priors are taken from Smets and Wouters (2003) while the value of the prior for $\gamma = 0.3$ is consistent with the calibration in BGG. In order to remain agnostic about this value I set a very loose prior with mean equal zero, while this value is set to 0.5 in BGG. I use a prior of $\phi_\pi = 1.3$ as the central bank's policy parameters.

Note that the prior for σ is very tight. This is necessary because I only consider determined solutions, and a fairly higher value of σ easily renders the rational expectations system undetermined. It is a fairly well-known fact that such a model is unable to match the data well.

The key parameter of the model presented so far is ν, the marginal costs' elasticity to stock prices, wich is estimated to be very close to zero. It furthermore is not well identified with a standard deviation of the posterior of about 0.5. Accordingly, changing the prior for ν results in corresponding changes of the posterior mean, but without significantly improving identification or data fit. Furthermore, due to the nature of first order Taylor approximation, such model will be unable to capture the tail distribution of asset price movement with respect to interest rates.

As expected, exogenous fluctuations in asset prices provide a better data fit than the additional cost push shock. The cost push shock enters the model similarly (though by far not identically) to the productivity shock while with a stock market shock it is possible to completely steer the stock price dynamics exogenously. This however is not the intention of this work, also does a strong exogenous component (and given the results in the appendix those would be tremendous) lack economic intuition and empirical evidence. With exogenous fluctuations in asset prices the vast majority of the asset price dynamics are exogenous. This does not link well to any known story, including news shocks. Moreover, an exogenous shock to stock prices does lack economic intuition. While news shocks could explain exogenous movements to some extend, the fact that stock prices appear to be driven by highly persistent news shocks in combination with interest rate setting does not seem very convincing.

C Solving for the rational expectations equilibrium

In expectations it has to hold that

$$E_t\tilde{v}_{t+1}^{\pi} = \rho_\pi \tilde{v}_t^{\pi} \tag{C.1}$$
$$E_t\tilde{v}_{t+1}^{y} = \rho_y \tilde{v}_t^{y}. \tag{C.2}$$

Using this form, I can write the PLM using the system of equations (3.1) – (3.5) and bring all expectations to the LHS:

$$\beta E_t \pi_{t+1} = \pi_t - \kappa x_t - \tilde{v}_t^{\pi} \tag{C.3}$$
$$E_t y_{t+1} = \sigma^{-1} r_{t+1} + y_t - \tilde{v}_t^{y} \tag{C.4}$$
$$\beta E_t s_{t+1} = s_t + r_{t+1} \tag{C.5}$$
$$0 = -x_t + \eta y_t + i_t - \nu s_t \tag{C.6}$$
$$0 = -i_t + \phi_\pi \pi_t + \phi_s s_t \tag{C.7}$$
$$E_t\tilde{v}_{t+1}^{\pi} = \rho_\pi \tilde{v}_t^{\pi} \tag{C.8}$$
$$E_t\tilde{v}_{t+1}^{y} = \rho_y \tilde{v}_t^{y} \tag{C.9}$$

Using (C.6) and (C.7) to substitute out i_t and x_t and rewriting as a matrix yields the System (4.2). Let me for rewrite this system as

$$\mathbf{\tilde{P}} E_t \tilde{\mathbf{x_{t+1}}} = \mathbf{M}\mathbf{\tilde{x}_t}$$

and $\mathbf{\tilde{N}} = \tilde{\mathbf{M^{-1}}}\mathbf{\tilde{P}}$ is the 5×5 matrix which summarizes the dynamics of the perceived law of motion of rational agents. I use eigenvector/eigenvalue decomposition to obtain $\mathbf{\Gamma\Lambda\Gamma^{-1}} = \mathbf{\tilde{N}^{-1}}$, where $\mathbf{\Lambda}$ is the diagonal matrix $\mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_5)$ of the eigenvalues of $\mathbf{\tilde{N}^{-1}}$ ordered by size (smallest in modulus first) and $\mathbf{\Gamma}$ the associated eigenvectors, columns ordered in the same fashion. I can then rewrite the expectation system as

$$\mathbf{\Gamma^{-1}} E_t \mathbf{x_{t+1}} = \mathbf{\Lambda\Gamma^{-1}x_t}.$$

Let me denote the sub-matrix of $\mathbf{\Lambda}$ that only contains unstable eigenvalues as $\mathbf{\Lambda_u}$, and the associated eigenvectors I will likewise call $\mathbf{\Gamma_u}^{-1}$. I then know that $\mathbf{\Gamma_u^{-1}} E_t \mathbf{x_{t+1}} = \mathbf{0}$ if I want to be consistent with transversallity or feasibility constraints. Using this fact I can solve for $E_t\mathbf{x_{t+1}}$:

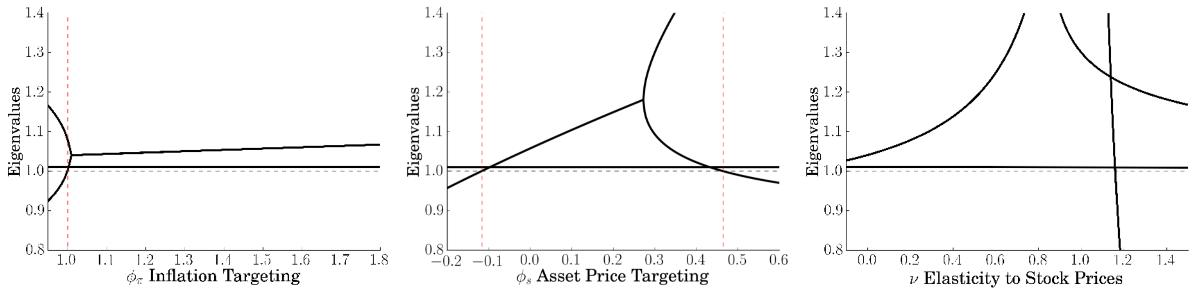
$$E_t \mathbf{x_{t+1}} = \mathbf{\Gamma_{u,1:3}^{-1}\Gamma_{u,4:5}} E_t \mathbf{\tilde{v}_{t+1}} = \mathbf{\Gamma_{u,1:3}^{-1}\Gamma_{u,4:5}}\boldsymbol{\rho}\mathbf{\tilde{v}_t}$$

Note that the requirement that $\mathbf{\Gamma_{u,1:3}}$ is invertible implies the Kuhn-Tucker condition: I impose that $\mathbf{\Gamma_{u,1:3}}$ is a square matrix with full rank. This means that the number of forward looking variables has to equal the number of unstable eigenvalues $\lambda > 1$ of $\mathbf{\tilde{N}^{-1}}$. Accordingly, I do only look at policy parameters $\boldsymbol{\phi}$ that guarantee that this condition is met. Let me define $\mathbf{\bar{\Omega}} = \mathbf{\Gamma_{u,1:3}^{-1}\Gamma_{u,4:5}}$. The solution from the main body is then $\mathbf{\bar{\Omega}_{1:2,1:2}}$.[16]

[16] This implies that rational agents do not take asset prices into account when forming expectations. However, a more general approach including the adjustment for measurement errors of projecting three endogenous variables on two shock terms (stochastic indeterminacy) approximately lead to the same Ω. Assuming that agents use OLS to regress $\mathbf{x_t}$ on $\mathbf{\tilde{v}_t}$, $\mathbf{\tilde{\Omega}} = \left(\mathbf{\bar{\Omega}^T\bar{\Omega}}\right)\mathbf{\bar{\Omega}^T} \in \mathbb{R}^{3\times 2}$ and $\mathbf{\tilde{\Omega}_{1:2,1:2}} \approx \mathbf{\Omega}$.

D Eigenvalues of the Rational Expectations System

In (3.6) I define the matrix $\mathbf{N}$, which represents the dynamics under Rational Expectations and is crucial for the dynamics under quasi-rational expectations. For both cases the Blachard-Kahn condition for a determined and unique solution of the rational expectations is that the eigenvalues λ of $\mathbf{N^{-1}}$ are lying outside the unit circle. Although an analytical representation exists, I opt for the numeric representation for reasons of clearer presentability. Figure 5 shows the eigenvalues depending on both Taylor-Rule parameters and key parameter ν, the elasticity of external finance premium with respect to equity. The dashed red line marks the point in parameter space where the relevant eigenvalues turn negative.



(a) Eigenvalues depending on ϕ_π (b) Eigenvalues depending on ϕ_s (c) Eigenvalues depending on ν

Figure 5: Eigenvalues of $\mathbf{N^{-1}}$ as a function of policy parameters and the elasticity of the external finance premium

E Fitting the model to match the statistical moments from Table 1

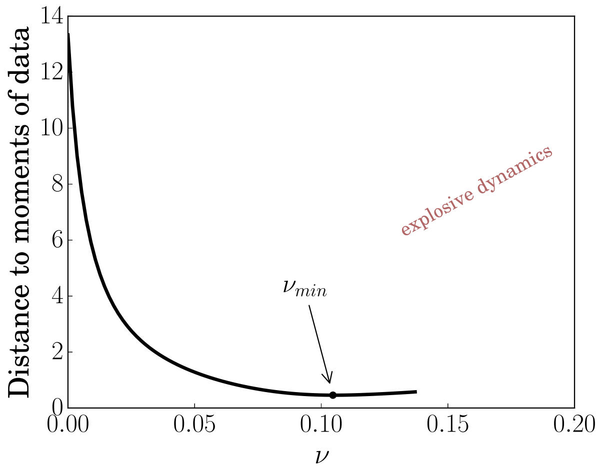


Figure 6: Matrix norm as a function of the marginal cost elasticity to stock prices

Let me denote the 2×3 matrix that summarizes the standard deviations and cross correlations between the endogenous variable as $\mathbf{\Xi}$. I run simulations of 20,000 periods and denote the same statistical moments of a simulation by $\tilde{\mathbf{\Xi}} : (\nu, \boldsymbol{\sigma}, \mathbf{\Delta}) \rightarrow \mathbb{R}^{2\times3}$. $\boldsymbol{\sigma}$ denotes the standard deviation of both shock terms and $\mathbf{\Delta}$ includes the behavioral parameters α and γ. As the key fitness measure I use the Frobenius norm $\chi = ||\mathbf{\Xi}/\tilde{\mathbf{\Xi}} - 1||$ of the deviation of both matrices. Due to the nonlinearity and particularly the non-continuity I can not make use of standard optimization or data fitting algorithms. The simplest solution is to use bruit force in combination with a grid. I construct a grid of 5000 points for every parameter value. Each iteration, I only take one parameter (let me use ψ as the placeholder for a parameter) and calculate a vector $\boldsymbol{\chi}_\psi$ for each point its while leaving the other parameters. At the end of the iteration I set $\psi = \arg\min \bar{\chi}_\psi$ and repeat the process and cycle through the parameter space until χ and parameters converge.

This will, with finite precision, approximate a local minimum of the non-continuous, non-smooth mapping. To find the global minimum I checked by creating a sparse grid of initial parameter values, however the algorithm always converged to the parameter set in Table 4.

Figure 6 plots the fitness measure as a function of ν. While for low values the distance if very high, fit increases tremendously for values larger ≈ 0.07. For values larger than ≈ 0.14 dynamics are explosive and no measure is available.