

Contagion

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Challenges in Risk and Insurance

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by

Prof. dr. Roger J. A. Laeven

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OSSIUS PERS UvA

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*Mevrouw de Rector Magnificus,
Mijnheer de Decaan,
Waarde Collega's en Studenten,
Afgevaardigden van de KNAW en NWO,
Afgevaardigden van het Verbond van Verzekeraars,
Familie, vrienden,
Allen die door uw aanwezigheid blijk geeft van uw belangstelling,*

Mijn moeder verzocht me om vanmiddag een eenvoudige lezing tot vertier en vermaak te geven. Vroeger was me dat misschien nog wel gelukt, toen ik als kleine Urbanus schuine verhalen over ‘madammen met nen bontjas’ verkondigde. Nu ben ik een serieuze wetenschapper geworden, die zijn verhaal ook nog eens in het Engels gaat vertellen, voor alle buitenlandse collega’s hier vandaag aanwezig. Mocht ik daarom niet helemaal aan jouw verwachtingen voldoen, dan bij voorbaat hierbij mijn verontschuldigingen. Voor het overige, just sit back, relax and enjoy your stay. Dat was nog wel eenvoudig hè?

1 Introduction

Coincidentally or not, in September 2005, a biblical seven years ago with accuracy up to two weeks,¹ I publicly defended in this same exquisite Auditorium my PhD thesis. It was titled *Essays on Risk Measures and Stochastic Dependence, with Applications to Insurance and Finance*.² Reminiscent of that special day, with four generations of my family present,³ it is an honor and pleasure to stand here again, in front of so many of you for the next phase.

In the meantime, much has happened, both in my profession and in my own professional life. Today I want to share with you some of these developments. I will do so in a hopefully accessible manner and from a broad perspective: I start with a brief history and outline of the basic principles of the field of Risk and Insurance. Next, I discuss the challenges the field is nowadays facing. Finally, I point at expected future developments in the industry (economy) and in research and teaching, and sketch some of my plans going forward, for the years to come.

2 A Brief History of Risk and Insurance

The year was 1691 when Jacob (or James) Bernoulli⁴ informed his younger brother Johann (or Jean) Bernoulli⁵ that he had obtained a result that is nowadays⁶ known as the Law of Large Numbers, or Bernoulli's Theorem. The Bernoulli family is indisputably the most important and remarkable family in the early history of Risk and Insurance, and, in fact, much more broadly. The family produced several of the greatest mathematicians ever, including Jacob, Johann and Johann's son Daniel,⁷ which we will encounter later.⁸

The (Weak) Law of Large Numbers⁹ (and its proof) was eventually published posthumously in the book *Ars Conjectandi* (the Art of Conjecturing) in 1713, after editorial work by Nicolaus Bernoulli, the son of Jacob's middle brother Nicolaus. Loosely speaking, Jacob's 'Golden Theorem', which we will discuss in more detail later today, states that the average loss (or gain) in an expanding pool of risks eventually becomes certain (or predictable). In our context of Risk and Insurance, it will imply that pooling risks can serve as a basic risk mitigation technique.

At the same time of proving the Law of Large Numbers, Jacob Bernoulli also published other work on probability theory, in the *Journal des Scavans* and the *Acta Eruditorum*. The history of the latter learned journal, coincidentally or not, has been the subject of my father's PhD thesis.¹⁰ Furthermore, the Law of Large Numbers was discussed in some detail with Gottfried Wilhelm Leibniz in letter exchanges in the period 1703 to 1705.

In general, one can say that, until the end of the seventeenth century, correspondences were the far most important vehicle for the exchange of ideas between scholars. In the second half of the seventeenth century, the first learned journals came into existence, as there were the *Journal des Scavans*, *The Philosophical Transactions* and the *Acta Eruditorum*. Nevertheless, correspondences continued to play a major role for quite some time, most certainly when matters had not yet crystallized out or were still speculative. And one must bear in mind that letters between leading scholars, and especially between giants like Leibniz and Bernoulli, reached a much wider circle than the two correspondents only, because it was common practice to distribute copies. Thus, a wider circle of colleagues was involved in the scholarly dispute.¹¹

From the 1680's onwards, more and more products of the pen found their way into the fast growing number of learned journals, certainly when a claim of results was at stake. In this way, from its start, the periodical became the preferred means to claim 'the right of invention'. As far as the *Acta Eruditorum* was concerned, an additional reason for its success was the fact that Leibniz was one of the bearing forces of this periodical. He considerably contribut-

ed to the esteem and dissemination of it by his own contributions as well as by his efforts to involve others.¹² Otherwise, it appears that Leibniz, in the course of time, insisted on the publication of the *Ars Conjectandi* as a book.¹³ From that, one may conclude, that in this phase of transition, the periodical had not yet displaced the book for the final presentation of research results. Apparently, just like the correspondences, the periodical mainly offered a forum for the exchange of ideas, the submission of critical notes and the judgment of fellow experts. Differently from today, for the final presentation of results in more or less substantial treatises, the primacy still stayed with the book, that ‘monumentum aere perennius’, as Horace called it.

Jacob’s Law of Large Numbers is at the basis of the economics of risk and insurance. Amazingly, the work *Specimen Theoriae Novae de Mensura Sortis* (Exposition of a New Theory on the Measurement of Risk) that his nephew Daniel Bernoulli first presented in 1731¹⁴ constitutes one of the most fundamental steps in the development of the theory of risk measurement, which is one of the most important cornerstones of the economics of risk and insurance.

In a nutshell, Daniel Bernoulli argues in this essay that expectations, which are simply probability-weighted averages of possible outcomes, are no proper descriptions of risk in the context of individual decision-making. He argues that, beyond the objective elements of probabilities and outcomes that are common to all individuals,¹⁵ subjective elements play a role in individual decision-making. These subjective elements (utilities) differ for different individuals.¹⁶

His idea is illustrated by the famous so-called St. Petersburg paradox, to which also his cousin Nicolaus had contributed in letter exchanges. It describes a game of chance with infinite expected payoff but for which no ‘reasonable’ individual would be willing to pay a high (not to mention infinite) amount: Individual decision-making cannot be solely based on simple expectations. The ‘solution’ proposed by Daniel is to assign subjective utilities to the possible outcomes, rendering a finite and reasonable amount that a reasonable individual is willing to pay for the game of chance.¹⁷

3 Principles of Risk and Insurance¹⁸

Next to the already announced Law of Large Numbers, basic principles of the economics of risk and insurance include Pareto optimality (efficiency) and the related Mutuality Principle of Borch (1962). As we will see now, these principles imply that risk pooling and risk spreading, based on the Law of Large

Numbers and (possibly) mutuality, are important techniques for risk mitigation.¹⁹

Risk pooling and risk spreading (sharing) should be distinguished from one-sided risk transfer, relocating risk from a risk averse individual to a less risk individual, or rather to a less risk averse (or even risk neutral) (re)insurer. Furthermore, risk pooling should be distinguished from risk diversification. The former, the core of insurance, adds up (and eventually spreads) risks without affecting the total risk of the pool; the latter, the core of optimal portfolio choice in finance, is concerned with reduction of the total risk of the portfolio. In the following, I try to avoid economic and mathematical subtleties.²⁰

3.1 Law of Large Numbers

Yes, (a modern version of)²¹ Jacob Bernoulli's (Weak) Law of Large Numbers is one of the key principles in the economics of risk and insurance. But despite playing such a central role in the economics of risk and insurance, its exact implications are often only superficially and vaguely treated – or even poorly understood –, except for some noticeable exceptions.²²

Loosely speaking, the Law of Large Numbers says that upon expanding the number of risks in a pool, the average loss (or gain) in the pool eventually becomes certain (or predictable). More specifically, the Weak Law of Large Numbers says that the probability that the average loss in a pool of risks deviates from the expected loss by more than a specified amount, no matter how small, eventually, upon expanding the pool of risks, becomes zero. While this version of the law does not exclude the possibility that deviations from the expected loss become arbitrarily large infinitely often, the Strong Law of Large Numbers, whenever valid, says that this does not occur with probability one; see Feller (1971b, Sec. VII.8) for further details.

Notice that the Law of Large Numbers is a statement about the deviation or, henceforth loosely, variability of the average loss of the pool of risks, not about the aggregate loss of the pool of risks. Notice furthermore that it considers the situation in which the pool of risks becomes large, eventually pooling infinitely many risks. Upon expanding the pool of risks, the variability of the average loss, or the average risk per individual induced by risk pooling, is eventually annihilated.

Consider the following example.

Example 1: Suppose that the probability of a total loss car accident for a representative driver is 1% a year. This means that the expected number of total loss car accidents for the representative driver is 1 in every 100 years, so a bit less

than 1 in an expected lifetime.^{23,24} Suppose furthermore that, upon occurrence of the accident, the loss (always) equals EUR 10,000. Each individual thus faces the risk that with probability of 99% per annum he / she experiences no loss but with probability of 1% per annum he / she incurs a loss of size EUR 10,000; the maximum loss per driver is EUR 10,000 per annum. That is a big risk to bear for a single individual with a standard income,²⁵ even though the expected loss is (only) $1\% \times \text{EUR } 10,000 = \text{EUR } 100$ per annum.

Probability	99%	1%
Loss	EUR 0	EUR 10,000

Suppose now that we consider a pool of 1,000 such drivers. The expected loss per driver is (still) EUR 100 per annum. However, the variability of the average loss, or the average risk per individual induced by risk pooling, is much reduced. With only a bit of high school mathematics, one may compute that with probability of only about 0.001% per annum (a 1-in-100,000-years event) the average loss per individual is larger than EUR 250. This may be compared to the 1% probability per annum of incurring a EUR 10,000 loss in the previous situation without risk pooling. The maximum annual loss per driver is (still) EUR 10,000.

Probability	99.999%	0.001%
Average Loss	$\leq \text{EUR } 250$	$> \text{EUR } 250$

The variability of the average loss per individual in a pool of 1,000,000 such drivers would be even further reduced: One may compute that with probability of only 0.001% per annum the average loss per individual is now larger than EUR 104.26. Large deviations from the expected loss of EUR 100 become highly unlikely. The maximum annual loss per driver is (still) EUR 10,000.

Probability	99.999%	0.001%
Average Loss	$\leq \text{EUR } 104.26$	$> \text{EUR } 104.26$

$[\Omega]$

Example 1 illustrates that while the loss of a single individual may be highly unpredictable, the average loss of an individual, averaged over an expanding pool of risks, eventually becomes predictable (specifically, EUR 100, in Example 1). Alternative but similar examples with e.g., ‘deaths’ (or ‘births’) rather than ‘car accidents’ (and pools of ‘people’ or ‘animals’ rather than pools of ‘car drivers’) are straightforward to construct.

More formally, Khintchine’s (weak version of the) Law of Large Numbers says that the sample mean (or average) of an expanding pool of independent risks with identical probability distribution eventually converges (in probability) to the true (or population) mean, whenever the risks have finite expectation (Feller, 1971b, p. 235).²⁶ In the setting of Example 1, this means that the average loss of the expanding pool of risks eventually converges to EUR 100.

In what follows, I will elaborate on three fallacies regarding the Law of Large Numbers, which are subsequently:

1. The Law of Large Numbers does not say anything about the aggregate loss of the pool of risks.²⁷
2. The Law of Large Numbers is not applicable if there is a common component to all risks²⁸ and hence does not apply to systematic risk.²⁹
3. The Law of Large Numbers is a limiting relation, while in reality the number of risks in a pool does not expand to infinity.

3.1.1 Average versus Aggregate

Reconsider the example.

Example 2: *In contrast to the variability of the average loss, which is much reduced by risk pooling as Example 1 illustrates, the variability of the aggregate loss due to the pool of 1,000 drivers is substantial. The expected aggregate annual loss of the pool is $1,000 \times \text{EUR } 100 = \text{EUR } 100,000$, but one may compute (again with a bit of high school mathematics) that with probability of about 5% per annum the aggregate loss of the pool is larger than to EUR 150,000, a deviation of EUR 50,000 when compared to the expected value. The maximum annual loss of the pool is $1,000 \times \text{EUR } 10,000 = \text{EUR } 10,000,000$.*

Probability	95%	5%
Aggregate Loss	$\leq \text{EUR } 150,000$	$> \text{EUR } 150,000$

$[\Omega]$

Example 2 makes explicit that pooling of risks does not lead to risk reduction on the aggregate level of the pool. In fact, on the contrary, pooling of risks, which effectively means adding up risks, leads to increased risk on the aggregate level of the pool. As a result, the Law of Large Numbers cannot be interpreted to imply that pooling of risks leads to risk reduction. As Samuelson (1963, abstract) writes in crystal-clear language: ‘This valid property of large numbers is often given an invalid interpretation. Thus people say that an insurance company reduces its risk by increasing the number of ships it insures.’

3.1.2 Independent versus (Completely)³⁰ Dependent

Reconsider again the example.

Example 3: Suppose now that all 1,000 drivers in the pool work in the same village, exploiting the fertile soil on the edge of a volcano. In case of a sudden volcanic eruption, the workers are evacuated by helicopter and are not allowed to drive their cars. It will lead to a total car loss for all drivers in the pool. Suppose that the probability of a sudden volcanic eruption is 0.1% per annum, meaning that a sudden volcanic eruption is expected to occur once in 1,000 years.³¹

The probability of a total loss for a representative driver is still supposed to be 1% a year, 0.1% being due to the possibility of a sudden volcanic eruption and 0.9% being due to other sources (traffic exposure). Upon occurrence of the eruption or an otherwise total loss, the loss incurred (still) equals EUR 10,000 per car. For a single individual, without risk pooling, the risks of Example 1 and the present example are comparable: A big risk to bear for a single individual with a standard income. The expected average annual loss is (still) EUR 100.

Probability	99%	0.1%	0.9%
Loss	EUR 0	EUR 10,000	EUR 10,000

The variability of the average loss, or the average risk per individual induced by risk pooling, is, however, no longer comparable; it is now not reduced as much as in Example 1. In particular, with probability of 0.1% per annum (the probability of a sudden volcanic eruption), the average loss per individual is EUR 10,000: Upon eruption, all 1,000 drivers bear a total car loss of EUR 10,000. The maximum annual loss per driver is (still) EUR 10,000.

Probability	0.1%
Average Loss	EUR 10,000

The variability of the aggregate loss due to the pool of 1,000 drivers is now even more substantial. The maximum annual loss of the pool is (still) EUR 10,000,000.

[Ω]

Example 3 illustrates that the Law of Large Numbers does not work if there is a common component to all risks: Systematic risk is not annihilated by risk pooling. Clearly, Example 3 is perhaps a little stylized to bring the matter out into the limelight, but systematic risks are abundant in the insurance industry. A canonical example of a systematic insurance risk is longevity risk. The Law of Large Numbers does apply to the (Bernoulli) risk of life and death of single individuals in an expanding pool of risks, but it breaks down when there is a common component, such as longevity, affecting the probabilities of life and death of all individuals in the pool. Other examples are provided e.g., by interest rate risk, to which many insurance liabilities are exposed.

3.1.3 Finite versus Infinite

While the Law of Large Numbers is theoretically valid, the expanding pool of risks, eventually pooling infinitely many risks, only exists in the mathemati-

cian's imagination. In practice, the pool inevitably consists of only finitely many risks. Does this mean that the Law of Large Numbers is useless?

The true answer is 'yes and no'. Let's start with the positive 'no'. As a description of a phenomenon ('the variability of the average loss in an expanding pool of risks diminishes'), the law is perfectly legitimate and useful, as we will see later. Next, the negative 'yes'. Because the pool of risks contains at most a finite number of risks, the variability of the average loss per individual induced by risk pooling is not fully annihilated, and the average loss does not become fully predictable.

The fact that the average loss in a pool of a large but finite number of risks is not (yet) fully predictable means that the remaining variability cannot be ignored and needs to be dealt with. In an insurance context, this will give rise to a safety (or risk) loading embedded in insurance premia and buffer (or solvency) capital to be held by the insurer, even for risks that are, in principle, non-systematic.

Feller (1971a, p. 251) writes: '[...] the law of large numbers is operationally meaningless unless many millions of trials are involved. Now all fire, automobile, and similar insurance is of the described type; the risk involves a huge sum, but the corresponding probability is very small. [...] As for the company, it plays a large number of games, but because of the large variance the chance fluctuations are pronounced. The premiums must be fixed so as to preclude a huge loss in any specific year, and hence the company is concerned with the ruin problem rather than the law of large numbers'.

Feller is right as regards the fact that a large number of risks need to be pooled in order for the Law of Large Numbers to become operationally useful (and note that in reality the pool inevitably pools only finitely many risks). The remaining variability then needs to be accounted for (Feller's ruin problem). But Feller ignores a crucial aspect, that of risk spreading.

3.2 Risk Pooling and Risk Spreading

Suppose now that the pool of risks is placed with an insurance company. The insurance company, responsible for the pool of risks, is primarily concerned with the aggregate risk of the pool, not with the average risk per individual induced by risk pooling. But we have seen earlier that pooling of risks does not lead to risk reduction on the aggregate level of the pool, in fact, on the contrary. If that is not the case, then the question arises why the Law of Large Numbers is at the core of the economics of risk and insurance.

The reason is that (the underlying premise is that) the expanding pool of risks is owned by a similarly expanding number of insurer shareholders. That

is, the expanding pool of risks gets spread over (or subdivided among) an expanding numbers of owners. It is the average loss of the pool that gets attributed to an individual owner, the variability of which, as we have seen earlier, by virtue of the Law of Large Numbers, is indeed much reduced by risk pooling. The more risks in the pool, the more owners subdividing the aggregate risk, and the more the average risk, borne by a single owner, gets reduced.

An alternative view is that the expanding pool of risks is placed with an expanding number of cooperating insurance companies. The cooperating insurance companies redistribute the aggregate risk. In the case of ‘identical’ insurance companies, upon redistribution, each individual insurer bears the average loss of the pool of risks. This, in fact, follows from the so-called Mutuality Principle of Borch (1962): In a pool of risks it will be Pareto efficient to allocate risks based on the total and systematic risk of the pool. Again, the more risks in the pool, the more cooperating insurance companies redistributing (and subdividing) the aggregate risk, and the more the variability of the average loss, allocated to a single insurer, gets reduced.

In sum, it is ‘the law of a large number of subdivisions’ that is at the core of the economics of risk and insurance! Of course, the Law of Large Numbers is only one possible description of what happens to a pool of risks upon expansion. Other descriptions are provided by the Central Limit Theorem, the Law of the Iterated Logarithm and the Theory of Large Deviations. We then find ourselves in the core of the fascinating field of stochastics, which plays a key role in my research.

4 Risk and Insurance: Stochastics and Economics

So in the utopian situation in which an insurer pools infinitely many independent risks, subdividing losses among an infinite number of shareholders, running the insurance business would, at least from a risk management perspective (and neglecting the investment decision – and neglecting the operational risk that the administration of such infinite records would unavoidably be exposed to), become a triviality. Annually charge EUR 101 per policy and subdivide the average annual loss, which is fully predictable to be EUR 100, among the shareholders: A moneymaker. Unfortunately (or fortunately!), this is not the case.

So far utopia, let’s get real. In reality, the pool inevitably consists of only a finite number of risks, with losses subdivided among only a finite number of owners, and with a systematic component (due to common risk factors such as longevity risk, interest rate risk, equity risk, operational risk, and credit risk)

that cannot be nullified. As a result, the insurer (and policymaker and academic alike) needs to deal with the fundamental and interrelated questions of:

1. How to set capital requirements and how to allocate risk capital (the ‘regulatory and economic capital problem’), dealing with model uncertainty and imperfect financial markets that give rise to sound and important risk management considerations. More generally, we face the question of how to measure risk (the development of a theory of measures of risk).
2. How to determine risk premia (risk loadings) and how to (market-consistently, or not) value insurance liabilities, dealing with the inherent non-hedgeable nature of insurance risks (mortality, catastrophe, inflation). More generally, we face the question of how to conduct asset pricing in incomplete markets.
3. How to deal with the dependences and common components among the risks, in risk management, pricing and portfolio choice?

These questions have been at the core of my research agenda, right from the start, now about 10 years ago.

Proper answers to these questions require the rich interplay between economics and mathematics: From probability, mathematical statistics, financial and insurance mathematics and econometrics to economic theory, and applied financial and insurance economics. ‘Risk and stochastics, idea and language, are inseparable.’³²

5 Challenges at the Core of Risk and Insurance

5.1 Risk Measurement, Model Uncertainty and Implications

In a series of papers, commencing with papers in my PhD thesis (Laeven, 2005, see end notes) with Marc Goovaerts and surmounting in a recently accepted paper with Mitja Stadje (Laeven and Stadje, 2012a), I have studied the question of how to measure risk. More specifically, we have systematically analyzed classes of risk measures that satisfy certain economic (additivity) properties. The fundamental question we have tried to address, a challenge in which we have largely succeeded, is: What is the mathematical shape of risk measures that satisfy the economic properties (or axioms) of, so-called, translation invariance,³³ convexity³⁴ and additivity (independent or comonotonic)?³⁵ For example, the axiom of convexity reflects the effect that the total risk of a portfolio is reduced by diversification.

Such ‘axiomatic characterizations’ translate economic properties of risk measures on the one hand, which can be justified or rejected, into a mathematical representation of risk measures on the other hand. Implications for risk management and capital requirements, pricing in incomplete markets and portfolio choice have also been studied.³⁶

While several important steps have been made in this realm, many challenges remain. For example, I will continue to study, with colleagues, the question of how to measure risk in a setting in which there is uncertainty about the right model to measure risk, the typical situation, I would argue. Implications of this setting for risk management and capital requirements, pricing and portfolio choice will also be studied.

5.2 *Financial Contagion and Systemic Risk*

When the (class of) risk measure(s) has deliberately been selected on the basis of such results, one still needs to estimate and compute it. This may be a true Hercules challenge.³⁷ It is this step at which it becomes critical to account for the phenomenon of (financial) ‘contagion’ – the title of this lecture. Linguistically, ‘contagion’ is synonymous with infection. In a loose and wide sense, one might use the term contagion to refer to such risks as common financial economic shocks, biometric shocks, common damage of reputation (image risk, ‘woekerpolis’) and common operational risk. Such risks constitute the main challenge the field of Risk and Insurance is nowadays facing.

In a less wide but still broad sense, financial contagion refers to the transmission of financial economic shocks.³⁸ Such transmission of shocks takes place in space – across countries or regions of the world – and in time – successive shocks in affected countries –, in good times but, in particular, in bad times. The mathematical modeling of such shocks and the statistical inference on such models, with dependences in both the time series dimension and the cross-sectional dimension, poses important challenges to the econometrician. Such dependent shocks also pose challenges to the Law of Large Numbers, if you will, hence to the fundamentals of risk management.

In Aït-Sahalia, Cacho-Díaz and Laeven (2012) and Aït-Sahalia and Laeven (2012), we design new models and testing and inference procedures for precisely this purpose of capturing dependences among shocks, both in the time series dimension and the cross-sectional dimension. In our setup, the occurrence of shocks in one country or region of the world increases the likelihood of observing new shocks, not only in the affected areas but also in other areas. Shocks in financial markets generated from our model thus self-excite and cross-excite, mimicking the patterns in the data that we want to capture. A

prototypical example in the data is provided by the week following October 3, 2008, in which the major US bailout plans were assessed by the world's financial markets.

An analogy with earthquakes and their aftershocks is quickly drawn: Long and calm periods are startled by a sudden shock, which is then not unlikely to be quickly followed by aftershocks. However, while clusters of earthquakes are typically local phenomena, this is not true for contagious financial economic shocks, which may develop into global phenomena.

Indeed, financial crises don't consist of a single shock in a local area but rather of infections in time and space. Such shocks are the essence of systemic risk: Eventually the entire system gets infected. It is the nature of the challenge the field of Risk and Insurance is facing: '*Non in cauda sed in caudis venenum.*'

As we indicate in Aït-Sahalia and Laeven (2012), adoption of proper models for contagion has important implications for risk management and capital requirements, pricing and portfolio choice. For example, it makes explicit that the mathematical models that are typically used for financial risk management do not properly describe the contagion phenomenon.³⁹ Both the 'time and space' dimensions of contagion are inadequately captured.⁴⁰ Under the commonly adopted mathematical models, financial economic shocks occur rarely, clusters of shocks in time occur more rarely and clusters of shocks in time and space occur even more rarely, or not at all, in spite of reality.

As a result, some financial institutions will not be adequately protected against contagious shocks or generate too much systemic risk. In that case, clients and tax payers partially bear the risks. It may also happen that financial institutions are, in fact, not sufficiently rewarded for an effective risk management strategy and hold too much costly capital.

Furthermore, while traditional financial economic theory stipulates that shocks are rare and independent over time, our analysis rejects this assumption. It means that the benefit of diversifying investments across assets and regions, which traditional financial economic theory acknowledges, is overestimated. This matters because the risk management technique of diversification fails to be rewarding when it is needed most urgently. Upon the first signs of crises, investors should therefore relocate their portfolios. Again, while several important steps have been made, challenges remain. On the economic side, we are developing an economic theory of diversification and an asset pricing theory of contagion. On the mathematical statistical side, we are developing sophisticated goodness-of-fit testing procedures.

6 Future of Risk and Insurance

Insurers face turbulent times. The financial economic downturn we have witnessed has resulted in low consumer confidence in the Dutch economy (and more broadly), reaching a historical low in June 2012.⁴¹ As a result, insurers have seen decreased demand for insurance products; many insurance contracts are usually entered into in tandem when people buy cars or real estate, which is what consumers currently postpone. In addition, damaged reputation and increased competition from the banking sector ('banksparen') make it more difficult for Dutch insurers to sell their products.⁴² Low market interest rates further depresses solvency capital ratios. And upcoming Solvency II may put further pressure on required solvency capital, apart from the major implementation costs that this massive reform generates for the sector.

Fortunately, prosperous opportunities exist too. While in the current economic environment, most insurers will primarily be concerned with keeping up existing business, important opportunities exist in the product development area.

6.1 *Insurers and Pensions*

Insurance companies may face turbulent times, but the Dutch pension funds' business model, while in a better shape than in many other countries, is facing the sword of Damocles. The Dutch pension market will likely see long terms of no indexation and may even see unanticipated cuts of nominal pension benefits. Ageing and increased life expectancy as well as intransparency and inflexibility with respect to the effects of shocks on financial and biometric markets have put high pressure on the system. Furthermore, the current level of pension premiums is high already.

The Labour Foundation (Stichting van de Arbeid, StAr) has recommended in 2010 to focus attention on the so-called real pension ambition and not on nominal pension guarantees. The StAr argues that nominal pension guarantees do not maintain purchasing power and are furthermore costly. The choice of the StAr to focus attention on the real pension ambition ignores the fact that consumers value unconditional promises and guarantees.

Against this backdrop, there are important opportunities for insurers to develop transparent and intelligent pension contracts, with unconditional promises and guarantees. In research conducted in collaboration with Theo Nijman and our PhD student Servaas van Bilsen (van Bilsen, Laeven and Nijman, 2012), we propose such an alternative pension contract. This contract is referred to as 'escalating guarantees'. The contract has on the one hand the am-

bition to maintain purchasing power but offers on the other hand explicit nominal guarantees.

More specifically, the contract with escalating guarantees offers nominal guarantees that are designed such that, in expectation, they grow with the level of (wage or price) inflation. Nominal guarantees are financed by nominal bonds so as to safeguard the unconditional character of these promises. The indexation of the nominal guarantees is financed by risky assets. The level of indexation actually assigned is determined annually on the basis of the investment returns. The nominal guarantees annually grow with the assigned indexation which then becomes unconditional: The nominal guarantees ‘escalate’. In this alternative pension contract, participants never face nominal pension cuts. Once built up means always retained.

In our research, we have compared the contract with escalating guarantees to the pension contract with a return adjustment mechanism (rendementsaanpassingsmechanisme, RAM) as proposed by StAr.⁴³ The results show that (i) the probability of cuts in nominal pension benefits, which is virtually zero by construction in the contract with escalating guarantees, is considerable in the contract proposed by StAr; (ii) the possibility of an extremely low pension benefit is not excluded under the StAr contract, in contrast to under the contract with escalating guarantees; and (iii) only a limited portion of the upward potential needs to be traded off against downside risk reduction.⁴⁴

While the contract with escalating guarantees has been designed with great care, it is ‘merely’ one possible alternative to the StAr contract, and various variations on this contract are conceivable. The primary message is that there are eminent opportunities for insurers to design transparent and intelligent pension contracts, with unconditional promises and guarantees. Under such contracts, the younger generation will commonly take more risk pre-retirement than the older generation. Furthermore, some risk, albeit very limited, will commonly be assumed post-retirement. These features are, for the time being, not in line with current Dutch second pillar pension legislation but do not restrain insurers from offering such products in the third pension pillar. Further research in this direction⁴⁵ is urgently needed, and will be carried out.

6.2 Solvency and Supervision: Discounting and Long Term Liabilities

Another hot topic in the current policy debate is that of discounting insurance liabilities to calculate technical provisions: What discount rate is to be applied for this purpose? In meetings with EIOPA (the European Insurance and Occupational Pensions Authority) and the Dutch Central Bank and in writings with

an international group of scholars from a.o. the University of Amsterdam, the London School of Economics and ETH Zurich, we considered important aspects of this question (see Ayadi, Danielsson, Laeven, Pelsser, Perotti and Wüthrich, 2012, Danielsson, de Jong, Laeven, Laux, Perotti and Wüthrich, 2011, and Laeven, 2011).

The most controversial issue in this respect is, in a nutshell, whether so-called liquidity (or matching) and countercyclical premia should be included on top of the discount rate. Another issue, not discussed here, is the question of how to determine the ultimate forward rate (UFR) to be used for discounting very long term liabilities.

The new European supervisory framework for insurers (Solvency II) proposes to add a matching adjustment to the term structure of interest rates when discounting specific forms of insurance liabilities. This adjustment is suggested by the perceived possibility of replicating insurance liabilities by illiquid assets (illiquid bonds), which earn a liquidity premium when held to maturity. It further proposes to discount all remaining liabilities by a countercyclical top-up in times of distressed market conditions, to be declared by EIOPA.

Without going into too many details (I refer to the writings cited above for further details)⁴⁶, I want to send out the message here that regulators (and the industry and academia alike) face a relevant issue. In times of distressed financial markets, with illiquid assets and excess volatility, hence, unreliable asset values for the purpose of market valuation, there exists a need to counteract effects of ‘exaggeratedly low’ solvency figures for insurers with unredeemable long term matched liabilities.

Of course, solvency supervision should be concerned with countercyclically building up buffers in good times, which may be run down in bad times. The countercyclical aspect is important, not the least because it reduces the need to overreact in bad times, imposing a high burden on the industry which can easily hamper or slow down recovery. In addition, it should also be concerned with macroprudentially monitoring the systemic risk individual financial institutions generate; risk that cannot be nullified in insurance markets as AIG has recently illustrated. To this end, models such as those described in Section 5.2 above (or simplified versions thereof) should be adopted.

Then, to counteract the effects of exaggeratedly low solvency figures in distressed financial markets, regulators may provide leniency in downturns, adjusting the ladder of intervention for insurers with long term matched liabilities. Such insurers have more time to recover, without directly jeopardizing policyholders. Different from the banking industry, insurer policyholders can (and need) typically not run away easily and without costs. The time dimen-

sion should be acknowledged and more explicitly incorporated in solvency supervision. Level playing field considerations, with the banking industry, with the pension funds industry and with the international insurance industry, are also relevant in this respect. Maintaining transparency and market-consistent valuation is essential in view of the key role played by the third pillar of solvency supervision. Further research on the exact implementation of such supervisory measures is urgently needed, and will be conducted.

7 Education in Risk and Insurance

An important change that I envision in the education in Risk and Insurance, and specifically in Actuarial Science and Mathematical Finance, is that integrated approaches to Risk and Insurance, and specifically Integrated Risk Management, will become a(n even more) central part of the education programs. This integrated approach no longer purports a separate treatment of the Life, Pension and Non-Life domains, as was common use traditionally, but goes beyond sub disciplines.

As such, I am happy teaching an introductory course on the economics of risk and insurance in the UvA Executive MSc Insurance Studies, covering applications to life insurance, pensions, non-life insurance, and also financial economics. The course also provides an incentive to write an introductory textbook on Risk and Insurance, which I hope to complete within the next biblical seven years. Furthermore, the MSc course Asset Liability Management 1, which in the beginning of the millennium 2000 provided a strong impulse to financial economic and risk management techniques within the actuarial program and which I taught a few times during my PhD studies about 10 years ago, has now been lifted to an integrated course on Risk Management for Insurers and Pensions. I am happy teaching this course, starting this week, as part of the current MSc program in Actuarial Science and Mathematical Finance at the University of Amsterdam.

I expect that this development towards integrated risk management, in which the actuary plays a crucial role in my view, will persist in the next few years, both in academia (research and teaching) and in practice. Good contacts with the Actuarial Society (AG-AI), to monitor and tune these developments, remain vital to us.

Topics that are key to integrated risk management include: Solvency and economic capital, capital allocation, regulation, aggregation and diversification, model risk, valuation of insurance and pension liabilities, ALM and portfolio choice. Some of these topics will also be touched upon in the PhD course

on Asset Pricing that I will be teaching in the Tinbergen Institute Graduate School this spring.

Acknowledgements / Tot slot

Het succes van een wetenschappelijke loopbaan is co-monotoon met de ondersteuning van nogal wat mensen en instanties. Ik zal, nu we aan het eind van deze lezing zijn gekomen, slechts enkele namen hier in het bijzonder noemen.

Allereerst dank ik het College van Bestuur van de Universiteit van Amsterdam, de Decaan van de Faculteit Economie en Bedrijfskunde, en allen die zich hebben ingespannen voor mijn benoeming tot gewoon hoogleraar op de nieuw gestichte leerstoel *Risk and Insurance*, voor het in mij gestelde vertrouwen. Ik zal mij ten volle inzetten voor een internationale voortrekkersrol van de Universiteit van Amsterdam op het gebied van onderzoek en onderwijs in *Risk and Insurance*. Mocht ik bij het verwezenlijken van die ambitie een lastige, veeleisende hoogleraar zijn, dan hoop ik dat u me dat zult vergeven en me de benodigde ondersteuning zult blijven bieden.

De Amsterdam School of Economics onder leiding van Roel Beetsma en Peter Boswijk en de (ex)-Afdeling Kwantitatieve Economie onder leiding van Cars Hommes dank ik voor het bieden van alle mogelijkheden voor een uiterst vruchtbare broedplaats. Ik dank Rob Kaas en Michel Vellekoop en de sectie Actuariele Wetenschappen en Financiële Wiskunde voor een ongelooflijk warm welkom eerder dit jaar. Ik zie uit naar een voortzetting van de aangename samenwerking.

De Nederlandse Organisatie voor Wetenschappelijk Onderzoek financiert via Veni- en Vidi-subsidies het wetenschappelijk onderzoek van mij en mijn groep, waarvoor ik zeer erkentelijk ben. Verder ben ik dank verschuldigd aan de Koninklijke Nederlandse Academie voor Wetenschappen en de Stichting Christiaan Huygens Wetenschapsprijs voor het waarderen en stimuleren van mijn wetenschappelijk onderzoek.

Veel dank is ook verschuldigd aan het Verbond van Verzekeraars dat mijn huidige leerstoel *Risk and Insurance* sponsort. Ik zie uit naar een vruchtbare samenwerking met u. Ik hoop dat mijn lezing u ervan overtuigd heeft dat er gezamenlijke kansen en uitdagingen te over zijn.

Ook dank ik het Amsterdam Center for Insurance Studies (ACIS), niet in de laatste plaats voor het mogelijk maken van het mini symposium dat eerder vandaag heeft plaats gevonden bij de Koninklijke Nederlandse Academie voor

Wetenschappen; en ik dank de vooraanstaande sprekers op het symposium van vanmiddag voor hun bijdrage.

De titel van deze lezing luidt in goed Nederlands ‘Besmettingsgevaar’. Dat verwijst niet alleen naar een economisch, overigens ook medisch, verschijnsel, maar ook naar de aanstekelijkheid van wetenschappelijk onderzoek. Wat betreft het voeden van die aanstekelijkheid, wil ik twee buitengewoon inspirerende mentoren hier noemen. Allereerst dank ik mijn promotor Marc Goovaerts, thans emeritus hoogleraar (KU Leuven en UvA), vermoedelijk de meest dynamische promotor denkbaar. Sinds het begin van mijn academische loopbaan, zelfs nog na uw emeritaat, bent u een onuitputtelijke bron van inspiratie. Daarnaast dank ik Yacine Aït-Sahalia van Princeton University die mij na mijn promotie verder heeft bekwaamd in het ontwikkelen van geavanceerde wiskundige technieken voor enkele van de meest relevante economische problemen van deze tijd.⁴⁷

Mijn internationale onderzoekssamenwerking met o.a. Leuven, Londen, Princeton en Zürich hoop ik nog lang te mogen onderhouden. In Tilburg heb ik met veel plezier samen mogen werken met enkele vooraanstaande specialisten op hun vakgebied. Ook van die samenwerking hoop ik nog lang de vruchten te mogen plukken. In Amsterdam, beyond the radius of the Department of Quantitative Economics, I am looking forward to maintaining close contacts with the Finance Group, with the Macro Group, and with the Stochastics Group of the Korteweg-de Vries Institute for Mathematics.

I would also like to mention my rapidly growing group of postdocs and PhD students Mitja Stadje, Andrea Krajina, Umut Can, Servaas van Bilsen, Zhenzhen Fan, Gijs Kloek, Frans de Weert and Xiye Yang. One of the privileges of being a professor is that it allows me to guide you, and many undergraduate students alike, on your academic explorative expedition.

Daarnaast ben ik veel dank verschuldigd aan Eurandom, het European Institute for Statistics, Probability, Stochastic Operations Research and Its Applications, waarvan ik sinds 2008 één van de onderzoeksgroepen heb mogen leiden. Dit heeft geleid tot uiterst vruchtbare samenwerking en voor mij belangrijke disseminatie van mijn onderzoeksresultaten.

Tot slot wil ik een woord van dank richten tot een paar bijzondere mensen. Allereerst dank ik mijn geliefde ouders, Hub. en Lucy. Het moet voor jullie een hele geruststelling zijn dat met mijn benoeming ook jullie jongste zoon, ‘der Witte’, van de straat lijkt te zijn. Dan kunnen jullie nu onbezorgd van jullie pensioen gaan genieten, afgezien dan van het uitblijven van indexatie van dat pensioen en van de mogelijke nominale korting. Au sérieux hoop ik dat mijn kinderen net zo’n thuishaven mogen ervaren als ik: dan heb ik het goed gedaan. Daarnaast dank ik mijn geliefde Guus en Maria: ik kan me geen dier-

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Mijn collega Robbert Dijkgraaf dacht, met zijn benoeming tot hoogleraar aan de Universiteit van Amsterdam op dezelfde 32-jarige leeftijd als ik, ‘het hoogst haalbare in het vakgebied bereikt te hebben’,⁴⁹ maar realiseerde zich later dat het met die benoeming eigenlijk pas echt was begonnen. Laat me daarom nu, nou vooruit na de borrel dan, aan de slag gaan.

Uit duurzaamheidoverwegingen is de oratiegalerij van de Universiteit van Amsterdam tegenwoordig digitaal. De volledige tekst van deze lezing is te vinden via mijn webpagina www.rogerlaeven.com. Rest mij om u te suggereren de rij bij de receptie niet langer dan 10 personen te laten worden.

Dixi (Ik heb gezegd).

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Nederlandse Samenvatting

Besmettingsgevaar

Financiële markten staan bloot aan besmettelijke schokken. De wiskundige modellen die doorgaans worden gebruikt voor financieel risicomanagement geven geen adequate beschrijving van dit besmettingsgevaar. Ook in de traditionele economische theorie speelt dit besmettingsgevaar een te bescheiden rol. De ontwikkeling van wiskundige modellen en economische theorie voor het meten van financiële risico's *inclusief besmettingsgevaar* staat centraal in deze oratie. Actuele beleidkwesties zoals het prudentieel toezicht op schokken in verzekeringsverplichtingen en een ontwerp van een moderne pensioenverzekerings die polishouders beschermt tegen financiële schokken – een contract dat uitgelezen kansen biedt voor verzekeraars – komen ook aan de orde.

Prof. dr. Roger J. A. Laeven (22 januari 1979) is Gewoon Hoogleraar (Leerstoel *Risk and Insurance*) aan de Universiteit van Amsterdam. In 2004 was hij gastonderzoeker aan de London School of Economics en van 2007-2012 is hij gastonderzoeker aan Princeton University. Van 2001-2005 werkte hij parttime bij Mercer Oliver Wyman en van 2007-2011 was hij Universitair Hoofddocent aan de Universiteit van Tilburg. Roger studeerde Actuariële Wetenschappen en Econometrie aan de Universiteit van Amsterdam (*Cum Laude*) en promoveerde in 2005 aan dezelfde universiteit (*Cum Laude*). Zijn proefschrift ontving de Christiaan Huygens Wetenschapsprijs 2007 van de Koninklijke Nederlandse Academie van Wetenschappen. Roger ontving een Veni-subsidie (2006) en een Vidi-subsidie (2009) van de Nederlandse Organisatie voor Wetenschappelijk Onderzoek. De leerstoel *Risk and Insurance* die Roger bekleedt, wordt gesponsord door het Verbond van Verzekeraars.

Quote

Besmettingsgevaar in financiële markten: cruciale uitdaging en aanstekelijke problematiek.

(Financial contagion: crucial challenge and exciting research.)

Notes

1. Wednesday, September 21, 2005.
2. Laeven, Roger J.A. (2005). Essays on Risk Measures and Stochastic Dependence, with Applications to Insurance and Finance, Tinbergen Institute Research Series 360. [Awarded the Christiaan Huygens Prize 2007 by the Royal Netherlands Academy of Arts and Sciences.]
3. In fact, celebrating the birthday (September 21, 1922) of my grandfather.
4. Born: 27 December 1654, Basel, Switzerland; died: 16 August 1705, Basel, Switzerland.
5. Born: 6 August 1667, Basel, Switzerland; died: 1 January 1748, Basel, Switzerland.
6. In more general versions.
7. Born: 8 February 1700, Groningen, The Netherlands; died: 17 March 1782, Basel, Switzerland.
8. Sources: Bernstein (1996), Feller (1971a,b), <http://www.cs.xu.edu/math/Sources/> and the references therein.
9. In a special version.
10. Laeven, A. Hubertus (1990). The ‘Acta Eruditorum’ under the Editorship of Otto Mencke (1644-1707): The History of an International Learned Journal Between 1682 and 1707, trans. L. Richards, Amsterdam, APA-Holland University Press.
11. Source: A.H. Laeven, personal communication.
12. The reason of success of the other two mentioned periodicals, The Philosophical Transactions and the Journal des Scavans, was comparable. The first one was the official journal of the Philosophical Society. The other one, though not the journal of the Académie Française, operated very closely to this circle.
13. See: <http://www.cs.xu.edu/math/Sources/JakobBernoulli/JakobB.html>.
14. The corresponding paper was published in 1738 in the Papers of the Imperial Academy of Sciences in St. Petersburg. An English translation is available in Bernoulli (1954).
15. When probabilities are given, at least.
16. Sources: Bernoulli (1954), Bernstein (1996), <http://www.cs.xu.edu/math/Sources/> and the references therein.
17. This history is clearly largely incomplete and by no means an attempt to mention, let alone to discuss, all historical key steps in the development of the basic principles of Risk and Insurance. The interested reader is referred to Bernstein (1996) and the references therein for a much more complete overview.
18. This section is largely based on my book in preparation, with working title Risk and Insurance: An Introduction.
19. See also Arrow and Lind (1970) in the context of public investment decisions.
20. One may consult Seog (2010) or the less specialized Eeckhoudt, Gollier and Schlesinger (2005) and Gollier (2001) for further details (though not covering all aspects mentioned here).

21. Such as Khintchine's; see Feller (1971b), p. 235; or better, a version of the Strong Law of Large Numbers, which is of more practical relevance; see Feller (1971b), Section VII.8.
22. See, in particular, Samuelson (1963), elaborating on the first 'fallacy' mentioned below.
23. Assuming a binomial distribution, resulting from summing a sequence of independent Bernoulli risks.
24. For simplicity, we ignore in this example the possibility of more than a single total loss car accident in one year, the probability of which is negligible.
25. The gross modal income in the Netherlands is EUR 33,000 (2012). Source: Economic Policy Analysis (CPB). A loss of EUR 10,000 therefore amounts to a loss of almost half of a standard net income.
26. Formally, let X_i be independent risks with a common distribution and finite expectation μ (mu). Then, for each fixed positive number ε (epsilon), the probability $\Pr[|(X_1 + \dots + X_n) / n - \mu| > \varepsilon]$ goes to zero as n tends to infinity.
27. See also the related Samuelson (1963).
28. Connected to the notion of comonotonicity.
29. This is a crucial fallacy in what follows.
30. Versions of the Law of Large Numbers for (incompletely) dependent variables exist.
31. Assuming, again, a binomial distribution, resulting from summing a sequence of independent Bernoulli risks. For simplicity, we (again) ignore in this example the possibility of more than a single loss event (in this case, volcanic eruption) in one year.
32. Source: Ragnar Norberg.
33. Laeven and Stadje (2012a).
34. Laeven and Stadje (2012a).
35. Goovaerts, Kaas, Laeven and Tang (2004), Goovaerts and Laeven (2008) and Goovaerts, Kaas and Laeven (2010a).
36. Goovaerts, Kaas and Laeven (2010b) and the references therein, Kaluszka, Laeven and Okolewski (2012) and Laeven and Stadje (2012b).
37. See Genest, Gerber, Laeven and Goovaerts (2009) for some of the issues involved.
38. There is a vivid discussion on the meaning of the term 'financial contagion'. See e.g., Ait-Sahalia and Laeven (2012) for details.
39. Examples are abundant, not only in finance but also in insurance, e.g., the standard model of Solvency II (the future European regulatory framework for insurers) which is inherently based on Gaussianity ('bell curve') and correlation.
40. See also Kaas, Laeven and Nelsen (2009) and Laeven (2009) for further details on the specific problem of risk aggregation for solvency purposes under the Value-at-Risk and Tail-Value-at-Risk measures of risk.
41. Source: Statistics Netherlands (CBS).
42. Consumer confidence in Dutch insurers recently increased slightly. (In the second quarter of 2012 consumers assigned a grade of 6.1. In the third quarter of 2012 consumers assigned a grade of 6.3.) Source: Dutch Association of Insurers, August 3, 2012.
43. And also to a canonical ('traditional') pension contract with return guarantees.

44. We admit that the contract with escalating guarantees does not spread the risk of inflation intergenerationally and does not automatically focus the asset allocation choice on inflation risk hedging. Furthermore, individual pension products carry, as yet, typically more expenses.
45. And in particular on its connection to descriptive theories of choice such as Kahneman and Tversky's (1979) prospect theory, and on the exact treatment of un-hedgeable risks.
46. In a nutshell, we endorse the relevance of the issues meant to address, but we (and all scholars we know of support this view) do not advocate the specific approach to resolve the issues through adjustments to the term structure of interest rates. Fiddling around with the discount rate is theoretically unsound, can as yet not be based on a proper operational procedure and does not improve the much needed transparency in insurance markets. Such transparency is key to the success of 'third pillar solvency supervision', which is critically important (arguably paramount).
47. Ik wil op deze plaats ook Hans Gerber (HEC Lausanne), Ragnar Norberg (LSE) en Peter Wakker (UvA, nu EUR) vermelden die in een eerder stadium hebben bijgedragen aan mijn wetenschappelijke ontwikkeling.
48. Zoals mijn grootvader placht te zeggen.
49. Source: Intermediair, 30 januari 2008.